AP Calculus AB  Linearization/ Local Linearity  Section 5.5

AP Questions:

1998 AP Calculus AB Scoring Guidelines

4. Let \( f \) be a function with \( f(1) = 4 \) such that for all points \((x, y)\) on the graph of \( f \) the slope is given by \( \frac{3x^2 + 1}{2y} \).

(a) Find the slope of the graph of \( f \) at the point where \( x = 1 \).

(b) Write an equation for the line tangent to the graph of \( f \) at \( x = 1 \) and use it to approximate \( f(1.2) \).

1995 AB3

Consider the curve defined by \(-8x^2 + 5xy + y^3 = -149\).

(a) Find \( \frac{dy}{dx} \).

(b) Write an equation for the line tangent to the curve at the point \((4, -1)\).

(c) There is a number \( k \) so that the point \((4.2, k)\) is on the curve. Using the tangent line found in part (b), approximate the value of \( k \).

(d) Write an equation that can be solved to find the actual value of \( k \) so that the point \((4.2, k)\) is on the curve.

(e) Solve the equation found in part (d) for the value of \( k \).

AB?

Let \( f \) be the function defined by \( f(x) = (1 + \tan x)^\frac{3}{2} \) for \(-\frac{\pi}{4} < x < \frac{\pi}{2}\).

(a) Write an equation for the line tangent to the graph of \( f \) at the point where \( x = 0 \).

(b) Using the equation found in part (a), approximate \( f(0.02) \).
Let \( f \) be a function that is differentiable for all real numbers. The table above gives the values of \( f \) and its derivative \( f' \) for selected points \( x \) in the closed interval \(-1.5 \leq x \leq 1.5\). The second derivative of \( f \) has the property that \( f''(x) > 0 \) for \(-1.5 \leq x \leq 1.5\).

Write an equation of the line tangent to the graph of \( f \) at the point where \( x = 1 \). Use this line to approximate the value of \( f(1.2) \). Is this approximation greater than or less than the actual value of \( f(1.2) \)? Give a reason for your answer.

**Multiple Choice:**

The approximate value of \( y = \sqrt{4 + \sin x} \) at \( x = 0.12 \), obtained from the tangent to the graph at \( x = 0 \), is

(A) 2.00    (B) 2.03    (C) 2.06    (D) 2.12    (E) 2.24
AP Calculus AB  

Linearization/Local Linearity  Section 5.5

AP Questions:

1998 AP Calculus AB Scoring Guidelines

4. Let \( f \) be a function with \( f(1) = 4 \) such that for all points \((x, y)\) on the graph of \( f \) the slope is given by \( \frac{3x^2 + 1}{2y} \).

(a) Find the slope of the graph of \( f \) at the point where \( x = 1 \).
(b) Write an equation for the line tangent to the graph of \( f \) at \( x = 1 \) and use it to approximate \( f(1.2) \).

\[
\begin{align*}
3(1) + 1 &= \frac{4}{8} = \frac{1}{2} \quad &b) & y - 4 = \frac{1}{2}(x - 1) \\
L(x) &= \frac{1}{2}x + 3.5 \\
L(1.2) &= 4.1
\end{align*}
\]

1995 AB3

Consider the curve defined by \(-8x^3 + 5xy + y^3 = -149\). \[-16x + 5y + 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0\]

(a) Find \( \frac{dy}{dx} = \frac{16x - 5y}{5x + 3y^2} \).

(b) Write an equation for the line tangent to the curve at the point \((4, -1)\).

(c) There is a number \( k \) so that the point \((4.2, k)\) is on the curve. Using the tangent line found in part (b), approximate the value of \( k \). \( k = 3(4.2) - 13 = -2.5 \approx -4 \)

(d) Write an equation that can be solved to find the actual value of \( k \) so that the point \((4.2, k)\) is on the curve.

\[-8(4.2)^2 + 5(4.2) y + y^3 = -149\]

(e) Solve the equation found in part (d) for the value of \( k \).

Let \( f \) be the function defined by \( f(x) = (1 + \tan(x))^\frac{3}{2} \) for \(-\frac{\pi}{4} < x < \frac{\pi}{2}\).

(a) Write an equation for the line tangent to the graph of \( f \) at the point where \( x = 0 \).

(b) Using the equation found in part (a), approximate \( f(0.02) \).

\[
\begin{align*}
a) & f'(x) = \frac{3}{2} (1 + \tan(x))^\frac{1}{2} (\sec^2(x)) \quad y - 1 = \frac{3}{2}(x - 0) \\
& f'(0) = \frac{3}{2} (1 + 0)^\frac{1}{2} (\sec^2(0))^2 \\
& = \frac{3}{2} = 1.5 \\
b) & L(x) = \frac{3}{2}x + 1 \\
& L(0.02) = \frac{3}{2}(0.02) + 1 \\
& = 1.03 \\
f(0.02) & \approx 1.03
\end{align*}
\]
Let \( f \) be a function that is differentiable for all real numbers. The table above gives the values of \( f \) and its derivative \( f' \) for selected points \( x \) in the closed interval \(-1.5 \leq x \leq 1.5\). The second derivative of \( f \) has the property that \( f''(x) > 0 \) for \(-1.5 \leq x \leq 1.5\).

Write an equation of the line tangent to the graph of \( f \) at the point where \( x = 1 \). Use this line to approximate the value of \( f(1.2) \). Is this approximation greater than or less than the actual value of \( f(1.2) \)?

\[
(1, -4) \quad f'(1) = 5
\]
\[
y + 4 = 5(x - 1)
\]
\[
\underline{y = 5x - 9}
\]
\[
f(1.2) \approx L(1.2) = -3
\]

* The graph is concave up since \( f'' > 0 \) at \( x = 1.2 \) so the tangent line is below the graph making the approximation less than the actual value.

**Multiple Choice:**

The approximate value of \( y = \sqrt{4 + \sin x} \) at \( x = 0.12 \), obtained from the tangent to the graph at \( x = 0 \), is

(A) 2.00  (B) 2.03  (C) 2.06  (D) 2.12  (E) 2.24

\[
y' = \frac{1}{2} (4 + \sin x)^{-1/2} \cos x
\]
\[
y'(0) = \frac{1}{2} (4)^{-1/2} = 1
\]
\[
y - 2 = \frac{1}{4} (x - 0)
\]
\[
L(x) = \frac{1}{4} x + 2
\]
\[
L(-1.2) = \frac{1}{4}(-1.2) + 2 = \boxed{2.03}
\]