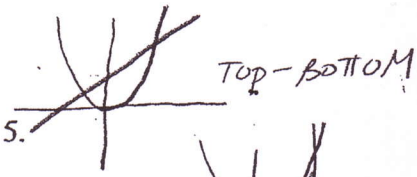
Solutions

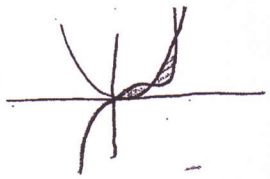
AP CALCULUS AB

Practice Problems – Areas and Volumes



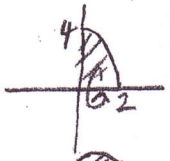
1. Find the area between the curves $f(x) = x^2 + 2x + 1$ and $g(x) = 2x + 5$.

2. Find the area between the curves $f(x) = x(x^2 - 3x + 3)$ and $g(x) = x^2$.



3. Find the area of the region bounded by $y = x^4 - 2x^2$; $y = 2x^2$; $x = 1$ and $x = 2$.

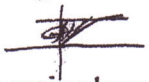
4. Find the area between $f(x) = 2 \sin x$ and $g(x) = \tan x$ for $-\pi/3 < x < \pi/3$.



5. Find the volume of the solid created by revolving the region in the first quadrant bounded by $y = 4 - x^2$; $x = 0$ and $y = 0$ about the x-axis.

6. Find the volume of the solid created by revolving the region bounded by $y = 4 - x^2/4$ and $y = 2$ (in the first and second quadrants) about the x-axis. $4 - \frac{x^2}{4} = 2$ $x^2 = 8$ $x = \pm\sqrt{8}$

7. Find the volume of the solid created by revolving the region in the first quadrant bounded by $y = x^{2/3}$; $y = 1$ and $x = 0$ about the y-axis. $x = y^{3/2}$



$\pi \int_0^1 (y^{3/2})^2 dy = \pi/4$

8. Find the volume of the solid created by revolving the region bounded by $y = 6 - 2x - x^2$ and $y = x + 6$ about the line $y = 3$. $6 - 2x - x^2 = x + 6$ $x = -3, 0$

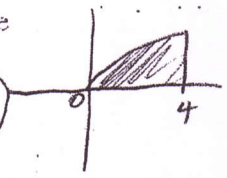
8) $\pi \int_{-3}^{0} (6 - 2x - x^2 - 3)^2 - (x + 6 - 3)^2 dy$

9. The region R is bounded by $y = \frac{1}{2}x^2$; $y = 0$ and $x = 6$. R is the base of a solid whose cross sections are perpendicular to the x-axis. Find the volume of this solid if its cross sections are (a) squares and (b) semi-circles.

$\frac{108\pi}{5} \approx 67.85$

a)

10. The region R is bounded by $y = \sqrt{x}$; $y = 0$ and $x = 4$. R is the base of a solid whose cross sections are perpendicular to the x-axis. Find the volume of this solid if its cross sections are (a) rectangles with a height of 10 and (b) equilateral triangles. $A = b \cdot h = 10b$ ↳ $A = \frac{s^2\sqrt{3}}{4}$ $s = \sqrt{x}$



1. $\int_{-2}^{2} (2x+5) - (x^2+2x+1)\, dx = 32/3$

2. $\int_0^1 (x^3 - 3x^2 + 3x - x^2)\, dx + \int_1^3 [x^2 - (x^3 - 3x^2 + 3x)]\, dx = 37/12$

3. $\int_1^2 [2x^2 - (x^4 - 2x^2)]\, dx = 47/15$

4. $\int_{-\pi/3}^{0} (\tan x - 2\sin x)\, dx + \int_0^{\pi/3} (2\sin x - \tan x)\, dx = .614$

5. $\pi \int_0^2 (4 - x^2)^2 dx = \frac{256\pi}{15}$

6. $\pi \int_{-\sqrt{8}}^{\sqrt{8}} \left[\left(4 - \frac{x^2}{4}\right)^2 - (2)^2\right] dx = 132.694$

⑦ $\pi \int_0^1 y^3 dy = \frac{\pi}{4}$

⑧ Above

⑨ a) $\int_0^6 \left(\frac{1}{2}x^2\right)^2 dx = 38808$

b) $\int_0^6 \frac{1}{2}\pi \left(\frac{1}{4}x^2\right)^2 dx = 152.681$

⑩ a) $\int_0^4 (\sqrt{x}) \cdot 10\, dx = 53.333$

b) $\int_0^4 \frac{(\sqrt{x})^2}{4}\sqrt{3}\, dx = 2\sqrt{3}$