Reverse classroom – known cross sections;

http://www.chaoticgolf.com/vodcasts/calc/lesson7_3_part1/lesson7_3_part1.html

7.3 VOLUMES H W - p 430 59, 60, 61

Just like in the last section where we found the area of one arbitrary rectangular strip and used an integral to add up the areas of an infinite number of infinitely thin rectangles, we are going to apply the same concept to finding volume. The key ... Find the volume of ONE arbitrary "slice", and use an integral to add up the volumes of an infinite number of infinitely thing "slices".

We will first apply this concept to the volume of a solid with a known cross section, then we will find the volumes of solids formed by revolving a region about a horizontal or vertical line. We will discuss three different methods of finding volumes of solids of revolution, but first ...

Day 1: *Volumes of Solids with Known Cross Sections*

First Question ... What is a cross section? Imagine a loaf of bread. Now imagine the shape of a slice through the loaf of bread. This shape would be a cross section. Technically a *cross section* of a three dimensional figure is the intersection of a plane and that figure. It would be like cutting an object and then looking at the face of where you just cut.

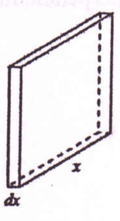
The cross sections we will be dealing are almost entirely perpendicular to the x – axis.

Here's the basic idea ... You will be given a region defined by a number of functions. We will graph that region on an x and y axis. Then we will lay they region flat and build upon that region a solid which has the same cross section no matter where you slice it.

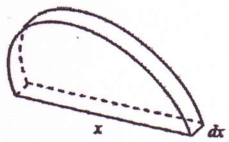
To see some animated views of this go to http://mathdemos.gcsu.edu/mathdemos/sectionmethod/sectiongallery.html

Second question ... How do we find the volume of this solid that has been created to have a similarly shaped cross section, even though each cross section may have a different size? We get to use calculus, of course! But first, we need to know how to find the Volume of a prism. Even though every shape may be different, we can find the volume of a prism by finding the area of the base times the "height". The "height" of our prisms will be the thickness of the slices. Once you know the volume of one slice, you just use an integral to add the volumes of all the slices to get the volume of the solid.

Example 1: Find the volume of the following square "slice". Since most of the "slices" we will be dealing with will have a thickness of dx, we will use that same thickness here.



Example 2: Find the volume of the following semicircular "slice".



You will also need to be able to find the volume of equilateral triangle cross sections, isosceles right triangle cross sections, and others. Remember, what you really need is a formula for the area of the base, which is just the cross sectional shape.

Example 3: The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x – axis. Each plane section of the solid perpendicular to the x – axis is a square. The volume of the solid is

A) $\frac{4\pi}{3}$

B) $\frac{16\pi}{5}$

C) $\frac{4}{3}$

D) $\frac{16}{5}$

E) $\frac{64}{5}$

Example 4: The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$. If the cross sections of the solid perpendicular to the x – axis are semicircles, what is the volume of the solid?

A) 12.566

B) 14.661

C) 16.755

D) 67.021

E) 134.041

Example 5: The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

A) $\pi\int_0^2 (2-y)^2\,dy$

B) $\int_0^2 (2-y)\,dy$

C) $\pi\int_0^{\sqrt{2}} (2-x^2)^2\,dx$

D) $\int_0^{\sqrt{2}} (2-x^2)^2\,dx$

E) $\int_0^{\sqrt{2}} (2-x^2)\,dx$