Given that a particle moves so that its velocity is $v = t^2 - 2t$ meters/sec.

(a) Determine the displacement during the interval $0 \le t \le 3$ seconds.

(b) Determine the total distance traveled during the first three seconds.

Answer:

(a) To determine the displacement, you need only find the integral over the interval.

$$\text{Displacement } = \int_0^3 v(t)\,dt = \int_0^3 \left(t^2 - 2t\right)dt = \left(\frac{t^3}{3} - t^2\right)\Bigg|_0^3 = 0$$

The particle winds up at the same position from which it started!

(b) To determine the total distance, we must first find where the particle turned around. These points would occur when the velocity is zero. So, set the velocity equal to zero and solve for t.

$t^2 - 2t = 0$

$t(t-2) = 0$

$t = 0$ and $t = 2$.

Since it turns around at these two points, we must find the distance from $t = 0$ to $t = 2$ and then the distance from $t = 2$ to $t = 3$.

$$\text{Total Distance} = \left|\int_0^2 \left(t^2 - 2t\right)dt\right| + \left|\int_2^3 \left(t^2 - 2t\right)dt\right|$$

$$= \left|\left(\frac{t^3}{3} - t^2\right)\Bigg|_0^2\right| + \left|\left(\frac{t^3}{3} - t^2\right)\Bigg|_2^3\right|$$

$$= \left|\left(\frac{2^3}{3} - 2^2\right) - \left(\frac{0^3}{3} - 0^2\right)\right| + \left|\left(\frac{3^3}{3} - 3^2\right) - \left(\frac{2^3}{3} - 2^2\right)\right|$$

$$= \left|\frac{8}{3} - 4\right| + \left|\left(\frac{27}{3} - 9\right) - \left(\frac{8}{3} - 4\right)\right|$$

$$= \left|-\frac{4}{3}\right| + \left|0 - \left(-\frac{4}{3}\right)\right|$$

$$= \frac{4}{3} + \frac{4}{3} = \frac{8}{3} \text{ meters.}$$

5. Example