Chain Rule Used for finding the derivative of a composite function

If y = f(u) is a differentiable function of u AND u = g(x) is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and its derivative, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Eww! That is not real helpful!

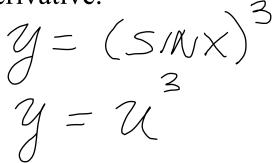
Maybe this is a little bit more helpful:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \bullet g'(x)$$
This can also be thought of as: Let $g(x) = U$
Then $\frac{d}{dx} f(u) = f'(u) \frac{du}{dx}$

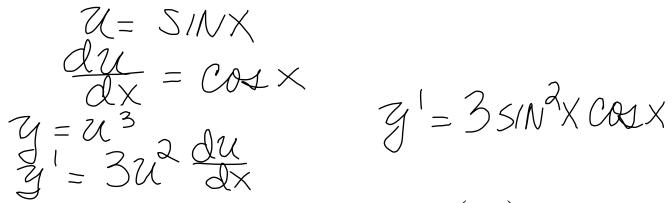
Many people think of this as the derivative of the "outside" times the derivative of the "inside"

Let's see it at work!

Let $y = \sin^3 x$. Let us rewrite this first and then find the derivative.



Now define what "u" should be.



K=11×

Now let us find the derivative of $y = \sin(11x)$.

Give me a "u"!

$$\begin{aligned} y' &= SINU & \frac{du}{dx} = l \\ y' &= l & Cosu \frac{du}{dx} \\ y' &= l & Cosu(llx) \end{aligned}$$

Our goal most of the time will be to rewrite the function in order to see the "inside" or the g(x)- part of f(g(x)). It is useful to get rid of some grouping symbols such as the division bar or the $\sqrt{}$.

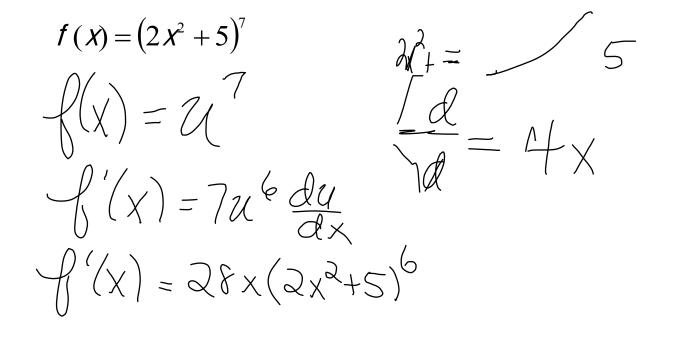
Let
$$y = \sqrt{3x^{2} + 17x}$$

Rewrite as $y = (3x^{2} + 17x)^{\frac{1}{2}}$
 $y = \pi^{-\frac{1}{2}} \qquad \pi = 3x^{2} + 17x$
 $y' = \sqrt{2}\pi^{-\frac{1}{2}} \qquad \frac{4\pi}{2} \qquad \frac{4\pi}{2} = 6x + 17$
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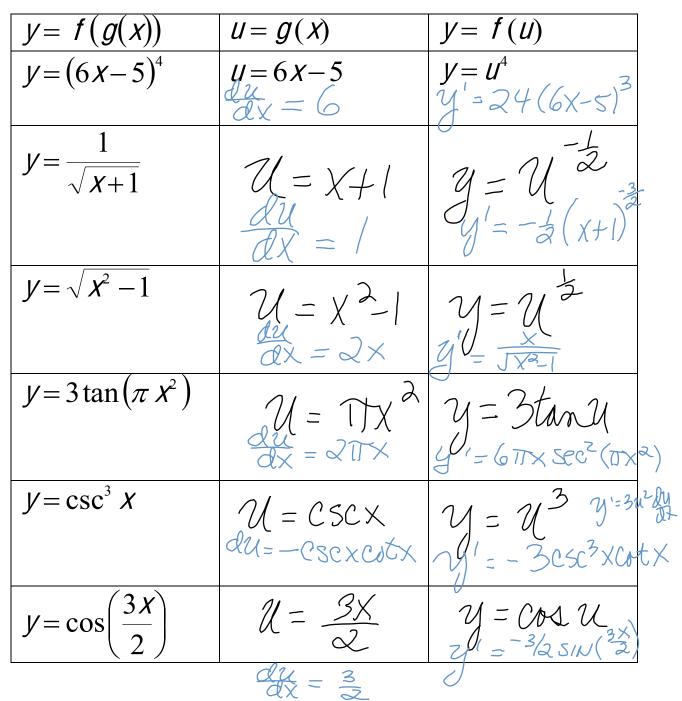
Let $h(x) = \frac{1}{x+13}$ $h(x) = (x+13)^{-1}$ $h(x) = u^{-1}$ $h'(x) = -lu^{-2} du$ h'(x) = -(x+13)²

U=X+(3 $\frac{du}{dv} = 1$

Try:



See page 137 #1-6



Now find the derivatives of each of these

More of the Chain Rule Give me a "u"! Let u be a differentiable function of x

 $\frac{d}{dx}f(u) = f'(u)\frac{du}{dx}$

Looking for patterns:

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$f(\mathbf{X})$	f'(X)
sin X	COLX
$ \sin(2x) \qquad \begin{array}{c} \mathcal{U} = \mathcal{Z} \times \\ \mathcal{U} \\ \mathcal{A} \times = \mathcal{Z} \end{array} $	2 Cos (2x)
$\sin(3x)$	3 cos (3x)
$\sin(4x)$	4 cos (4x)
$\sin(nx), n \in \text{Reals}, n \neq 0$	n cos(nx)
sin U	A XXXX
$\frac{\partial}{\partial x} \cos(nx) = -n \sin(nx)$	

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У	У'
VX OLX 1/2	2d×
$ \sqrt{3x} ol (3x)^{\frac{1}{2}} \\ \qquad \qquad$	3 2J3X
$ \sqrt{7x} o (7x)^{\frac{1}{2}} \\ $	 7J7X
$\sqrt{11x}$ or $(11x)^{2}$	VIIX 11
$\sqrt{g(x)} on \left(g(x)\right)^{\frac{1}{2}}$ $\mathcal{U} = g(x) \frac{du}{dx} g'(x)$	$\frac{1}{2} \mathcal{U}^{-\frac{1}{2}} \frac{d\mathcal{U}}{d\mathcal{X}}$ $= \frac{g'(\mathcal{X})}{2}$ $= \sqrt{2} \sqrt{g(\mathcal{X})}$
$ \begin{array}{l} \sqrt{g(x) + h(x)} \\ \mathcal{U} = g(x) + h(x) \\ \frac{Qu}{dx} = g'(x) + h'(x) \end{array} $	g'(x) + h'(x) $Q \int g(x) + h(x)$
$y = \sqrt{u}$ where <i>u</i> is a differentiable function of <i>x</i>	2521 pt 1848

$g(\mathbf{X})$	g'(x)
$\left \frac{1}{x}\right \mathcal{O} \mathcal{L} \times^{-1}$	$-1 \times -2 $ -1 $\times -1$ $\times 2$
$\frac{1}{2x} \qquad \begin{array}{c} (2\chi)^{-1} \\ $	$-1 \mathcal{U}^{-2} \frac{\partial \mathcal{U}}{\partial x} = -\frac{1}{\partial x^2}$
$\frac{1}{3x} \begin{array}{c} (3x)^{-1} \\ \overline{3x} \\ \mathcal{U} = 3x \\ \overline{3x} \\ \end{array} = 3$	- <u>(</u> 3X ²
$\frac{1}{f(x)} \left(\begin{array}{c} f(x) \\ \mathcal{U} = f(x) \end{array} \right)^{-1} \\ \mathcal{U} = f(x) \begin{array}{c} \frac{du}{dx} = f(x) \\ \frac{du}{dx} = f(x) \end{array}$	$ \begin{array}{r} -1 & (f(x))^{-2} & f'(x) \\) = \frac{-f'(x)}{f^{2}(x)} \end{array} $
$\frac{1}{f(x) + h(x)}$	$-\left(f'(x)+h'(x)\right)$
$\mathcal{U} = -f_1(x_1) + h(x_1)$	$(f(x) + h(x))^{2}$
$\frac{1}{u}$ where <i>u</i> is a	$-\frac{1}{u^2}\frac{dy}{dx}$ $-\frac{dy}{dx}$
differentiable function of <i>x</i>	AAA qu

Sometimes you have to use the Chain Rule more than once. For instance, let $y = \sin^2 (3x)$ We could think of this as: $y = (\sin(3x))^2$ If we did, then $y = u^2$ where $u = \sin(3x)$ If $u = \sin(3x)$, then $\frac{du}{dx}$ would need to be found using the Chain Rule. Good thing we already know this. If $u = \sin(3x)$, then $\frac{du}{dx} = 3\cos(3x)$ Now we can find y' [We can do this!!!] $y = (\sin(3x))^2$ $y = u^2$ $y' = 2u \cdot \frac{du}{dx}$ $\frac{du}{dx} = 3CMS(3x)$

$$y' = 2 \underbrace{\sin(3x)}_{\mathcal{U}} \cdot \underbrace{3 \cos(3x)}_{\mathcal{U}_{X}}$$
$$\underbrace{\frac{d\mathcal{U}}{d\mathcal{U}_{X}}}_{\mathcal{U}_{X}}$$
$$y' = 6 \sin(3x) \cos(3x)$$

Now let us consider $y = \sqrt{\tan(4x)}$ Stupid $\sqrt{}$ Let us rewrite

 $\begin{array}{l} \mathcal{Y} = \left(\tan\left(4x\right)\right)^{\frac{1}{2}} \\ \mathcal{Y} = \mathcal{U}^{\frac{1}{2}} \end{array}$ dù dx 25U y' = $Z' = \frac{25ec^{2}(4x)}{5tan(4x)}$

 $= 4 \sec^2(4x)$

Beware of poor reading skills! [When would we need the Chain Rule to find *y*'?]

 $y = \cos 3x^2$ is read as $y = \cos(3x^2)$

 $y = (\cos 3)x^2$ is read as $y = (\cos 3) \bullet x^2$ where cos3 is a constant

$$y = \cos(3x)^2$$
 is read as $y = \cos(9x^2)$

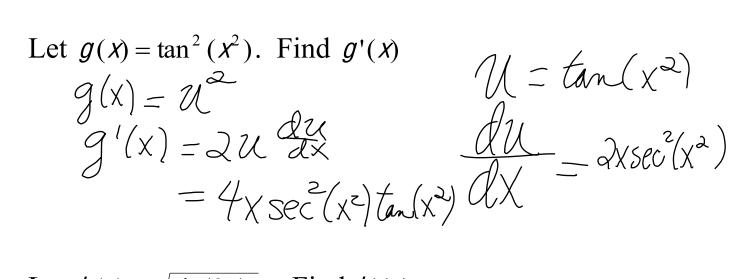
$$y = \cos^2 x$$
 is read as $y = (\cos x)^2$

$$y = \cos^2(3x^2)$$
 is read as $y = [\cos(3x^2)]^2$

$$y = \sqrt{\cos x}$$
 can be read as $y = (\cos x)^{\frac{1}{2}}$

Try: Let $f(x) = \sin(\sqrt{x})$. Find f'(x) $\int_{1}^{1} f(x) = Sin U$ $\int_{1}^{1} f(x) = Cos U \frac{du}{dx}$ $\int_{1}^{1} f(x) = \frac{Cos(\sqrt{x})}{\sqrt{2}\sqrt{x}}$

$$\begin{array}{l}
\mathcal{U} = JX \\
\frac{d\mathcal{U}}{d\mathcal{X}} = J \\
\frac{d\mathcal{U}}{d\mathcal{X}} \\
\frac{d\mathcal{U}}{d\mathcal{X}} \\
\end{array}$$



Let $h(x) = \sqrt{\sin(2x)}$. Find h'(x)

$$h(x) = \sqrt{\mathcal{U}}$$

$$h'(x) = \frac{1}{2\sqrt{\mathcal{U}}} \frac{\partial \mathcal{U}}{\partial x}$$

$$-h'(x) = \frac{\mathcal{R}coe(2x)}{\sqrt{\sqrt{5in(ex)}}}$$

$$\mathcal{U} = SIN(2x)$$

$$\mathcal{U}$$

$$\mathcal{U} = 2cor(2x)$$

Homework: Read 2.4, do page 137 #7, 11, 13, 17, 21 25, 26, 27 41, 43, 45, 47, 51, 53 [Please state your *u* and $\frac{du}{dx}$ and clearly show your steps using standard mathematical notation, blah, blah, blah, ...] If all you have is the function and the derivative, then

NO points will be awarded.

Note: Some problems requires the Chain Rule AND the Product Rule, or they require the Chain Rule AND the Quotient Rule