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## Lesson #1- accumulation “rates”

Cw- problems below

HW- #1,2 of packet

Drive 2 hours at 70 mph how far did I travel- set up the integral and solve

Oil is leaking from a storage tank at a rate of

$$r(t) = 4000e^{-.3t} \text{ liter/day.}$$

Set up an integral and use your calculator to get the answer

1. How much oil will leak in the first 5 days?

2. How much oil in the next 5 days

3. How much oil in first 50 days

### Sandy Beach Problem

The tide removes sand from Sandy Point Beach at a rate modeled by the function  $R$ , given by

$$R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function  $S$ , given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both  $R(t)$  and  $S(t)$  have units of cubic yards per hour and  $t$  is measured in hours for  $0 \leq t \leq 6$ . At time  $t = 0$ , the beach contains 2500 cubic yards of sand.

- (a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
- b) Write an expression for  $y(t)$ , the total number of cubic yards of sand on the beach at time  $t$
- c) Find the rate at which the total amount of sand on the beach is changing at time  $t=4$
- d) For time  $[0,6]$  at what time  $t$  is the amount of sand on the beach a minimum? What is the minimum? Justify your answers.

(a)  $\int_0^6 R(t) dt = 31.815$  or  $31.816 \text{ yd}^3$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b)  $Y(t) = 2500 + \int_0^t (S(x) - R(x)) dx$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c)  $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$  or  $-1.909 \text{ yd}^3/\text{hr}$

1 : answer

(d)  $Y'(t) = 0$  when  $S(t) - R(t) = 0$ .

The only value in  $[0, 6]$  to satisfy  $S(t) = R(t)$  is  $a = 5.117865$ .

3 :  $\begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

$t$	$Y(t)$
0	2500
$a$	2492.3694
6	2493.2766

The amount of sand is a minimum when  $t = 5.117$  or  $5.118$  hours. The minimum value is  $2492.369$  cubic yards.

## Lesson #2- accumulation “rates”

Cw- 3,4 problems below

HW- #5,6 of packet

## Lesson #3- accumulation “rates” PVA

CW p. 390-391 #1,3,no calculator

HW #12-16,19,21 CW- #7 from packet

### Do Now

The rate at which wizards pass through the wall onto Platform  $9\frac{3}{4}$ , measured in wizards per minute, is modeled by the function  $F(t) = 24 + 2\sin\left(\frac{t}{2}\right)$  for times the platform is open  $0 \leq t \leq 30$  in minutes. There are initially 50 wizards on the platform. The rate of wizards leaving the platform is represented by  $G(t) = 3 + \frac{6}{t+4}$  also for  $0 \leq t \leq 30$  in minutes.

To the nearest whole number, how many wizards enter onto the platform during the 30 minute period?

Is the amount of wizards on the platform increasing or decreasing at  $t=10$  minutes. Justify

How many total wizards are on the platform at  $t=30$

At what time is the amount of wizards on the platform at a maximum, Justify?

#### Lesson #4

Go over ch 7 test- explain last month extra help sessions

390-391 #1,3,7,11-16,19,21    392 #31-36 calculator

HW\_ Leckie video -area

#### **Lesson #5**

Area between two curves- to show finding the area with horizontal rectangles

CW/HW - p. 399 #1-6 . #1,2 with calculator 3-6 without calc

p. 401 # 52,53,55 without a calc 54 with calculator

$$\int_{x(\text{values})}^{x(\text{values})} (\text{top} - \text{bottom}) dx \quad (\text{x in the equation})$$

$$\int_{y(\text{values})}^{y(\text{values})} (\text{right} - \text{left}) dy \quad (\text{y in the equation})$$

1. Find the area of the region between the curves

$$F(x) = 2 - x^2 \quad g(x) = x$$

2. Find the area of the region between the curves

$$f(x) = x^2 \quad \text{and} \quad g(x) = e^x \quad \text{on the interval } [0,3]$$

3. Find the area of the regions between the curves

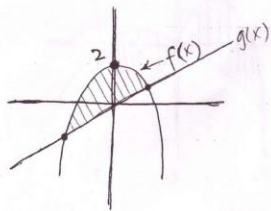
$$f(x) = 3x^3 - x^2 - 10x \quad \text{and} \quad g(x) = -x^2 + 2x$$

4. Find the area of the region in the first quadrant bounded by the curves  $y = \sqrt{x}$ ,  $y = x - 2$  and the  $x$ -axis

~~page~~ Lesson #1

~~3/16~~ Area Between two Curves

- ① Find the area of the region between the curves  $f(x) = 2 - x^2$  and  $g(x) = x$ .



points of intersection

$$2 - x^2 = x$$

$$x = 1, x = -2$$

on calculator  
 $y = 2 - x^2 - x$   
 then  $\int_{-2}^1 f'(x) dx$

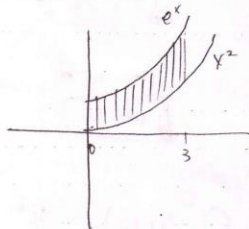
$$\int f(x) - g(x) = \int f(x) - \int g(x)$$

$$\int_{-2}^1 (2 - x^2 - x) dx = 2x - \frac{x^3}{3} - \frac{x^2}{2} \Big|_{-2}^1 = \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right) = 4\frac{1}{2}$$

- ② Find the area of the ~~triangle~~ region between the curves  $y = x^2$  and  $y = e^x$  on the interval  $[0, 3]$ .

~~$\int_0^3 x^2 - e^x = \frac{x^3}{3} - e^x \Big|_0^3 = \frac{27}{3} - e^3 = 9 - e^3$~~

$$\int_0^3 e^x - x^2 dx = e^x - \frac{x^3}{3} \Big|_0^3 = e^3 - 9 - 1 = \boxed{e^3 - 10}$$



which is on top?  
 $f(0) = e^0 = 1$  on top  
 $g(0) = -x^2 = 0$

Et 3

Find the area of the regions between the curves  
 $f(x) = 3x^3 - x^2 - 10x$  and  $g(x) = -x^2 + 2x$

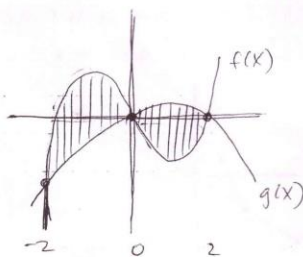
1. find where they intersect  
 which is on top

$$3x^3 - x^2 - 10x = -x^2 + 2x$$

$$3x^3 - 12x = 0$$

$$x(3x^2 - 12) = 0$$

$$x = 0 \quad x = 2 \quad x = -2$$

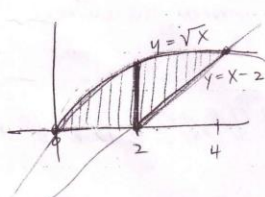


$$\int_{-2}^0 (3x^3 - x^2 - 10x) - (-x^2 + 2x) dx + \int_0^2 (-x^2 + 2x) - (3x^3 - x^2 - 10x) dx$$

$$12 + 12 = 24$$

Et 4

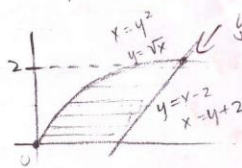
Find the area of the region in the first quadrant bounded by the curves  $y = \sqrt{x}$ ,  $y = x - 2$ , and  $x$ -axis



$$\int_0^2 \sqrt{x} dx + \int_2^4 \sqrt{x} - (x - 2) dx$$

$$= \left( \frac{2}{3} \right) = 3.333$$

Change the variable to y



$$\int_0^2 (y+2) - y^2 dy$$

$$= \left[ \frac{1}{2}y^2 + 2y - \frac{y^3}{3} \right]_0^2$$

$$= 6 - \frac{8}{3} = \left( \frac{10}{3} \right) = 3.333$$

$y = \sqrt{x}$  solve for x

$$y^2 = x$$

$$y = x - 2$$

$$\int_0^2 (y+2) - y^2$$

to use with video -

[http://www.chaoticgolf.com/vodcasts/calc/lesson7\\_2/lesson7\\_2.html](http://www.chaoticgolf.com/vodcasts/calc/lesson7_2/lesson7_2.html)

**Area of a Region Between Two Curves**

If  $f$  and  $g$  are continuous on  $[a, b]$  and  $g(x) \leq f(x)$  for all  $x$  in  $[a, b]$ , then the area of the region bounded by the graphs of  $f$  and  $g$  and the vertical lines  $x = a$  and  $x = b$  is

$$A = \int_a^b [f(x) - g(x)] dx$$

*Example 5:* Find the area of the region bounded by the graphs of  $y = x^2 + 2$ ,  $y = -x$ ,  $x = 0$ , and  $x = 1$ .

Step 1: Draw a picture and shade the desired region.

Step 2: Draw an arbitrary rectangular strip.

Step 3: Using the area of the rectangular strip as a guide, set up and solve an integral to find the area between the curves.

*Example 6:* Find the area of the region bounded by the graphs of  $x = 3 - y^2$  and  $x = y + 1$ .



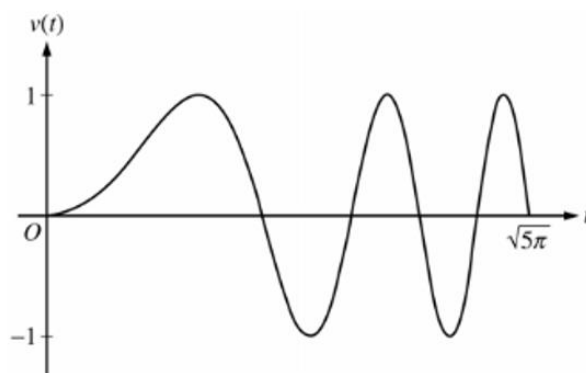
## Lesson #6

### AREA

CW-AP problems 2001 parts A,B, 2002 part A, 2007 on top entire question, bottom one parts A,B  
HW-p. 400 #9,10 no calc 13,calc 15,36,38, (no calculator)

### 2007 form B #2

A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = \sin(t^2)$ . The graph of  $v$  is shown above for  $0 \leq t \leq \sqrt{5\pi}$ . The position of the particle at time  $t$  is  $x(t)$  and its position at time  $t = 0$  is  $x(0) = 5$ .



- Find the acceleration of the particle at time  $t = 3$ .
- Find the total distance traveled by the particle from time  $t = 0$  to  $t = 3$ .
- Find the position of the particle at time  $t = 3$ .
- For  $0 \leq t \leq \sqrt{5\pi}$ , find the time  $t$  at which the particle is farthest to the right. Explain your answer.

(a)  $a(3) = v'(3) = 6 \cos 9 = -5.466$  or  $-5.467$

(b) Distance  $= \int_0^3 |v(t)| dt = 1.702$

OR

For  $0 < t < 3$ ,  $v(t) = 0$  when  $t = \sqrt{\pi} = 1.77245$  and

$$t = \sqrt{2\pi} = 2.50663$$

$$x(0) = 5$$

$$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t) dt = 5.89483$$

$$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t) dt = 5.43041$$

$$x(3) = 5 + \int_0^3 v(t) dt = 5.77356$$

$$|x(\sqrt{\pi}) - x(0)| + |x(\sqrt{2\pi}) - x(\sqrt{\pi})| + |x(3) - x(\sqrt{2\pi})| = 1.702$$

(c)  $x(3) = 5 + \int_0^3 v(t) dt = 5.773$  or  $5.774$

- (d) The particle's rightmost position occurs at time  $t = \sqrt{\pi} = 1.772$ .

The particle changes from moving right to moving left at those times  $t$  for which  $v(t) = 0$  with  $v(t)$  changing from positive to negative, namely at  $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$  ( $t = 1.772, 3.070, 3.963$ ).

Using  $x(T) = 5 + \int_0^T v(t) dt$ , the particle's positions at the times it

changes from rightward to leftward movement are:

$$T: 0 \quad \sqrt{\pi} \quad \sqrt{3\pi} \quad \sqrt{5\pi}$$

$$x(T): 5 \quad 5.895 \quad 5.788 \quad 5.752$$

The particle is farthest to the right when  $T = \sqrt{\pi}$ .

1 :  $a(3)$

2 :  $\begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

## LESSON #7

Volume- disk and washer method over x and y axis

use the animation examples

Night before- watch video on disk method- and complete worksheet

[http://www.chaoticgolf.com/vodcasts/calc/lesson7\\_3\\_part2/lesson7\\_3\\_part2.html](http://www.chaoticgolf.com/vodcasts/calc/lesson7_3_part2/lesson7_3_part2.html)

p. 411 #7-15 - set up but use calculator to solve

Horizontal Axis of Revolution  $V = \pi \int_a^b [R(x)]^2 dx$

Vertical Axis of Revolution  $V = \pi \int_a^b [R(y)]^2 dy$

### More animation

**EX 1-** <http://archives.math.utk.edu/visual.calculus/5/volumes.5/index.html>

- Finding the volume of a rotated graph.  
Discussion [[Using Flash](#)] [[Using Java](#)]
- Examples:

\*\*\*\*\*Area bounded by the graph of  $f(x) = x - x^2$  and the x-axis.

Solution [[Using Flash](#)] [[Using Java](#)]

[[Graph](#)]

**EX 2-----\*\*\*\*\***Area bounded by the graphs of  $f(x) = x^3 - x + 1$ ,  $x = -1$ ,  $x = 1$  and the x-axis.

Solution [[Using Flash](#)] [[Using Java](#)]

[[Graph](#)]

Notes to be filled out while watching video

Day 2: Volumes of Solids of Revolution: The Disc Method

In finding the area of a region, we drew an arbitrary representative rectangle. Keeping with the same idea, if we revolve a rectangle around a line, it forms a cylinder, as shown below. The key to using the disc method will be twofold: 1) The rectangular strip must be connected to the axis of revolution (no matter where you draw it), and 2) the rectangular strip must be perpendicular to the axis of revolution.



*Example 6:* What is the volume of the cylinder shown if the height of the rectangle is considered  $R$  and the width of the rectangle is  $dx$ ?

Just like we did in finding the area, as we increase the number of rectangles to infinity, the width of each rectangle becomes infinitely small and we denote this  $dx$  (if it is a vertical strip) or  $dy$  (if it is a horizontal strip). We then use an integral to sum the volume of every one of these infinitely thin cylinders. This concept leads to the following:

**The Disc Method**

To find the volume of a solid of revolution with the *disc method*, use one of the following;

**HORIZONTAL AXIS OF REVOLUTION**

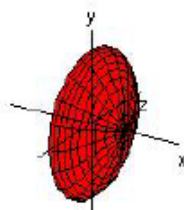
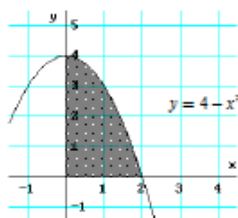
$$V = \pi \int_a^b [R(x)]^2 dx$$

**VERTICAL AXIS OF REVOLUTION**

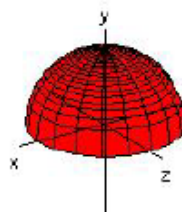
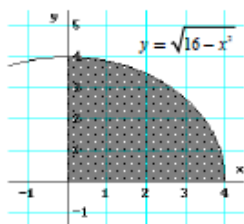
$$V = \pi \int_a^b [R(y)]^2 dy$$

where  $R(x)$  and  $R(y)$  are the "heights" of your representative rectangular strips.

*Example 7:* Draw an appropriate rectangular strip and find the volume of the solid formed by revolving the region about the  $x$ -axis.



**Example 8:** Find the volume of the solid formed by revolving the region about the  $y$ -axis. (Draw a representative rectangular strip)



**Example 9:** Find the volume of the solid generated by revolving the region bounded by the graphs of the equations  $xy = 6$ ,  $y = 2$ ,  $y = 6$ , and  $x = 6$  about the indicated lines. Sketch the region formed, and draw a representative rectangular strip for each solid.

a) the line  $x = 6$ .

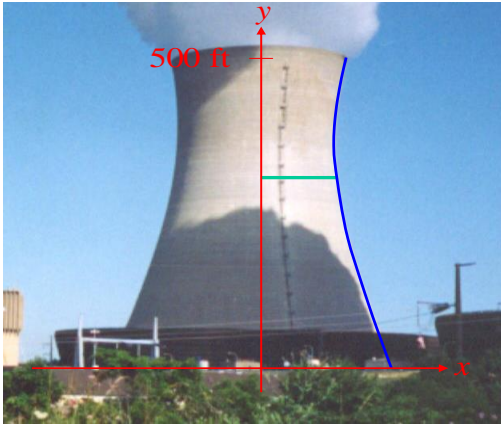
b) the line  $y = 6$ .

## LESSON #8 day 2 of volume disk method

Go over chapter test- p. 411 #29 b,c and #31 A

Cw- problems below

HW- watch leckie washer method



The natural draft cooling tower shown at left is about 500 feet high and its shape can be approximated by the graph of this equation revolved about the y-axis:

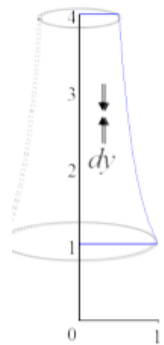
$$x = .000574y^2 - .439y + 185$$

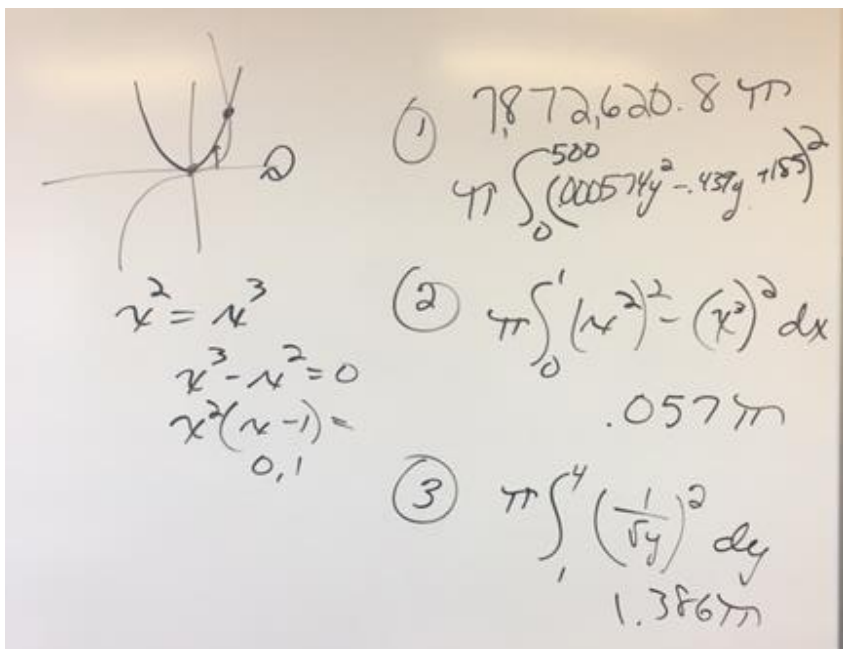
Find the volume- set up the integral and use your calculator

1. Sketch and set up the integral to find the volume of the region bounded by  $y = \sqrt{x}$  and  $x=4$  and the x axis, about the line  $y=-2$

2. Set up the integral to find the volume of the region bounded by the curve

$x = \frac{1}{\sqrt{y}}$   $y = 1$  and  $y = 4$  and the y-axis is revolved about the y-axis.





## LESSON #9

Disk and Washer method over other axis-

CW. p. 411 #29 b,31

HW- worksheet -  $e^x$

- 1 Find the volume of the solid created by revolving the region bounded by  $y=e^x$  and  $y=1$  for  $0 < x < 3$ , about the x-axis

$$(\text{Outer volume})^2 - (\text{inner volume})^2$$

$$R^2 - r^2$$

$$V = \pi \int_0^3 (e^x)^2 - 1^2 dx = \pi \int_0^3 e^{2x} - 1 dx = \pi \left( \frac{e^{2x}}{2} - x \right) \text{ evaluate from } 0-3$$

without a calculator

- 1 Find the area of the region bounded by  
 $f(x) = x^2$  and  $g(x) = x$

2. Find the volume when rotated over the  $y = -1$  axis

Then  $y=2$  and  $x=3$

<http://archives.math.utk.edu/visual.calculus/5/volumes.5/index.html>

Find the volume of the solid obtained by rotating the area bounded by  $f(x) = x^2$  and  $g(x) = x$  about the following lines:

4.  $y = -1$ .  
Solution [[Using Flash](#)] [[Using Java](#)]  
[[Graph](#)]
5.  $y = 2$ .  
Solution [[Using Flash](#)] [[Using Java](#)]  
[[Graph](#)]
6.  $y$ -axis.  
Solution [[Using Flash](#)] [[Using Java](#)]  
[[Graph](#)]

HW- AP Problems 2001 -c,2002b,2007 c

## LESSON #10

Volume of solids with known cross sections (6.2) ex 6  
Animation cross section different bases

2014-used Mr Leckies video for 3 minutes, then did his examples with the class. –

<http://mathdemos.gcsu.edu/mathdemos/sectionmethod/sectiongallery.html>

Cool videos:

<http://www.youtube.com/watch?v=omQSp2uMYTk&feature=plcp&context=C34d7b1eUDOEgsToPDskK8ZAxQf79xm6D4JsiCtZgZ>

<http://www.youtube.com/watch?v=3yIW9FgKXKU&feature=context&context=C34d7b1eUDOEgsToPDskK8ZAxQf79xm6D4JsiCtZgZ>

<http://www.youtube.com/watch?v=97pCh8A0rJQ&feature=context&context=C34d7b1eUDOEgsToPDskK8ZAxQf79xm6D4JsiCtZgZ>

cw\_ p. 430 – 431 # 59,60,61-old book

HW- part C of 2010 and 2001, and 9,10 of previous review

We are not revolving around an axis.

### AP TIP

When working in the calculator section of the exam the expectation is that you will write the integral in the proper form (above) and then use your calculator to find the value of the definite integral. Although you may want to show that you can do it by hand, that will not only take valuable time, but you could also make an error along the way.

In this section a solid is described to you and you want to take a cross section (in a cylinder that cross section would be a circle, in a cube it would be a square, that will be predetermined. If you take the **sum of the areas of those cross sections**, then you should get the volume of the solid

Within this concept you will be asked to find the volume of a solid formed with a given base but with different cross sections. The solutions will vary by the type of cross sections because a different solid is formed.

Find the volume of the solid whose base is bounded by the circle  $x^2 + y^2 = 9$  with the indicated cross sections taken perpendicular to the x-axis.

Here, the base is circular in each case but the area you will sum will vary. However this volume formula is just

$\int_a^b A(x)dx$  for a cross section perpendicular to the x-axis and  $\int_c^d A(y)dy$  for cross sections perpendicular to the y-

axis where  $A(x)$  and  $A(y)$  are the areas of the cross sections. The base that is given is used to find the length of one side of the cross sections which is then used to find the equation for the area of the cross sections



cw\_p. 430 – 431 # 59,60,61-old book

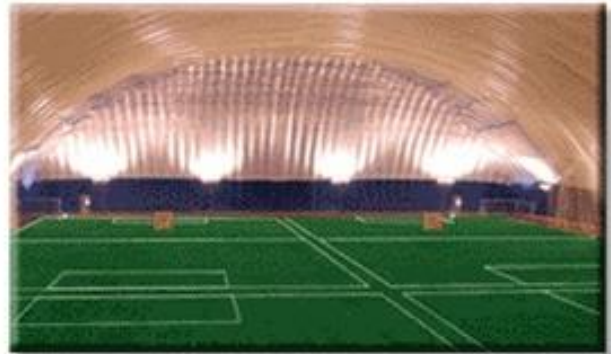
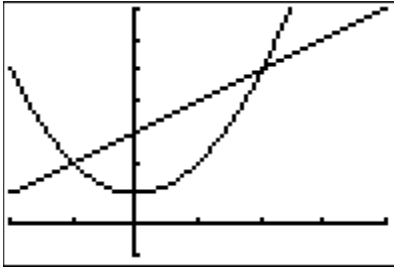
HW- part C of 2010 and 2001, and 9,10 of previous review

We are not revolving around an axis.

EXAMPLE #1 Use a calculator

$$F(x) = x + 3$$

$$G(x) = x^2 + 1$$



**Find the volume of the solid if the cross sections are:**

Limits of integration are where  $f(x)$  and  $g(x)$  intersect

Square

$$A = b^2$$

Rectangle with height  $h$

$$A = bh$$

Semi-circle

$$A = \frac{1}{8}\pi b^2$$

Equilateral triangle

$$A = \frac{\sqrt{3}b^2}{4}$$

Isosceles rt triangle hypotenuse on the base (1/4 of a square)

$$A = \frac{b^2}{4}$$

Isosceles rt triangle leg on the base (1/2 of a square)

$$A = \frac{b^2}{2}$$

## LESSON #11

Review for test- calculators allowed

CW- p. 437 #55 (2002) amusement park problem as Do Now- exchange papers and grade multiple choice (2 sided)

Book p. 415 1,3,4

## LESSON #12

Go over HW

p. 437 #53-54

p. 413 #65-68

p 412 #39

3. [Calculator] A region in the first quadrant is enclosed by the graphs of  $y = e^{2x}$ ,  $x = 1$ , and the coordinate axes. If the region is rotated about the  $y$ -axis, what is the volume of the solid generated?

When finding area if you use absolute value it doesn't matter which is on top- do not just add the abs value bars at the end , must be in the integral

When working in the calculator section of the exam the expectation is that you will write the integral in the proper form and then use your calculator to find the value of the definite integral. Although you may want to show that you can do it by hand, that will not only take valuable time, but you could also make an error along the way

## Lesson 13

AP Problems

2010,2001,2012,2013

3. [Calculator] A region in the first quadrant is enclosed by the graphs of  $y = e^{2x}$ ,  $x = 1$ , and the coordinate axes. If the region is rotated about the  $y$ -axis, what is the volume of the solid generated?

### **calculator**

Find the area of the region bounded by  $f(x) = x^2$  and  $g(x) = \frac{-3}{2}x + \frac{17}{2}$

Find the volume if it is rotated about the  $y=15$

### **Test-**