Chapter Three Stuff That I Must Know Cold

Critical values/numbers

If $f'(c)=0$ or $f'(c)$ is undefined but $f(c)$ is defined, then the graph of f has a critical point at the point $(c,\ f(c))$.

If a function has a relative extreme value at a point, then that point must be a critical point. [But not vice versa]

To justify a relative maximum at the point $(c,\ f(c))$, you must state that at $x=c$, $f'(x)$ changes from positive to negative values, hence $f(c)$ is the relative maximum value.

To justify a relative minimum at the point $(c,\ f(c))$, you must state that at $x=c$, $f'(x)$ changes from negative to positive values, hence $f(c)$ is the relative minimum value.

Always read the question carefully. If the question asks for ***where*** a function has a relative extreme value, then it is asking for the $x-$value. If the question asks for ***what*** is the relative extreme value, then is it asking for the function's value [the $y-$value].

Points of Inflection

If $f''(d)=0$ or $f''(d)$ is undefined but $f(d)$ is defined, then f MAY have a point of inflection at the point $(d,\ f(d))$.

To justify [using the second derivative] a point of inflection at the point $(d,\ f(d))$ you must state one of the following:

At $x=d$ $f''(x)$ changes from positive to negative values, hence $(d,\ f(d))$ is a point of inflection for the graph of f.

OR

At $x=d$ $f''(x)$ changes from negative to positive values, hence $(d,\ f(d))$ is a point of inflection for the graph of f.

Using the correct interval notation: [for a differentiable function]

If $f'(x) > 0$ for every x in the open interval (a, b), then $f(x)$ is increasing on the closed interval $[a, b]$.

If $f'(x) < 0$ for every x in the open interval (a, b), then $f(x)$ is decreasing on the closed interval $[a, b]$.

If $f''(x) > 0$ for every x in the open interval (a, b), then $f(x)$ is concave up for the open interval (a, b). [$f'(x)$ is increasing on $[a, b]$]

If $f''(x) < 0$ for every x in the open interval (a, b), then $f(x)$ is concave down for the open interval (a, b). [$f'(x)$ is decreasing on $[a, b]$]

Using the first derivative graph

If the graph of $f'(x)$ is increasing on the interval $[a, b]$, then $f(x)$ is concave up for the open interval (a, b).

If the graph of $f'(x)$ is decreasing on the interval $[a, b]$, then $f(x)$ is concave down for the open interval (a, b).

If the graph of $f'(x)$ changes from increasing to decreasing at the point $(d,\ f(d))$, then the graph of f has a point of inflection at $(d,\ f(d))$.

If the graph of $f'(x)$ changes from decreasing to increasing at the point $(d,\ f(d))$, then the graph of f has a point of inflection at $(d,\ f(d))$.

If the graph of $f'(x)$ changes from positive to negative values at the point $(c,\ f(c))$, then f has a relative maximum value at $x = c$.

If the graph of $f'(x)$ changes from negative to positive values at the point $(c,\ f(c))$, then f has a relative minimum value at $x = c$.

Absolute Extrema

An absolute maximum value or absolute minimum value may only occur if you are given a closed interval. To justify an absolute extrema you must [usually] do "candidate testing".

First find any critical values for the graph of f. Then compare the function's values at the endpoints and at the critical values. You must clearly identify your candidates and clearly show their function values.

We will do exceptions to this in class.

How to justify using the Mean Value Theorem

By the Mean Value Theorem, here is a $c, a < c < b,$ such that $f'(c) = \frac{f(b) - f(a)}{b-a}$.

Now show why this supports your solution.

You must provide the values for $a, b, and\ c$ and clearly show how the theorem supports your solution. Be sure to write your justification in the manner that was covered in class.

Since Rolle's Theorem is just a special case of the Mean Value Theorem, you can always just use MVT.

Tangents

Horizontal tangents occur at $x = c$ if $f(c)$ is defined and $f'(c) = 0$.

Vertical tangents occur at $x = c$ if $f(c)$ is defined and $f'(c)$ is undefined.

If $f'(x)$ is of the form $f'(x) = \frac{g(x)}{h(x)}$, then f will have a horizontal tangent if $g(x) = 0$ and $h(x) \neq 0$ and f is defined. f will have a vertical tangent if $g(x) \neq 0$ and $h(x) = 0$ and $x = c$ if $f(c)$ is defined

Asymptotes

$y = b$ is a horizontal asymptote of f if $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

$x = a$ is a vertical asymptote of f if $\lim_{x\to a} f(x) = \infty$ or $\lim_{x\to a} f(x) = -\infty$

Using limits to find horizontal asymptotes or using "end behavior"

If $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = 0$ if the degree of $f(x)$ < degree of $g(x)$ and hence, $y = 0$ is a horizontal asymptote of the function.

If $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = \infty$ or does not exist if degree of $f(x)$ > degree of $g(x)$ and hence the function does not have a horizontal asymptote.

If $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = b$ if degree of $f(x)$ = degree of $g(x)$ and hence $y = b$ is horizontal asymptote of the function.

The Second Derivative Test for Extrema

(1) If $f''(c) > 0$, then f has a relative minimum value at the point $(c, f(c))$

(2) If $f''(c) < 0$, then f has a relative maximum value at the point $(c, f(c))$

(3) If $f''(c) = 0$, then the Second Derivative Test fails and you must resort to the First Derivative Test to find your extrema.