Stuff to remember:

Critical values/numbers are where $f'(x)$ is either equal to zero OR $f'(x)$ is undefined [but $f(x)$ is defined]
Possible points of inflection are where $f''(x)$ is either equal to zero OR $f''(x)$ is undefined [but $f(x)$ is defined]
If a function is differentiable at a point, then the function is continuous at that point.

How to justify: PLEASE LOOK AT THE INTERVAL NOTATION
[Assume that the function is continuous and differentiable]
If $f'(x) > 0$ for every $x$ in $(a, b)$, then $f(x)$ is increasing on $[a, b]$
If $f'(x) < 0$ for every $x$ in $(a, b)$, then $f(x)$ is increasing on $[a, b]$
If $f''(x) > 0$ for every $x$ in $(a, b)$, then $f(x)$ is concave up on $(a, b)$ AND $f'(x)$ is increasing on $[a, b]$
If $f''(x) < 0$ for every $x$ in $(a, b)$, then $f(x)$ is concave down on $(a, b)$ AND $f'(x)$ is decreasing on $[a, b]$
If $f'(x)$ is increasing on $(a, b)$, then $f(x)$ is concave up on $(a, b)$ AND $f''(x) > 0$ on $(a, b)$
If $f'(x)$ is decreasing on $(a, b)$, then $f(x)$ is concave down on $(a, b)$ AND $f''(x) < 0$ on $(a, b)$

Let $x = a$ be a critical value for $f(x)$ [in other words, $f'(a) = 0$ OR $f'(a)$ is undefined but $f(a)$ is defined]

To show that $(a, f(a))$ is a relative/local minimum:
“At $x = a$, $f'(a)$ changes from negative to positive values. Hence, $f$ has a relative minimum at $x = a$.”

To show that $(a, f(a))$ is a relative/local maximum:
“At $x = a$, $f'(a)$ changes from positive to negative values. Hence, $f$ has a relative maximum at $x = a$.”
Let $f''(b) = 0$ or $f''(b)$ be undefined but $f(b)$ is defined. To show that $(b, f(b))$ is a point of inflection you must justify with one of the following statements:

“At $x = b$, $f''(b)$ changes from positive to negative values. Hence, $f$ has a point of inflection at $x = b$.”

OR

“At $x = b$, $f''(b)$ changes from negative to positive values. Hence, $f$ has a point of inflection at $x = b$.”

If given the graph of the first derivative, then a point of inflection will occur at $x = a$ if at $x = a$ the graph of $f'(x)$ changes from either increasing to decreasing OR decreasing to increasing.

NOTE: When you justify, you must be SPECIFIC. Do NOT state a rule but rather, explain what is occurring at some specific value of $x = a$.

**To justify an absolute maximum or absolute minimum**

The candidates are the critical values of $f(x)$ AND the endpoints! First justify any relative min/max AND find their function values. Then compare the function values to the endpoint values and decide which is the absolute min/max.

**Extreme Value Theorem**

If a function, $f$, is continuous on $[a, b]$, then $f$ has both a maximum and a minimum value on $[a, b]$.

**Mean Value Theorem**

Your justification statement should look like:

By the Mean Value Theorem there is a $c$, $a < c < b$, such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

You need to supply all of the values!
Rolle’s Theorem [A “special” case of MVT]
Your justification statement should look like:
By Rolle’s Theorem, since f is continuous on [a, b] and
differentiable on (a, b) AND f(a)=f(b), then there is a c, a < c < b,
such that f’(c) = 0.
Note: You can always just use MVT.

Horizontal TANGENTS occur at x = a if f’(a) 0= 0
Vertical TANGENTS occur at x = a if f’(a) is undefined but f(a) is defined

If f’(x) is of the form \( f'(x) = \frac{g(x)}{h(x)} \), then f will have a horizontal
tangent when g(x) = 0 AND f will have a vertical tangent if h(x) = 0. [Of course f(x) must be defined for those values]

Horizontal Asymptote
y = b is a horizontal asymptote of the graph of y = f(x) if either
\[ \lim_{x \to \infty} f(x) = b \quad \text{OR} \quad \lim_{x \to -\infty} f(x) = b \]

Limits of Rational Functions as \( x \to \pm \infty \) [End Behavior]
\[ \lim_{x \to \pm \infty} \frac{f(x)}{g(x)} = 0 \quad \text{if the degree of } f(x) < \text{ the degree of } g(x) \]
\[ \lim_{x \to \pm \infty} \frac{f(x)}{g(x)} = \infty \quad \text{or “dne” if the degree of } f(x) > \text{ the degree of } g(x) \]
\[ \lim_{x \to \pm \infty} \frac{f(x)}{g(x)} \quad \text{is finite [there is a horizontal asymptote] if} \]
\[ \text{degree of } f(x) = \text{degree of } g(x) \]
Remember: A graph is NOT justification. You must use Calculus to justify any extrema, points of inflection, horizontal asymptotes, etc.

Remember:
If $s(t)$ is the position function, then
Velocity, $v(t) = s'(t)$
Acceleration, $a(t) = v'(t) = s''(t)$
Speed is $|v(t)|$

The speed of a particle is increasing at $x = c$ if $v(c)$ and $a(c)$ are either both positive or both negative.
The speed of a particle is decreasing at $x = c$ if $v(c)$ and $a(c)$ have different signs.

**To justify that the speed of a particle is increasing at** $x = c$, you need to clearly state that $v(c) > 0$ AND $a(c) > 0$
OR $v(c) < 0$ AND $a(c) < 0$ in a statement.

**To justify that the speed of a particle is decreasing at** $x = c$, you need to clearly state that $v(c) > 0$ AND $a(c) < 0$
OR $v(c) < 0$ AND $a(c) > 0$ in a statement.

**OPTIMIZATION**
If necessary, you might want to draw a picture of what the problem is about.
Figure out what you are trying to optimize.
Find a primary equation that fits what you are trying to optimize.
If necessary, find a secondary equation that will allow you to re-write your primary equation in terms of just ONE VARIABLE.
Now just find the min or max using standard Calculus min/max techniques.