Lesson 4

Watch Mr Leckie from the 9:22 minute marker to the end
Follow along with attached worksheets- then do page 3
http://www.chaoticgolf.com/tutorials_calc_ch5.htmlesson 4

6.2 Definite Integrals

Notation for Definite Integrals

The limit notation we used last is the form we will use to develop Integral notation. As the number of rectangles goes to infinity, the width of each rectangle, Δx, goes to zero. As we did in the section on differentials, we are going to use the notation *dx* to represent this infinitely tiny distance.

The summation notation of sigma is going to be replaced with an *Integral Sign*, $\int$, which looks somewhat like a giant "S" for sum.

The $f(c_k)$ which represented a different function value for each interval is going to be replaced with $f(x)$ since the x – values are going to be soooooooo close together it's almost as if we are evaluating the function at EVERY x – value in the interval $[a, b]$. Combining all of this we have the following notation:

$h \cdot b$

$$\int_a^b f(x)\,dx$$

$y \cdot \Delta x$ ← Add up infinitely many rectangles w/ small width Δx

We read the notation above as "***The Integral of f of x from a to b***"

Important ♫ (Actually it's a Theorem): IF the function is continuous, THEN the Definite Integral will exist. However, the converse, while true some of the time is NOT ALWAYS true.

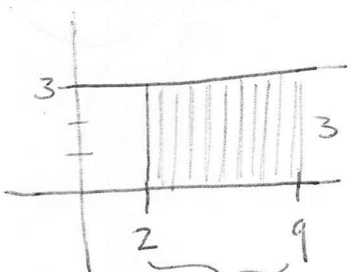
Using Definite Integrals as Area

We can define the **area under the curve** $y = f(x)$ **from** a **to** b as an *integral* from a to b ...
... AS LONG AS THE CURVE IS NONNEGATIVE AND INTEGRABLE on the closed interval $[a, b]$.

Drawing a picture and using geometry is still a valid method of finding areas in this class!

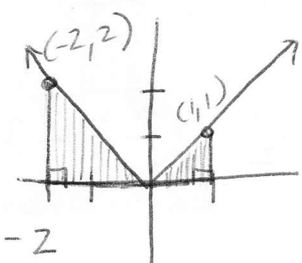
Example 1: For each of the following examples, sketch a graph of the function, shade the area you are trying to find, then use geometric formulas to evaluate each integral.

a) $\int_2^9 3\,dx$



7
rectangle
$7 \cdot 3 = 21$

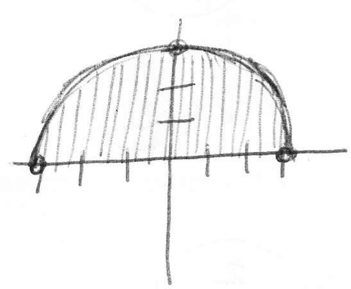
b) $\int_{-2}^{1} |x|\,dx$.



2 Δ's
$\frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$
$2 + \frac{1}{2} = 2.5$
$\frac{5}{2}$

c) $\int_{-3}^{3} \sqrt{9-x^2}\,dx$

$y = \sqrt{9-x^2}$
$y^2 = 9 - x^2$
$x^2 + y^2 = 9$
circle
$C(0,0)$
$r = 3$
since $+\sqrt{\ }$.
top half

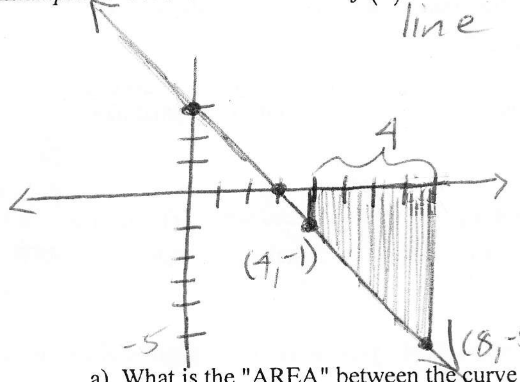


semicircle
$\frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$

So … what happens if the “area” is below the x-axis … as I mentioned before, “area” is inherently positive, but a Riemann sum … and therefore an Integral can have negative values if the curve lies below the x-axis.

Example 2: Consider the function $f(x)=3-x$. Sketch a graph of this function.

line



a) What is the "AREA" between the curve and the x – axis between $x = 4$ and $x = 8$? Trapezoid $= \frac{1}{2}h(b_1+b_2)$

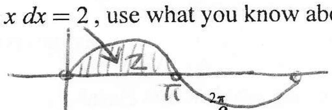
$\frac{1}{2}(4)(1+5) = 12$

b) Evaluate $\int_4^8 (3-x)\,dx$ – integral is estimated by adding up rectangles those rectangles would have neg y's so $\int = -$ b/c it's below x-axis

-12

Add $\int_0^8 (3-x)dx = -8$ split pos & neg area

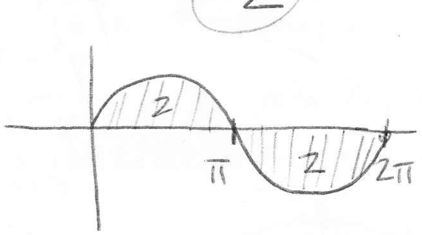
Example 3: Given $\int_0^{\pi} \sin x\,dx = 2$, use what you know about a sine function to evaluate the following integrals.

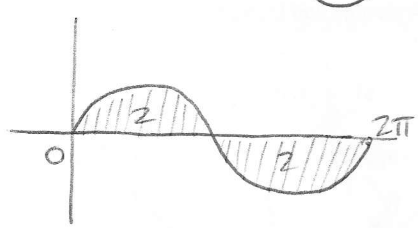


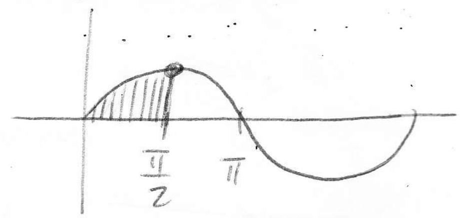
a) $\int_{\pi}^{2\pi} \sin x\,dx = -2$

b) $\int_0^{2\pi} \sin x\,dx = 2-2 = 0$

c) $\int_0^{\pi/2} \sin x\,dx \quad \frac{1}{2}(2) = 1$



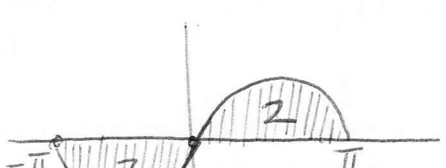


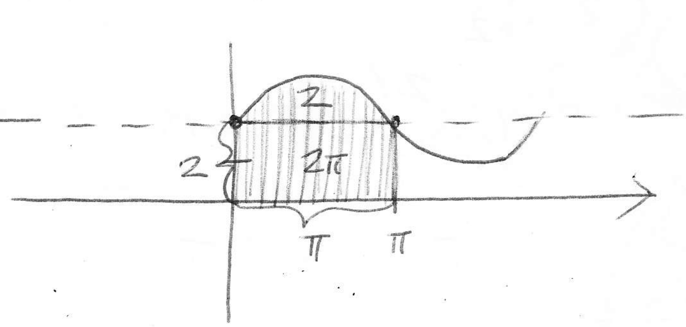


d) $\int_{-\pi}^{\pi} \sin x\,dx = 2-2 = 0$

e) $\int_0^{\pi} (2+\sin x)\,dx = 2+2\pi$

up 2



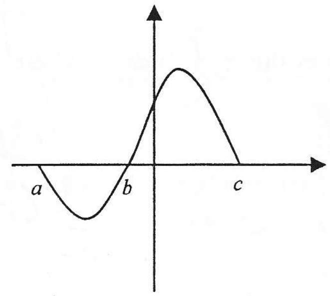


2

1. Graphically speaking, if $f(x)$ is always above the x-axis, what does $\int_a^b f(x)\,dx$ mean?

Area under the curve.

2. Given the graph of $f(x)$ below, answer the following questions:



a) Is $\int_a^b f(x)\,dx$ positive, negative, or zero? Why?

negative b/c its area is below the x-axis

b) Is $\int_b^c f(x)\,dx$ positive, negative, or zero? Why?

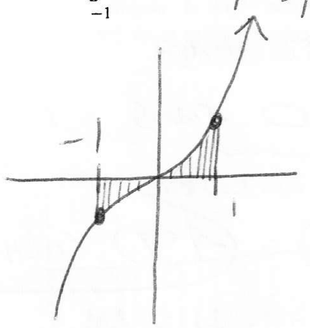
positive b/c its area is above the x axis

c) Is $\int_a^c f(x)\,dx$ positive, negative, or zero? Why?

positive b/c positive area of b to c is bigger than the neg. area from a to b.

3. Use your knowledge of the graph of $y = x^3$, your understanding of area, and the fact that $\int_0^1 x^3\,dx = \frac{1}{4}$ to answer the following: (Draw a sketch for each one!)

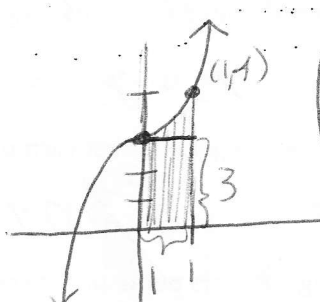
a) $\int_{-1}^{1} x^3\,dx = \frac{1}{4} - \frac{1}{4} = 0$



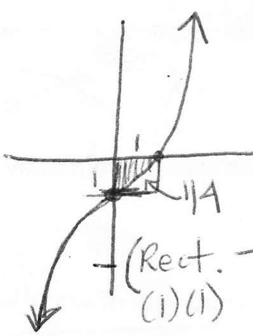
b) $\int_0^1 (x^3+3)\,dx = \frac{1}{4} + 3$

3.25

$\frac{13}{4}$



c) $\int_0^1 (x^3-1)\,dx = -\frac{3}{4}$



below x-axis

(Rect. $-\frac{1}{4}$) $= \left(1 - \frac{1}{4}\right)$

(1)(1)

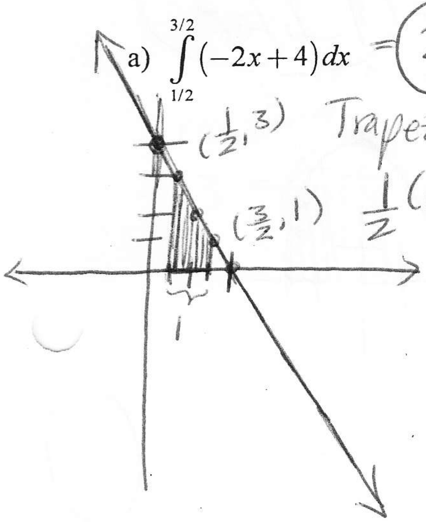
4. Draw a sketch and shade the "area" indicated by each integral, then use geometry to evaluate each integral.

a) $\int_{1/2}^{3/2} (-2x+4)\,dx = 2$

$(\frac{1}{2}, 3)$ Trapezoid

$(\frac{3}{2}, 1)$ $\frac{1}{2}(1)(1+3)$

2

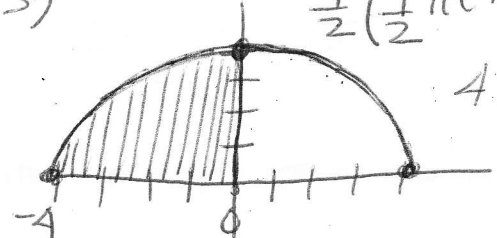


b) $\int_{-4}^{0} \sqrt{16-x^2}\,dx = 4\pi$

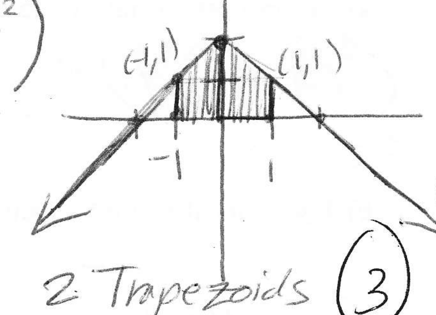
$\frac{1}{2}$ Semicircle

$\frac{1}{2}\left(\frac{1}{2}\pi(4)^2\right)$

4π



c) $\int_{-1}^{1} (2-|x|)\,dx = 3$

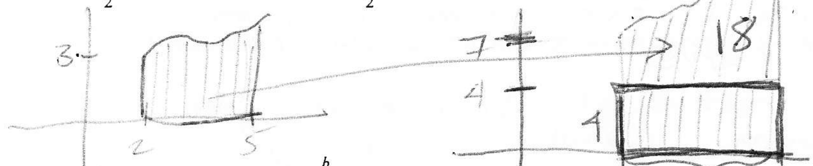


2 Trapezoids 3

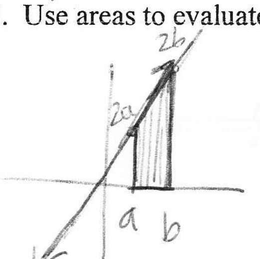
$\frac{1}{2}(1)(1+2) + \frac{1}{2}(1)(1+2)$

$3/2 + 3/2 = 3$

5. If $\int_2^5 f(x)\,dx = 18$, then $\int_2^5 (f(x)+4)\,dx = ?$ $\quad 18 \quad$ Adds rectangle $4(3)$ $\quad = 18 + 12 = 30$



6. Use areas to evaluate $\int_a^b 2s\,ds$, where a and b are constants and $0 < a < b$



Trapezoid

$\frac{1}{2}(b-a)(2a+2b) = (b-a)(a+b) = b^2 - a^2$

7. Which of the following quantities would NOT be represented by the definite integral $\int_0^8 70\,dt$?

A) The distance traveled by a train moving 70 mph for 8 minutes ← needs to be hours
B) The volume of ice cream produced by a machine making 70 gallons per hour for 8 hours ✓
C) The length of a track left by a snail traveling at 70 cm per hour for 8 hours ✓
D) The total sales of a company selling $70 of merchandise per hour for 8 hours ✓
E) The amount the tide has risen 8 min after low tide if it rises at a rate of 70 mm per minute during that period ✓

8. Express the desired quantity as a definite integral and then evaluate using geometry.

a) Find the distance traveled by a train moving at 87 mph from 8:00 AM to 11:00 AM

$\int_8^{11} 87\,dt \qquad 87 \cdot 3 = 261$ mi

b) Find the output from a pump producing 25 gallons per minute during the first hour of its operation.

$\int_0^{60} 25\,dt = \qquad 25 \cdot 60 = 1500$ gals

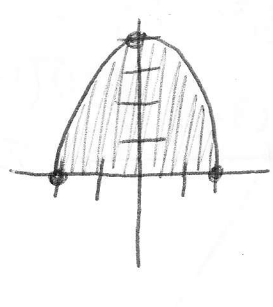
c) Find the calories burned by a walker burning 300 calories per hour between 6:00 PM and 7:30 PM.

$\int_{18}^{19.5} 300\,dt \qquad 300 \cdot (1.5) = 450$ calories

d) Find the amount of water lost from a bucket leaking 0.4 liters per hour between 8:30 AM and 11:00 AM.

$\int_{8.5}^{11} 0.4\,dt \qquad 0.4(2.5)$ $\frac{\text{liters}}{\text{hr}}$ hr. $= 1$ liter

9. Draw a sketch for the area enclosed between the x-axis and the graph of $y = 4 - x^2$ from $x = -2$ to $x = 2$.



a) Set up a definite integral to find the area of the region.

$\int_{-2}^{2} (4 - x^2)\,dx$

b) Use your calculator to evaluate the integral expression you set up in part a.

$10.\overline{6} \approx 10.667 = \frac{32}{3}$