

When I was 5 years old, my mother always told me that happiness was the key to life. When I went to school, they asked me what I wanted to be when I grew up. I wrote down 'happy'. They told me I didn't understand the assignment, and I told them they didn't understand life. —John Lennon

### Strategy for Solving Related Rate Problems


1. Understand the problem. In particular, identify the variable whose rate of change you seek and the variable (or variables) whose rate of change you know.
2. Develop a mathematical model of the problem. Draw a picture (many of these problems involve geometric figures) and label the parts that are important to the problem. Be sure to distinguish constant quantities from variables that change over time. Only constant quantities can be assigned numerical values at the start.
3. Write an equation relating the variable whose rate of change you seek with the variable(s) whose rate of change you know. The formula is often geometric, but it could come from a scientific application.
4. Differentiate both sides of the equation implicitly with respect to time  $t$ . Be sure to follow all the differentiation rules. The Chain Rule will be especially critical, as you will be differentiating with respect to the parameter  $t$ .
5. Substitute values for any quantities that depend on time. Notice that it is only safe to do this after the differentiation step. Substituting too soon "freezes the picture" and makes changeable variables behave like constants, with zero derivatives.
6. Interpret the solution. Translate your mathematical result into the problem setting (with appropriate units) and decide whether the result makes sense.

pgs 255-  
256  
3, 9, 11, 13, 17,  
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Hmwk #1  
Answers

③ a)  $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$       b)  $\frac{dV}{dt} = 2\pi r h \frac{dr}{dt}$

c)  $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r h \frac{dr}{dt}$

⑨   $\frac{dl}{dt} = -2 \text{ cm/sec}$        $\frac{dw}{dt} = 2 \text{ cm/sec}$

a)  $A = l \cdot w$        $\frac{dA}{dt} = w \frac{dl}{dt} + l \frac{dw}{dt} = (5)(-2) + (12)(2) = 14 \frac{\text{cm}^2}{\text{sec}}$

b)  $P = 2l + 2w$        $\frac{dP}{dt} = 2 \frac{dl}{dt} + 2 \frac{dw}{dt} = 2(-2) + 2(2) = 0 \text{ cm/sec}$

c)  $C^2 = l^2 + w^2$        $2C \frac{dC}{dt} = 2l \frac{dl}{dt} + 2w \frac{dw}{dt}$

$C^2 = 12^2 + 5^2$        $2(13) \frac{dC}{dt} = 2(12)(-2) + 2(5)(2)$        $\frac{dC}{dt} = \boxed{\frac{-14}{13} \text{ cm/sec}}$

$C = 13$

(11)  $\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$   
 $V = \frac{4}{3}\pi r^3$

Find  $\frac{dr}{dt}$  when  $r = 5$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$100\pi = 4\pi(25) \frac{dr}{dt}$

$1 \text{ ft}/\text{min} = \frac{dr}{dt}$

b)  $A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(5)(1) = 40\pi \text{ ft}^2/\text{min}$

(13) when  $s = 10$   $\frac{ds}{dt} = 300 \text{ mph}$  Find  $\frac{dx}{dt}$

$7^2 + x^2 = s^2$   
 $7^2 + x^2 = 10^2$   
 $0 + 2x \frac{dx}{dt} = 2s \frac{ds}{dt}$

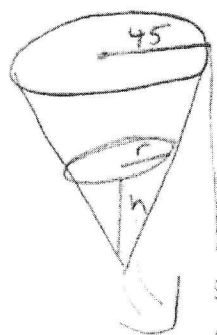
$x = \sqrt{51}$

$2(\sqrt{51}) \frac{dx}{dt} = 2(10)(300)$

$\frac{dx}{dt} = \frac{6,000}{2\sqrt{51}} = \frac{3000}{\sqrt{51}} \text{ mph}$

$42.0.084 \text{ mph}$

(17)



Similar  $\frac{r}{45} = \frac{h}{6}$

$6r = 45h$   
 $r = \frac{45h}{6} = \frac{15h}{2}$

$\frac{dV}{dt} = -50 \text{ m}^3/\text{min}$

Find  $\frac{dh}{dt}$  when  $h = 5$

Replace  $r = \frac{15h}{2}$

$V = \frac{1}{3}\pi r^2 h$   
 $V = \frac{1}{3}\pi \left(\frac{15h}{2}\right)^2 h$

$V = \frac{75}{4}\pi h^3$

$\frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi(5)^2 \frac{dh}{dt}$

$-\frac{8}{225\pi} \text{ m/min} = \frac{dh}{dt}$

or  $-\frac{32}{9\pi} \text{ cm/min}$

b)  $h = \frac{6r}{45} = \frac{2r}{15}$

When  $h = 5$   
 $r = \frac{15(5)}{2} = \frac{75}{2}$

$V = \frac{1}{3}\pi r^2 \cdot \frac{2r}{15} = \frac{2\pi}{45} r^3$

Replace

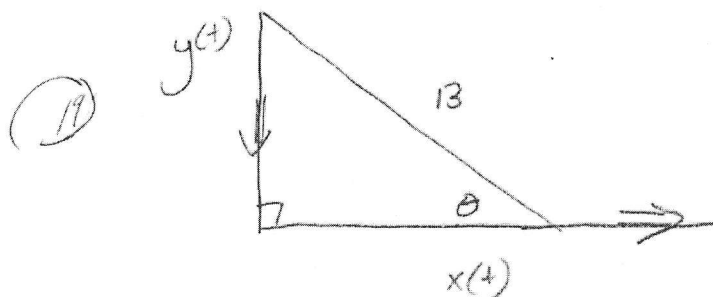
$h \text{ w/ } \frac{2r}{15}$   
 $\frac{dV}{dt} = \frac{6\pi}{45} r^2 \frac{dr}{dt}$

$-50 = \frac{6\pi}{45} \left(\frac{75}{2}\right)^2 \frac{dr}{dt}$

$-0.08488 \text{ m/min} = \frac{dr}{dt}$

or  $-8.488 \text{ cm/min}$

$\times \frac{100 \text{ cm}}{1 \text{ m}}$  to change to cm



a)  $x^2 + y^2 = 13^2$  Find  $\frac{dy}{dt}$  when  $x=12$  and  $\frac{dx}{dt} = 5$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$24(5) + 10 \frac{dy}{dt} = 0$$

$$12^2 + y^2 = 13^2 \rightarrow y = 5$$

$$\frac{dy}{dt} = \frac{-120}{10} = -12 \frac{\text{ft}}{\text{sec}}$$

b)  $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}y \frac{dx}{dt} + \frac{1}{2}x \frac{dy}{dt}$$

$$= \frac{1}{2}(5)(5) + \frac{1}{2}(12)(-12)$$

$$= \boxed{-\frac{119}{2} \text{ ft}^2/\text{sec}}$$

c)  $\sin \theta = \frac{y}{13}$

$$13 \sin \theta = y$$

$$13 \cos \theta \frac{d\theta}{dt} = \frac{dy}{dt}$$

$$13 \left( \frac{12}{13} \right) \frac{d\theta}{dt} = (-12)$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$$

$$\frac{d\theta}{dt} = \boxed{-1 \text{ radian/sec}}$$