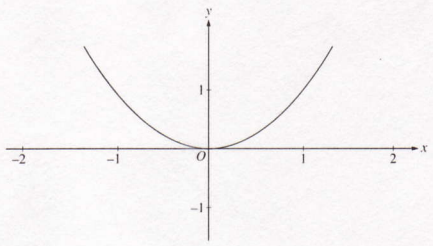
Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

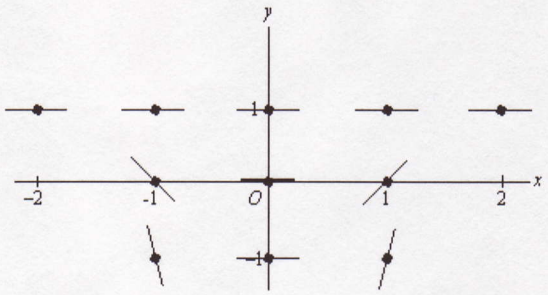
(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).



(a)



2 { 1 : zero slope at 7 points with $y = 1$ and $x = 0$; 1 : negative slope at $(-1,0)$ and $(-1,-1)$, positive slope at $(1,0)$ and $(1,-1)$, steeper slope at $y = -1$ than $y = 0$ }

(b) The graph does not have slope 0 where $y = 1$.

– or –

The slope field shown suggests that solutions are asymptotic to $y = 1$ from below, but the graph does not exhibit this behavior.

1 : reason

(c) $\frac{1}{(y-1)^2}\,dy = x\,dx$

$-\frac{1}{y-1} = \frac{1}{2}x^2 + C$

$\frac{1}{2} = 0 + C; \quad C = \frac{1}{2}$

$-\frac{1}{y-1} = \frac{1}{2}(x^2+1); \quad y = 1 - \frac{2}{x^2+1}$

5 { 1 : separates variables; 1 : antiderivatives; 1 : constant of integration; 1 : uses initial condition $f(0) = -1$; 1 : solves for y; 0/1 if y is linear }

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(d) range is $-1 \le y < 1$

1 : answer

0/1 if –1 not in range