15. $\int (x + 3)\,dx$

16. $\int (5 - x)\,dx$

17. $\int (2x - 3x^2)\,dx$

18. $\int (4x^3 + 6x^2 - 1)\,dx$

19. $\int (x^3 + 2)\,dx$

20. $\int (x^3 - 4x + 2)\,dx$

21. $\int (x^{3/2} + 2x + 1)\,dx$

22. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$

23. $\int \sqrt[3]{x^2}\,dx$

24. $\int \left(\sqrt[4]{x^3} + 1\right)dx$

25. $\int \frac{1}{x^2}\,dx$

26. $\int \frac{1}{x^4}\,dx$

27. $\int \frac{x^2 + x + 1}{\sqrt{x}}\,dx$

28. $\int \frac{x^2 + 2x - 3}{x^4}\,dx$

29. $\int (x + 1)(3x - 2)\,dx$

30. $\int (2t^2 - 1)^2\,dt$

31. $\int y^2\sqrt{y}\,dy$

32. $\int (1 + 3t)t^2\,dt$

33. $\int dx$

34. $\int 3\,dt$

In Exercises 35–42, find the indefinite integral and check the result by differentiation.

35. $\int (2\sin x + 3\cos x)\,dx$

36. $\int (t^2 - \sin t)\,dt$

37. $\int (1 - \csc t \cot t)\,dt$

38. $\int (\theta^2 + \sec^2 \theta)\,d\theta$

39. $\int (\sec^2 \theta - \sin \theta)\,d\theta$

40. $\int \sec y(\tan y - \sec y)\,dy$

C H A P T E R 4
Integration

Section 4.1 Antiderivatives and Indefinite Integration

Solutions to Even-Numbered Exercises

2. $\dfrac{d}{dx}\left(x^4 + \dfrac{1}{x} + C\right) = 4x^3 - \dfrac{1}{x^2}$

4. $\dfrac{d}{dx}\left(\dfrac{2(x^2+3)}{3\sqrt{x}} + C\right) = \dfrac{d}{dx}\left(\dfrac{2}{3}x^{3/2} + 2x^{-1/2} + C\right)$

$$= x^{1/2} - x^{-3/2} = \dfrac{x^2-1}{x^{3/2}}$$

6. $\dfrac{dr}{d\theta} = \pi$

$r = \pi\theta + C$

Check: $\dfrac{d}{d\theta}[\pi\theta + C] = \pi$

8. $\dfrac{dy}{dx} = 2x^{-3}$

$y = \dfrac{2x^{-2}}{-2} + C = \dfrac{-1}{x^2} + c$

Check: $\dfrac{d}{dx}\left[\dfrac{-1}{x^2} + C\right] = 2x^{-3}$

	Given	*Rewrite*	*Integrate*	*Simplify*
10.	$\displaystyle\int \dfrac{1}{x^2}\,dx$	$\displaystyle\int x^{-2}\,dx$	$\dfrac{x^{-1}}{-1} + C$	$-\dfrac{1}{x} + C$
12.	$\displaystyle\int x(x^2 + 3)\,dx$	$\displaystyle\int (x^3 + 3x)\,dx$	$\dfrac{x^4}{4} + 3\left(\dfrac{x^2}{2}\right) + C$	$\dfrac{1}{4}x^4 + \dfrac{3}{2}x^2 + C$
14.	$\displaystyle\int \dfrac{1}{(3x)^2}dx$	$\dfrac{1}{9}\displaystyle\int x^{-2}dx$	$\dfrac{1}{9}\left(\dfrac{x^{-1}}{-1}\right) + C$	$\dfrac{-1}{9x} + C$

16. $\displaystyle\int (5 - x)\,dx = 5x - \dfrac{x^2}{2} + C$

Check: $\dfrac{d}{dx}\left[5x - \dfrac{x^2}{2} + C\right] = 5 - x$

18. $\displaystyle\int (4x^3 + 6x^2 - 1)\,dx = x^4 + 2x^3 - x + C$

Check: $\dfrac{d}{dx}[x^4 + 2x^3 - x + C] = 4x^3 + 6x^2 - 1$

20. $\displaystyle\int (x^3 - 4x + 2)\,dx = \dfrac{x^4}{4} - 2x^2 + 2x + C$

Check: $\dfrac{d}{dx}\left[\dfrac{x^4}{4} - 2x^2 + 2x + C\right] = x^3 - 4x + 2$

22. $\displaystyle\int \left(\sqrt{x} + \dfrac{1}{2\sqrt{x}}\right)dx = \int \left(x^{1/2} + \dfrac{1}{2}x^{-1/2}\right)dx = \dfrac{x^{3/2}}{3/2} + \dfrac{1}{2}\left(\dfrac{x^{1/2}}{1/2}\right) + C = \dfrac{2}{3}x^{3/2} + x^{1/2} + C$

Check: $\dfrac{d}{dx}\left(\dfrac{2}{3}x^{3/2} + x^{1/2} + C\right) = x^{1/2} + \dfrac{1}{2}x^{-1/2} = \sqrt{x} + \dfrac{1}{2\sqrt{x}}$

24. $\int \left(\sqrt[4]{x^3} + 1 \right) dx = \int (x^{3/4} + 1)\, dx = \frac{4}{7}x^{7/4} + x + C$

Check: $\frac{d}{dx}\left(\frac{4}{7}x^{7/4} + x + C \right) = x^{3/4} + 1 = \sqrt[4]{x^3} + 1$

26. $\int \frac{1}{x^4}\, dx = \int x^{-4}\, dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3x^3} + C$

Check: $\frac{d}{dx}\left(-\frac{1}{3x^3} + C \right) = \frac{1}{x^4}$

28. $\int \frac{x^2 + 2x - 3}{x^4}\, dx = \int (x^{-2} + 2x^{-3} - 3x^{-4})\, dx$

$= \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + C$

$= \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

Check: $\frac{d}{dx}\left[-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C \right] = x^{-2} + 2x^{-3} - 3x^{-4}$

$= \frac{x^2 + 2x - 3}{x^4}$

30. $\int (2t^2 - 1)^2\, dt = \int (4t^4 - 4t^2 + 1)\, dt$

$= \frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C$

Check: $\frac{d}{dt}\left(\frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C \right) = 4t^4 - 4t^2 + 1$

$= (2t^2 - 1)^2$

32. $\int (1 + 3t)t^2\, dt = \int (t^2 + 3t^3)\, dt = \frac{1}{3}t^3 + \frac{3}{4}t^4 + C$

Check: $\frac{d}{dt}\left(\frac{1}{3}t^3 + \frac{3}{4}t^4 + C \right) = t^2 + 3t^3 = (1 + 3t)t^2$

34. $\int 3\, dt = 3t + C$

Check: $\frac{d}{dt}(3t + C) = 3$

36. $\int (t^2 - \sin t)\, dt = \frac{1}{3}t^3 + \cos t + C$

Check: $\frac{d}{dt}\left(\frac{1}{3}t^3 + \cos t + C \right) = t^2 - \sin t$

38. $\int (\theta^2 + \sec^2 \theta)\, d\theta = \frac{1}{3}\theta^3 + \tan \theta + C$

Check: $\frac{d}{d\theta}\left(\frac{1}{3}\theta^3 + \tan \theta + C \right) = \theta^2 + \sec^2 \theta$

40. $\int \sec y(\tan y - \sec y)\, dy = \int (\sec y \tan y - \sec^2 y)\, dy$

$= \sec y - \tan y + C$

Check: $\frac{d}{dy}(\sec y - \tan y + C) = \sec y \tan y - \sec^2 y$
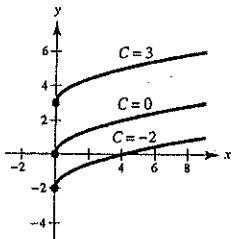
$= \sec y(\tan y - \sec y)$

42. $\int \frac{\cos x}{1 - \cos^2 x}\, dx = \int \frac{\cos x}{\sin^2 x}\, dx = \int \left(\frac{1}{\sin x} \right)\left(\frac{\cos x}{\sin x} \right) dx$

$= \int \csc x \cot x\, dx = -\csc x + C$

Check: $\frac{d}{dx}[-\csc x + C] = \csc x \cot x + \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

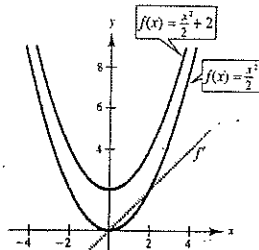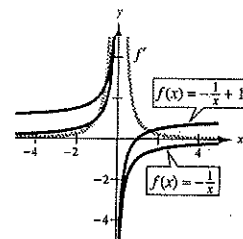$= \frac{\cos x}{1 - \cos^2 x}$

44. $f(x) = \sqrt{x}$

46. $f'(x) = x$

$f(x) = \frac{x^2}{2} + C$

48. $f'(x) = \frac{1}{x^2}$

$f(x) = -\frac{1}{x} + C$

C H A P T E R   4
Integration

Section 4.1   Antiderivatives and Indefinite Integration

Solutions to Odd-Numbered Exercises

1. $\dfrac{d}{dx}\left(\dfrac{3}{x^3} + C\right) = \dfrac{d}{dx}(3x^{-3} + C) = -9x^{-4} = \dfrac{-9}{x^4}$

3. $\dfrac{d}{dx}\left(\dfrac{1}{3}x^3 - 4x + C\right) = x^2 - 4 = (x - 2)(x + 2)$

5. $\dfrac{dy}{dt} = 3t^2$

$y = t^3 + C$

Check: $\dfrac{d}{dt}[t^3 + C] = 3t^2$

7. $\dfrac{dy}{dx} = x^{3/2}$

$y = \dfrac{2}{5}x^{5/2} + C$

Check: $\dfrac{d}{dx}\left[\dfrac{2}{5}x^{5/2} + C\right] = x^{3/2}$

	Given	*Rewrite*	*Integrate*	*Simplify*
9.	$\displaystyle\int \sqrt[3]{x}\,dx$	$\displaystyle\int x^{1/3}\,dx$	$\dfrac{x^{4/3}}{4/3} + C$	$\dfrac{3}{4}x^{4/3} + C$
11.	$\displaystyle\int \dfrac{1}{x\sqrt{x}}\,dx$	$\displaystyle\int x^{-3/2}\,dx$	$\dfrac{x^{-1/2}}{-1/2} + C$	$-\dfrac{2}{\sqrt{x}} + C$
13.	$\displaystyle\int \dfrac{1}{2x^3}\,dx$	$\dfrac{1}{2}\displaystyle\int x^{-3}\,dx$	$\dfrac{1}{2}\left(\dfrac{x^{-2}}{-2}\right) + C$	$-\dfrac{1}{4x^2} + C$

15. $\displaystyle\int (x + 3)\,dx = \dfrac{x^2}{2} + 3x + C$

Check: $\dfrac{d}{dx}\left[\dfrac{x^2}{2} + 3x + C\right] = x + 3$

17. $\displaystyle\int (2x - 3x^2)\,dx = x^2 - x^3 + C$

Check: $\dfrac{d}{dx}[x^2 - x^3 + C] = 2x - 3x^2$

19. $\displaystyle\int (x^3 + 2)\,dx = \dfrac{1}{4}x^4 + 2x + C$

Check: $\dfrac{d}{dx}\left(\dfrac{1}{4}x^4 + 2x + C\right) = x^3 + 2$

21. $\displaystyle\int (x^{3/2} + 2x + 1)\,dx = \dfrac{2}{5}x^{5/2} + x^2 + x + C$

Check: $\dfrac{d}{dx}\left(\dfrac{2}{5}x^{5/2} + x^2 + x + C\right) = x^{3/2} + 2x + 1$

23. $\displaystyle\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx = \dfrac{x^{5/3}}{5/3} + C = \dfrac{3}{5}x^{5/3} + C$

Check: $\dfrac{d}{dx}\left(\dfrac{3}{5}x^{5/3} + C\right) = x^{2/3} = \sqrt[3]{x^2}$

25. $\displaystyle\int \dfrac{1}{x^3}\,dx = \int x^{-3}\,dx = \dfrac{x^{-2}}{-2} + C = -\dfrac{1}{2x^2} + C$

Check: $\dfrac{d}{dx}\left(-\dfrac{1}{2x^2} + C\right) = \dfrac{1}{x^3}$

27. $\displaystyle\int \frac{x^2 + x + 1}{\sqrt{x}}\,dx = \int (x^{3/2} + x^{1/2} + x^{-1/2})\,dx = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C = \frac{2}{15}x^{1/2}(3x^2 + 5x + 15) + C$

Check: $\dfrac{d}{dx}\left(\dfrac{2}{5}x^{5/2} + \dfrac{2}{3}x^{3/2} + 2x^{1/2} + C\right) = x^{3/2} + x^{1/2} + x^{-1/2} = \dfrac{x^2 + x + 1}{\sqrt{x}}$

29. $\displaystyle\int (x + 1)(3x - 2)\,dx = \int (3x^2 + x - 2)\,dx$

$\qquad\qquad\qquad\qquad = x^3 + \dfrac{1}{2}x^2 - 2x + C$

Check: $\dfrac{d}{dx}\left(x^3 + \dfrac{1}{2}x^2 - 2x + C\right) = 3x^2 + x - 2$

$\qquad\qquad\qquad\qquad\qquad = (x + 1)(3x - 2)$

31. $\displaystyle\int y^2\sqrt{y}\,dy = \int y^{5/2}\,dy = \frac{2}{7}y^{7/2} + C$

Check: $\dfrac{d}{dy}\left(\dfrac{2}{7}y^{7/2} + C\right) = y^{5/2} = y^2\sqrt{y}$

33. $\displaystyle\int dx = \int 1\,dx = x + C$

Check: $\dfrac{d}{dx}(x + C) = 1$

35. $\displaystyle\int (2\sin x + 3\cos x)\,dx = -2\cos x + 3\sin x + C$

Check: $\dfrac{d}{dx}(-2\cos x + 3\sin x + C) = 2\sin x + 3\cos x$

37. $\displaystyle\int (1 - \csc t\cot t)\,dt = t + \csc t + C$

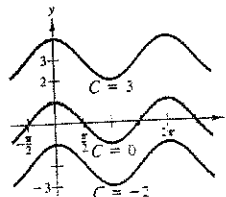Check: $\dfrac{d}{dt}(t + \csc t + C) = 1 - \csc t\cot t$

39. $\displaystyle\int (\sec^2 \theta - \sin \theta)\,d\theta = \tan \theta + \cos \theta + C$

Check: $\dfrac{d}{d\theta}(\tan \theta + \cos \theta + C) = \sec^2 \theta - \sin \theta$

41. $\displaystyle\int (\tan^2 y + 1)\,dy = \int \sec^2 y\,dy = \tan y + C$
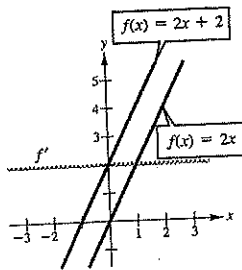
Check: $\dfrac{d}{dy}(\tan y + C) = \sec^2 y = \tan^2 y + 1$
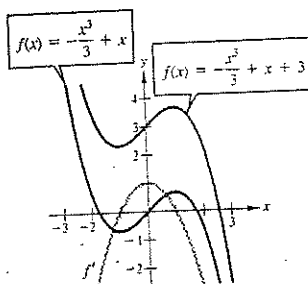
43. $f(x) = \cos x$

45. $f'(x) = 2$

$\qquad f(x) = 2x + C$

Answers will vary.

47. $f'(x) = 1 - x^2$

$\qquad f(x) = x - \dfrac{x^3}{3} + C$

Answers will vary.

49. $\dfrac{dy}{dx} = 2x - 1,\ (1, 1)$

$y = \displaystyle\int (2x - 1)\,dx = x^2 - x + C$

$1 = (1)^2 - (1) + C \Rightarrow C = 1$

$y = x^2 - x + 1$