AP Calculus AB **Linearization/ Local Linearity** **Section 5.5**

AP Questions:

1998 AP Calculus AB Scoring Guidelines

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

1995 AB3

Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\dfrac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(4, -1)$.

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

(e) Solve the equation found in part (d) for the value of k.

AB?

Let f be the function defined by $f(x) = (1 + \tan x)^{\frac{3}{2}}$ for $-\dfrac{\pi}{4} < x < \dfrac{\pi}{2}$.

(a) Write an equation for the line tangent to the graph of f at the point where $x = 0$.

(b) Using the equation found in part (a), approximate $f(0.02)$.

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

Multiple Choice:

The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

KEY

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

a) $\frac{3(1)+1}{2(4)} = \frac{4}{8} = 1/2$ b) $y-4 = 1/2(x-1)$ $L(x) = \frac{1}{2}x + 3.5$

$L(1.2) = \boxed{4.1}$

1995 AB3

Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$. $-16x + 5y + 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

(a) Find $\frac{dy}{dx}$. $= \frac{16x - 5y}{5x + 3y^2}$

(b) Write an equation for the line tangent to the curve at the point $(4, -1)$. $m = \frac{16(4)+5}{20+3} = 3$ $y+1 = 3(x-4)$ $\boxed{y = 3x - 13}$

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k. $k = 3(4.2) - 13 = \boxed{-2/5} = -.4$

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve. $-8(4.2)^2 + 5(4.2)y + y^3 = -149$

Graphing calculator

(e) Solve the equation found in part (d) for the value of k. $-141.12 + 21y + y^3 = -149$

$y^3 + 21y + 7.88 = 0$

$\boxed{y \approx -.373}$

AB?

Let f be the function defined by $f(x) = (1 + \tan x)^{\frac{3}{2}}$ for $-\frac{\pi}{4} < x < \frac{\pi}{2}$.

(a) Write an equation for the line tangent to the graph of f at the point where $x = 0$. $(0, f(0))$ $(0, 1)$

(b) Using the equation found in part (a), approximate $f(0.02)$.

a) $f' = \frac{3}{2}(1+\tan x)^{1/2}(\sec^2 x)$ $y - 1 = \frac{3}{2}(x - 0)$

$f'(0) = \frac{3}{2}(1+0)^{1/2}(\sec 0)^2$ $y = \frac{3}{2}x + 1$

$= \frac{3}{2}(1) = 3/2$

b) $L(x) = \frac{3}{2}x + 1$

$L(.02) = \frac{3}{2}(.02) + 1 = 1.03$

$f(.02) \approx 1.03$

Question 6

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Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

$\rightarrow$ concave up $\leftarrow$ tangent line

Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

$(1,-4) \quad f'(1) = 5$

$y+4 = 5(x-1)$

$\boxed{y = 5x - 9}$

$f(1.2) \approx L(1.2) = -3$

★ The graph is concave up since $f''>0$ at $x=1.2$ so the tangent line is below the graph making the approximation less than the actual value. ★

Multiple Choice:

The approximate value of $y=\sqrt{4+\sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

point $(0,2)$

$y' = \frac{1}{2}(4+\sin x)^{-1/2}(\cos x)$

$y'(0) = \frac{1}{2}(4)^{-1/2} \cdot 1$

$\frac{1}{2} \cdot \frac{1}{\sqrt{4}} = 1/4 = m_T$

$y-2 = \frac{1}{4}(x-0)$

$L(x) = \frac{1}{4}x + 2$

$L(.12) = \frac{1}{4}(.12) + 2 = \boxed{2.03}$ B