Answer Key #1

AP Calculus AB **Review Sec. 5.1-5.3**

Quiz - Applications of Derivatives -Topics

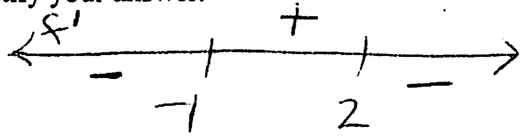
- Tangent Line Approximations
- Extreme Value Theorem
- Critical values
- Finding Absolute Min and Max
- Finding Relative Min and Max (First and Second Derivative Tests)
- Finding where a function is increasing or decreasing
- Concavity
- Inflection points
- Mean Value Theorem
- Rolle's Theorem

Do Now

1. Let $f(x) = k+12x+3x^2-2x^3$ where k is a constant

a) On what intervals is the function increasing. Justify your answer.

$$f'(x) = \frac{12+6x-6x^2}{-6} = \frac{0}{-6}$$



$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2, -1$$

f is increasing on $(-1,2)$ b/c $f'(x)>0$.

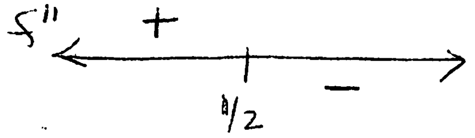
b) If the relative maximum value of f is 30, then what is the value of k

There is a rel. max at $x=2$ b/c f' changes from + to −.

$$f(2) = k+12(2)+3(2)^2-2(2)^3 = 30$$
$$k+24+12-16 = 30$$
$$k+20 = 30 \qquad \boxed{k=10}$$

c) Find the interval where the function is concave up. Justify.

$$f''(x) = 6-12x = 0$$
$$6 = 12x$$
$$x = \frac{1}{2}$$



f is concave up on $(-\infty, 1/2)$ b/c $f''(x) > 0$

2. Find the inflection point(s) and describe the concavity of the function.
$f(x) = 3x^4$

$f'(x) = 12x^3$

$f''(x) = 36x^2 = 0$ $\quad x = 0$

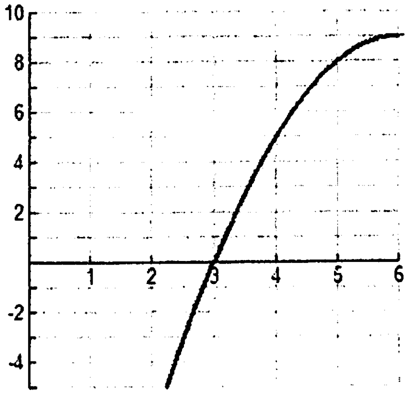
f'' + +

0

There are no points of inflection b/c the concavity never changes.

f is concave up $(-\infty, 0) \cup (0, \infty)$ b/c $f''(x) > 0$.

3. Given the graph of a function f at the right, what is...?
Answer with a 0, + or -



a) f (3) = 0

b) f'(3) = + inc.

c) f" (3) = — concave down OR decreasing slope

d) f (6) = + 9

b) f'(6) = 0 hor. tan

c) f" (6) = — or 0 depending on how graph continues

concave → down

inflection

4. Let $f(x) = \frac{2x^2}{x^2+4}$ on. $[-5,5]$.

(a) On what interval(s) is f increasing? Show the work that leads to your conclusion.

(b) On what interval(s) is f concave up? Justify your conclusion.

(c) At what values of x does f have an inflection point? Justify your conclusion.

5. Use the tangent line approximation at x=1 to estimate f(1.1) for the function $f(x) = 2x^4 + 3x$.

6. Find all critical values of the given function: $f(x) = \ln(9x^2 - 1)$

7. Find the relative max, min and absolute max, min of $f(x) = x^3 - 2x^2 + 3$ on the interval [-2,2]

8. Find the absolute extrema of the $f(x) = e^{-x} \sin x$ on the interval $[0, 2\pi]$.

9. $f(x) = e^x$ on [0,1]. Find all x-values for which MVT applies.

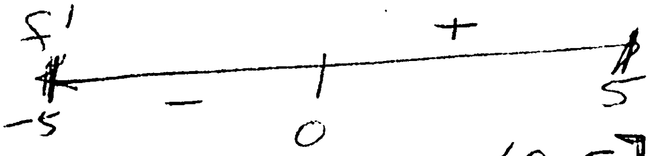
10. $f(x) = \ln x$ on the interval [½,2]. Find all x-values for which MVT applies.

4 $f(x) = \frac{2x^2}{x^2+4}$ $[-5,5]$

a) $f'(x) = \frac{4x(x^2+4) - 2x(2x^2)}{(x^2+4)^2} = \frac{4x^3+16x-4x^3}{(x^2+4)^2} = \frac{16x}{(x^2+4)^2}$

$f'(x)=0$
$16x=0$
$x=0$

$f'(x)=DNE$
$x^2+4=0$
$x^2=-4$
imaginary
None

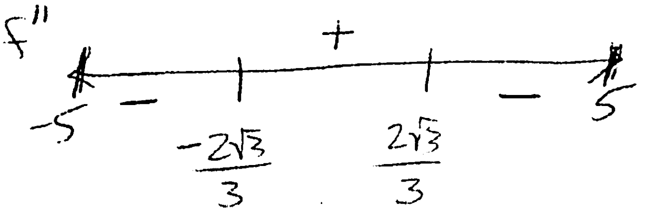


a) f is increasing on $(0,5]$
b/c $f'(x) > 0$.

b) $f''(x) = \frac{16(x^2+4)^2 - 2(x^2+4)(2x)(16x)}{(x^2+4)^4} = \frac{\cancel{(x^2+4)}(16(x^2+4)-2(2x)(16x))}{(x^2+4)^{\cancel{4}3}}$

$f''(x) = \frac{16x^2+64-64x^2}{(x^2+4)^3} = \frac{\overset{16(4-3x^2)}{64-48x^2}}{(x^2+4)^3}$

$f''=0$
$64-48x^2=0$
$x^2=\frac{64}{48}=\frac{4}{3}$
$x=\pm\sqrt{\frac{4}{3}}=\pm\frac{2}{\sqrt{3}}=\frac{\pm2\sqrt{3}}{3}$



$f''=DNE$ $x^2+4=0$
imag:
Never

b) f is concave up on $\left(\frac{-2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$
b/c $f''(x) > 0$

c) There are inflection points at $x=\frac{-2\sqrt{3}}{3}$ and $x=\frac{2\sqrt{3}}{3}$
b/c $f''(x)$ changes sign there. (OR f changes concavity)

⑤ $f(1.1) \approx f(1) + f'(1)\Delta x$
$f(1) + f'(1)(1.1-1)$
$5 + 11(0.1)$
$5+1.1 = \boxed{6.1}$

$f(1) = 2(1)^4+3(1)$
$f(1) = 5$

$f'(x) = 8x^3+3$
$f'(1) = 8(1)^3+3 = 11$

⑥ $f'(x) = \dfrac{18x}{9x^2-1}$

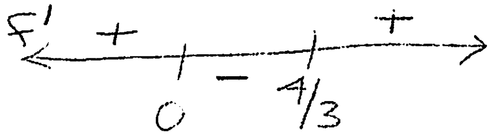
$f'(x) = 0$
$18x = 0$
$\boxed{x = 0}$

$f'(x) = \text{DNE}$
$9x^2 - 1 = 0$
$(3x-1)(3x+1) = 0$
$\boxed{x = \pm \frac{1}{3}}$

⑦ $f(x) = x^3 - 2x^2 + 3 \quad [-2, 2]$

$f'(x) = 3x^2 - 4x = 0$
$x(3x-4) = 0$
$x = 0, \frac{4}{3}$

Relative



There is a relative max of 3 at $x = 0$ b/c f' changes from pos to neg.
There is a relative min of $\frac{49}{27} \approx 1.815$ at $x = 4/3$ b/c f' changes from neg. to pos

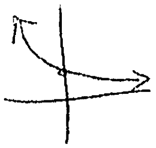
Absolute

x	$f(x) = x^3 - 2x^2 + 3$
-2	-13
0	3
$4/3$	$\frac{49}{27} \approx 1.815$
2	3

There is an absolute min of -13 at $x = -2$.
There is an absolute max of 3 at $x = 0, 2$

⑧ $f(x) = e^{-x} \sin x$ on $[0, 2\pi]$

Product Rule
$f'(x) = -e^{-x}(\sin x) + e^{-x}(\cos x)$
$f'(x) = e^{-x}(-\sin x + \cos x) = 0$



$e^{-x} = 0$
Never
Always pos.

$-\sin x + \cos x = 0$
$\cos x = \sin x$
$x = \frac{\pi}{4}, \frac{5\pi}{4}$
Q1 Q3
crit. pts.

x	$f(x) = e^{-x} \sin x$
0	0
$\pi/4$	$e^{-\pi/4} \cdot \frac{\sqrt{2}}{2} \approx 0.322$
$5\pi/4$	$e^{-5\pi/4}\left(\frac{-\sqrt{2}}{2}\right) \approx -0.01$
2π	0

Abs. Min ≈ -0.014 @ $x = \frac{5\pi}{4}$

Abs. Max ≈ 0.322 @ $x = \frac{\pi}{4}$

9 $f(x)=e^x$ $[0,1]$ MVT

f is cont. on $[0,1]$ and dif. on $(0,1)$ so by MVT there must exist a c on $(0,1)$ such that $f'(c)=\frac{f(1)-f(0)}{1-0}$

$$f'(c)=\frac{e^1-e^0}{1-0}=\frac{e-1}{1}=e-1$$

$f'(x)=e^x$ $\quad f'(c)\underset{\ln}{=}e^c\underset{\ln}{=}(e-1)$

$\boxed{c=\ln(e-1)}\approx 0.541$ ✓

⑩ $f(x)=\ln x$ on $[1/2,2]$ MVT

Since f is cont. on $[1/2,2]$ and dif. on $(1/2,2)$ by MVT there must exist a c on $(\frac{1}{2},2)$ such that $f'(c)=\frac{f(2)-f(1/2)}{2-1/2}$

$$f'(c)=\frac{\ln 2-\ln(1/2)}{\frac{3}{2}} \leftarrow \text{log rules} = \frac{\ln\left(\frac{2}{1/2}\right)}{\frac{3}{2}}=\frac{2}{3}\ln 4$$

$f'(x)=\frac{1}{x}$ $\quad f'(c)=\frac{1}{c}=\frac{2\ln 4}{3}$

$3=c(2\ln 4)$ $\quad \boxed{c=\frac{3}{2\ln 4}}$ ✓

≈ 1.082

⑪ $f(x)=x^3-7x+6=0$

$q=\pm1$ $\quad p=\pm1,\pm2,\pm3,\pm6$

a) Rational Root Thm.

$\frac{p}{q}=\pm1,\pm2,\pm3,\pm6$ ← possible rational roots

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$ ✓

$(x-1)(x^2+x-6)=0$

$(x-1)(x+3)(x-2)=0$

$\boxed{x=1,-3,2}$

⑪ b) $f(-1) = (-1)^3 - 7(-1) + 6$ $\quad (-1, 12)$

$-1 + 7 + 6 = 12$

$f'(x) = 3x^2 - 7 \quad f'(-1) = 3(-1)^2 - 7 = -4$

$$\boxed{y - 12 = -4(x+1)}$$

c) MVT on $[1,3]$

Since f is cont. and dif. on $[1,3]$ by MVT there must be a c on $(1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3-1} = \frac{12 - 0}{2} = 6$$

$f'(x) = 3x^2 - 7 \quad f'(c) = 3c^2 - 7 = 6$

$3c^2 = 13$

$c^2 = \frac{13}{3} \quad c = \pm\sqrt{\frac{13}{3}}$

↑ neg. not in interval

$$\boxed{c = \frac{\sqrt{39}}{3} \approx 2.082}$$

⑫ $f(x) = (1 + \tan x)^{3/2} \quad (-\pi/4, \pi/2)$

a) $f(0) = (1 + \tan 0)^{3/2} = 1^{3/2} = 1 \quad (0, 1)$

$f'(x) = \frac{3}{2}(1 + \tan x)^{1/2}(\sec^2 x)$

Chain Rule

$f'(0) = \frac{3}{2}\sqrt{1 + \tan 0}\left(\frac{1}{(\cos 0)^2}\right)$

$\frac{3}{2}\sqrt{1}(1) = \frac{3}{2}$

$$\boxed{\begin{gathered} y - 1 = \frac{3}{2}(x - 0) \\ y = \frac{3}{2}x + 1 \end{gathered}}$$

b) $f(0.02) \approx \frac{3}{2}(.02) + 1 \approx 3(.01) + 1 = \boxed{1.03}$

13) $f(1) = -4$

$f'(1) = 5$

$\boxed{y+4=5(x-1)} \rightarrow y=5(x-1)-4$

$f(1.2) \approx 5(1.2-1)-4 = \boxed{-3}$

$5(0.2) - 4$

Since it was given that $f''(x) > 0$ on $[-1.5, 1.5.]$ the graph of f is concave up. so the tangent line will underestimate the actual value. So the approximate is less than the actual value of $f(1.2)$

⑭ $y(0) = \sqrt{4+\sin 0} = \sqrt{4} = 2 \quad (0,2)$

$$y' = \frac{1}{2}(4+\sin x)^{-1/2} \cdot \cos x = \frac{\cos x}{2\sqrt{4+\sin x}}$$

$$y'(0) = \frac{\cos 0}{2\sqrt{4+\sin 0}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

Tan line $\quad y-2 = \frac{1}{4}(x-0)$

$$y = \frac{1}{4}x+2$$

$$y(0.12) \approx \frac{1}{4}(0.12)+2 = .03+2 = \boxed{2.03}$$

$\boxed{B}$