Related Rates- HW- worked out solutions on my webpage

1. A spherical balloon is inflated with gas at a rate of 500 cubic centimeters per minute. What is the rate of change of the radius when the radius is 30 centimeters? $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 500 \text{ cm}^3/\text{min}$

$r = 30$ cm

$\frac{dr}{dt} = ?$

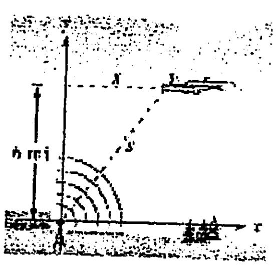
$\frac{dV}{dt} = \frac{4}{3}\pi(3)r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$500 = 4\pi(30)^2\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{36\pi}$ cm/min

2 An airplane is flying at an altitude of 6 miles and passes directly over a radar antenna (see figure below).When the plane is 10 miles away (s=10) the radar detects the distance *s* is changing at a rate of 240 miles per hour. What is the speed of the plane? (you need the Pythagorean thm)



$(6)^2 + x^2 = s^2$

$0 + 2x\frac{dx}{dt} = 2s\frac{ds}{dt}$

$2(8)\frac{dx}{dt} = 2(10)(240)$

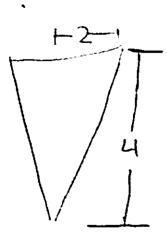
$\frac{dx}{dt} = 300$ mph

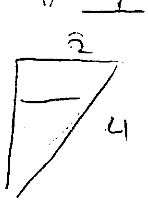
$s = 10$ mi

$\frac{ds}{dt} = 240$ mph

$x = 8$

3. A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of 2m³/min, find the rate at which the water level is rising when the water is 3m deep. The volume of a circular cone with radius 4 and height h is given by $V = \frac{1}{3}\pi r^2 h$





$\frac{2}{4} = \frac{r}{h}$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{12}\pi(3h^2)\frac{dh}{dt}$

$2 = \frac{1}{4}\pi(3)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{9\pi}$ m/min

$\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$

$h = 3$m

$\frac{dh}{dt} = ?$

RELATED RATES- CW-day 1

1. As a ball in the shape of a sphere is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1.5 inches. $V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\ in^3/sec$

$\frac{dr}{dt} = ?$

$r = 1.5\ in$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$

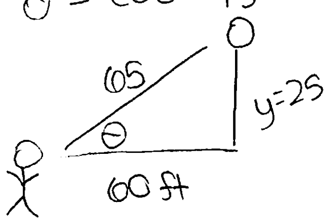
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4 = 4\pi (1.5)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{4}{9\pi}\ in/sec$

2. A balloon rises at the rate of 8 feet per second from a point on the ground 60 ft from an observer. Find the rate of change of the angle of elevation when the balloon is 25 ft above the ground.

$\theta = \cos^{-1}\frac{12}{13}$



$\tan\theta = \frac{y}{60}$

$\cos\theta = \frac{60}{65}$

$\frac{1}{\cos\theta} = \frac{65}{60} = \frac{13}{12}$

$\tan\theta = \frac{1}{60}(y)$

$\frac{d\theta}{dt}\sec^2\theta = \left(\frac{1}{60}\right)\left(\frac{dy}{dt}\right)$

$\frac{d\theta}{dt}\left(\frac{13}{12}\right)^2 = \frac{1}{60}(8)$

$\frac{d\theta}{dt} = \frac{8}{60}\left(\frac{144}{169}\right) = \frac{96}{845}\ rad/sec$

$\frac{dy}{dt} = 8\ ft/sec$

$y = 25\ ft$

3. A ladder 15 ft long is leaning against a building so that the end is on level ground. The ladder is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second. Find the rate at which the height is changing when the ladder is 9 feet from the building.

Ladder = 15 ft

$\frac{dx}{dt} = \frac{1}{2}\ ft/sec$

$\frac{dy}{dt} = ?$

$\cancel{y}x = 9$

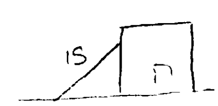
$a^2 + b^2 = c^2$

$x^2 + y^2 = (15)^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(9)(\frac{1}{2}) + 2(12)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-3}{8}\ ft/sec$



$(9)^2 + y^2 = (15)^2$

$y = 12$