103. $x(t) = t^3 - 6t^2 + 9t - 2$

$$x'(t) = 3t^2 - 12t + 9$$
$$= 3(t^2 - 4t + 3)$$
$$= 3(t-3)(t-1)$$

$$\text{Total distance} = \int_0^5 |x'(t)|\,dt$$
$$= \int_0^5 3|(t-3)(t-1)|\,dt$$
$$= 3\int_0^1 (t^2 - 4t + 3)\,dt - 3\int_1^3 (t^2 - 4t + 3)\,dt + 3\int_3^5 (t^2 - 4t + 3)\,dt$$
$$= 4 + 4 + 20$$
$$= 28 \text{ units}$$

105. $$\text{Total distance} = \int_1^4 |x'(t)|\,dt$$
$$= \int_1^4 |v(t)|\,dt$$
$$= \int_1^4 \frac{1}{\sqrt{t}}\,dt$$
$$= 2t^{1/2}\Big]_1^4$$
$$= 2(2-1) = 2 \text{ units}$$

Section 4.5 Integration by Substitution

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (5x^2+1)^2(10x)\,dx$	$5x^2 + 1$	$10x\,dx$
3. $\int \frac{x}{\sqrt{x^2+1}}\,dx$	$x^2 + 1$	$2x\,dx$
5. $\int \tan^2 x \sec^2 x\,dx$	$\tan x$	$\sec^2 x\,dx$

7. $\int (1+2x)^4\, 2\,dx = \frac{(1+2x)^5}{5} + C$

Check: $\frac{d}{dx}\left[\frac{(1+2x)^5}{5} + C\right] = 2(1+2x)^4$

9. $\int (9-x^2)^{1/2}(-2x)\,dx = \frac{(9-x^2)^{3/2}}{3/2} + C = \frac{2}{3}(9-x^2)^{3/2} + C$

Check: $\frac{d}{dx}\left[\frac{2}{3}(9-x^2)^{3/2} + C\right] = \frac{2}{3}\cdot\frac{3}{2}(9-x^2)^{1/2}(-2x) = \sqrt{9-x^2}(-2x)$

11. $\int x^3(x^4+3)^2\,dx = \frac{1}{4}\int (x^4+3)^2(4x^3)\,dx = \frac{1}{4}\frac{(x^4+3)^3}{3} + C = \frac{(x^4+3)^3}{12} + C$

Check: $\frac{d}{dx}\left[\frac{(x^4+3)^3}{12} + C\right] = \frac{3(x^4+3)^2}{12}(4x^3) = (x^4+3)^2(x^3)$

13. $\int x^2(x^3-1)^4\,dx = \frac{1}{3}\int (x^3-1)^4(3x^2)\,dx = \frac{1}{3}\left[\frac{(x^3-1)^5}{5}\right] + C = \frac{(x^3-1)^5}{15} + C$

Check: $\frac{d}{dx}\left[\frac{(x^3-1)^5}{15} + C\right] = \frac{5(x^3-1)^4(3x^2)}{15} = x^2(x^3-1)^4$

15. $\int t\sqrt{t^2+2}\,dt = \frac{1}{2}\int (t^2+2)^{1/2}(2t)\,dt = \frac{1}{2}\frac{(t^2+2)^{3/2}}{3/2} + C = \frac{(t^2+2)^{3/2}}{3} + C$

Check: $\frac{d}{dt}\left[\frac{(t^2+2)^{3/2}}{3} + C\right] = \frac{3/2(t^2+2)^{1/2}(2t)}{3} = (t^2+2)^{1/2}t$

17. $\int 5x(1-x^2)^{1/3}\,dx = -\frac{5}{2}\int (1-x^2)^{1/3}(-2x)\,dx = -\frac{5}{2}\cdot\frac{(1-x^2)^{4/3}}{4/3} + C = -\frac{15}{8}(1-x^2)^{4/3} + C$

Check: $\frac{d}{dx}\left[-\frac{15}{8}(1-x^2)^{4/3} + C\right] = -\frac{15}{8}\cdot\frac{4}{3}(1-x^2)^{1/3}(-2x) = 5x(1-x^2)^{1/3} = 5x\sqrt[3]{1-x^2}$

19. $\int \frac{x}{(1-x^2)^3}\,dx = -\frac{1}{2}\int (1-x^2)^{-3}(-2x)\,dx = -\frac{1}{2}\frac{(1-x^2)^{-2}}{-2} + C = \frac{1}{4(1-x^2)^2} + C$

Check: $\frac{d}{dx}\left[\frac{1}{4(1-x^2)^2} + C\right] = \frac{1}{4}(-2)(1-x^2)^{-3}(-2x) = \frac{x}{(1-x^2)^3}$

21. $\int \frac{x^2}{(1+x^3)^2}\,dx = \frac{1}{3}\int (1+x^3)^{-2}(3x^2)\,dx = \frac{1}{3}\left[\frac{(1+x^3)^{-1}}{-1}\right] + C = -\frac{1}{3(1+x^3)} + C$

Check: $\frac{d}{dx}\left[-\frac{1}{3(1+x^3)} + C\right] = -\frac{1}{3}(-1)(1+x^3)^{-2}(3x^2) = \frac{x^2}{(1+x^3)^2}$

23. $\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int (1-x^2)^{-1/2}(-2x)\,dx = -\frac{1}{2}\frac{(1-x^2)^{1/2}}{1/2} + C = -\sqrt{1-x^2} + C$

Check: $\frac{d}{dx}[-(1-x^2)^{1/2} + C] = -\frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}}$

25. $\int \left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt = -\int \left(1+\frac{1}{t}\right)^3\left(-\frac{1}{t^2}\right)dt = -\frac{[1+(1/t)]^4}{4} + C$

Check: $\frac{d}{dt}\left[-\frac{[1+(1/t)]^4}{4} + C\right] = -\frac{1}{4}(4)\left(1+\frac{1}{t}\right)^3\left(-\frac{1}{t^2}\right) = \frac{1}{t^2}\left(1+\frac{1}{t}\right)^3$

27. $\int \frac{1}{\sqrt{2x}}\,dx = \frac{1}{2}\int (2x)^{-1/2}\,2\,dx = \frac{1}{2}\left[\frac{(2x)^{1/2}}{1/2}\right] + C = \sqrt{2x} + C$

Check: $\frac{d}{dx}[\sqrt{2x} + C] = \frac{1}{2}(2x)^{-1/2}(2) = \frac{1}{\sqrt{2x}}$

29. $\displaystyle\int \frac{x^2+3x+7}{\sqrt{x}}\,dx = \int (x^{3/2}+3x^{1/2}+7x^{-1/2})\,dx = \frac{2}{5}x^{5/2}+2x^{3/2}+14x^{1/2}+C = \frac{2}{5}\sqrt{x}(x^2+5x+35)+C$

Check: $\displaystyle\frac{d}{dx}\left[\frac{2}{5}x^{5/2}+2x^{3/2}+14x^{1/2}+C\right] = \frac{x^2+3x+7}{\sqrt{x}}$

31. $\displaystyle\int t^2\left(t-\frac{2}{t}\right)dt = \int (t^3-2t)\,dt = \frac{1}{4}t^4 - t^2 + C$

Check: $\displaystyle\frac{d}{dt}\left[\frac{1}{4}t^4 - t^2 + C\right] = t^3 - 2t = t^2\left(t-\frac{2}{t}\right)$

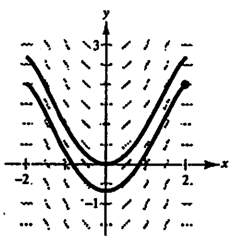
33. $\displaystyle\int (9-y)\sqrt{y}\,dy = \int (9y^{1/2}-y^{3/2})\,dy = 9\left(\frac{2}{3}y^{3/2}\right) - \frac{2}{5}y^{5/2} + C = \frac{2}{5}y^{3/2}(15-y)+C$

Check: $\displaystyle\frac{d}{dy}\left[\frac{2}{5}y^{3/2}(15-y)+C\right] = \frac{d}{dy}\left[6y^{3/2}-\frac{2}{5}y^{5/2}+C\right] = 9y^{1/2}-y^{3/2} = (9-y)\sqrt{y}$

35.
$$\begin{aligned} y &= \int\left[4x + \frac{4x}{\sqrt{16-x^2}}\right]dx \\ &= 4\int x\,dx - 2\int (16-x^2)^{-1/2}(-2x)\,dx \\ &= 4\left(\frac{x^2}{2}\right) - 2\left[\frac{(16-x^2)^{1/2}}{1/2}\right] + C \\ &= 2x^2 - 4\sqrt{16-x^2} + C \end{aligned}$$

37.
$$\begin{aligned} y &= \int \frac{x+1}{(x^2+2x-3)^2}\,dx \\ &= \frac{1}{2}\int (x^2+2x-3)^{-2}(2x+2)\,dx \\ &= \frac{1}{2}\left[\frac{(x^2+2x-3)^{-1}}{-1}\right] + C \\ &= -\frac{1}{2(x^2+2x-3)} + C \end{aligned}$$

39. (a)

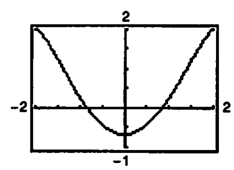


(b) $\displaystyle\frac{dy}{dx} = x\sqrt{4-x^2},\ (2, 2)$

$$\begin{aligned} y &= \int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int (4-x^2)^{1/2}(-2x\,dx) \\ &= -\frac{1}{2}\cdot\frac{2}{3}(4-x^2)^{3/2} + C = -\frac{1}{3}(4-x^2)^{3/2} + C \end{aligned}$$

$(2, 2)$: $2 = -\frac{1}{3}(4-2^2)^{3/2} + C \Rightarrow C = 2$

$$y = -\frac{1}{3}(4-x^2)^{3/2} + 2$$



41. $\displaystyle\int \pi\sin\pi x\,dx = -\cos\pi x + C$

43. $\displaystyle\int \sin 2x\,dx = \frac{1}{2}\int(\sin 2x)(2x)\,dx = -\frac{1}{2}\cos 2x + C$

45. $\displaystyle\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\,d\theta = -\int \cos\frac{1}{\theta}\left(-\frac{1}{\theta^2}\right)d\theta = -\sin\frac{1}{\theta} + C$

47. $\displaystyle\int \sin 2x \cos 2x\, dx = \frac{1}{2}\int (\sin 2x)(2\cos 2x)\, dx = \frac{1}{2}\frac{(\sin 2x)^2}{2} + C = \frac{1}{4}\sin^2 2x + C$ OR

$\displaystyle\int \sin 2x \cos 2x\, dx = -\frac{1}{2}\int (\cos 2x)(-2\sin 2x)\, dx = -\frac{1}{2}\frac{(\cos 2x)^2}{2} + C_1 = -\frac{1}{4}\cos^2 2x + C_1$ OR

$\displaystyle\int \sin 2x \cos 2x\, dx = \frac{1}{2}\int 2\sin 2x \cos 2x\, dx = \frac{1}{2}\int \sin 4x\, dx - \frac{1}{8}\cos 4x + C_2$

49. $\displaystyle\int \tan^4 x \sec^2 x\, dx = \frac{\tan^5 x}{5} + C = \frac{1}{5}\tan^5 x + C$

51. $\displaystyle\int \frac{\csc^2 x}{\cot^3 x}\, dx = -\int (\cot x)^{-3}(-\csc^2 x)\, dx$

$\displaystyle = -\frac{(\cot x)^{-2}}{-2} + C = \frac{1}{2\cot^2 x} + C = \frac{1}{2}\tan^2 x + C = \frac{1}{2}(\sec^2 x - 1) + C = \frac{1}{2}\sec^2 x + C_1$

53. $\displaystyle\int \cot^2 x\, dx = \int (\csc^2 x - 1)\, dx = -\cot x - x + C$

55. $\displaystyle f(x) = \int \cos\frac{x}{2}\, dx = 2\sin\frac{x}{2} + C$

Since $f(0) = 3 = 2\sin 0 + C$, $C = 3$. Thus,

$\displaystyle f(x) = 2\sin\frac{x}{2} + 3.$

57. $u = x + 2$, $x = u - 2$, $dx = du$

$$\int x\sqrt{x+2}\, dx = \int (u-2)\sqrt{u}\, du$$

$$= \int (u^{3/2} - 2u^{1/2})\, du$$

$$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$

$$= \frac{2u^{3/2}}{15}(3u - 10) + C$$

$$= \frac{2}{15}(x+2)^{3/2}[3(x+2) - 10] + C$$

$$= \frac{2}{15}(x+2)^{3/2}(3x - 4) + C$$

59. $u = 1 - x$, $x = 1 - u$, $dx = -du$

$$\int x^2\sqrt{1-x}\, dx = -\int (1-u)^2\sqrt{u}\, du$$

$$= -\int (u^{1/2} - 2u^{3/2} + u^{5/2})\, du$$

$$= -\left(\frac{2}{3}u^{3/2} - \frac{4}{5}u^{5/2} + \frac{2}{7}u^{7/2}\right) + C$$

$$= -\frac{2u^{3/2}}{105}(35 - 42u + 15u^2) + C$$

$$= -\frac{2}{105}(1-x)^{3/2}[35 - 42(1-x) + 15(1-x)^2] + C$$

$$= -\frac{2}{105}(1-x)^{3/2}(15x^2 + 12x + 8) + C$$