Solutions old book

p 101 − 103

(1) A) $m = 0$
b) $m = -3$

(2) A) $m = 4$
B) $m = 1$

(4) A) $>$
B) $<$

14) $\lim_{h \to 0} \dfrac{3(x+h) + 2 - (3x+2)}{h}$

$3x + 3h + 2 - 3x - 2 = \dfrac{3h}{h}$

$\lim_{h \to 0} \dfrac{3\cancel{h}}{\cancel{h}} = \boxed{3}$

17) $\lim_{h \to 0} \dfrac{2(x+h)^2 + (x+h) - 1 - (2x^2 + x - 1)}{h}$

$\boxed{4x+1}$

18) $\lim_{h \to 0} \dfrac{1 - (x+h)^2 - (1 - x^2)}{h}$

$1 - (x^2 + 2xh + h^2) - 1 + x^2$

$1 - x^2 - 2xh - h^2 - 1 + x^2$

$\dfrac{-2xh - h^2}{h}$

$\lim_{h \to 0} -2x - h = \boxed{-2x}$

39 B

40 D

41 A

42 C

43

44

#47 pt $(2,5)$

$f'(x) = 4-2x$

$m = 4-2x$

to find the x-values of the other pts

$\frac{5-y}{2-x} = 4-2x$

$8-8x+2x^2 = 5-y$

$5-(4x-x^2)$

$8-8x+2x^2 = 5-4x+x^2$

$3-4x+x^2$

$x^2+4x+3=0$

$(x-1)(x-3)$

$x = 1, 3$

at $(1,3)$

$f'(1) = 2$

$y-5 = 2(x-2)$

at $3,3$

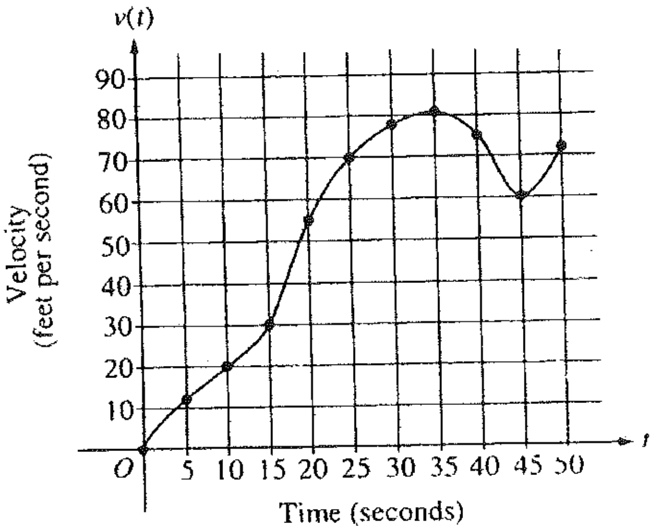
$f'(3) = -2$

$y-5 = -2(x-2)$

3

AP CALCULUS AB

QUESTIONS ON DERIVATIVES



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

because the Slope pos

0 to 35 seconds

45 to 50 seconds

(b) Find the average acceleration of the car, in ft/sec², over the interval $0 \le t \le 50$.

$$\frac{72-0}{50-0} = \frac{72}{50} = 1.44 \text{ ft/sec}$$

(c) Estimate the acceleration of the car when $t = 30$ seconds.

$$\frac{81-78}{35-30} = \frac{3}{5} \qquad \frac{78-70}{30-25} = \frac{8}{5} \qquad \frac{81-70}{35-25} = \frac{11}{10}$$

.6 ft/sec   1.6 ft/sec   1.1 ft/sec

page 4

ANS Key

Name________________________

Derivative concept & limit definition.

1) if f(2)=3 and $f'(2)=5$ find and equation of the tangent line to the graph of y=f(x).

$y - y = m(x - x)$

$y - 3 = 5(x - 2)$

2) Use the definition of derivative $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$ if $f(x)=\frac{1}{x}$ at x =3.

$f'(x) = \frac{-1}{x^2}$

$f'(3) = -1/9$

3) Find $f'(x)$ if a) f(x) = 3x – 12   b) f(x) = -2x   c) f(x)=-10

$f'(x) = 3$ $f'(x) = -2$ $f'(x) = 0$

4) The slope of the tangent line is the same as <u>Derivative</u>

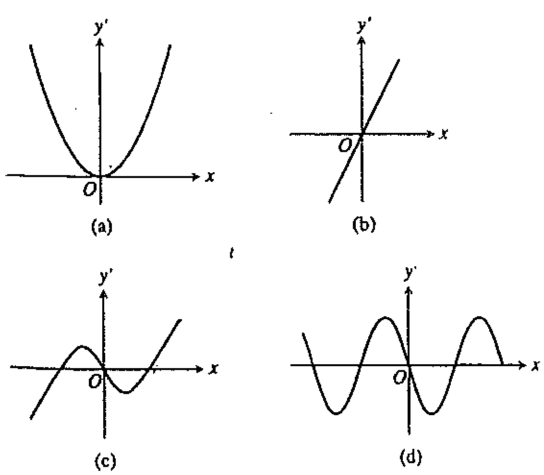
5) Find $\frac{d}{dx}(x^2)$.

$f'(x) = 2x$

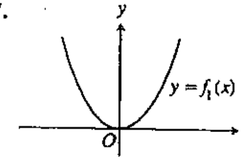
6) Find the slope of the tangent line to the parabola y=x²+1 at its vertex.

$y' = 2x$ $f'(0) = 0$

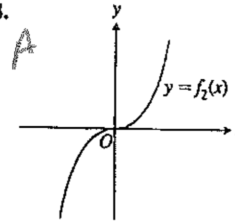
In Exercises 7–10, match the graph of the function with the graph of the derivative shown here:



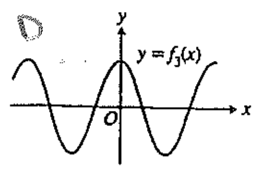
7. B



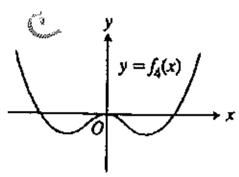
8. A



9. D



10. C

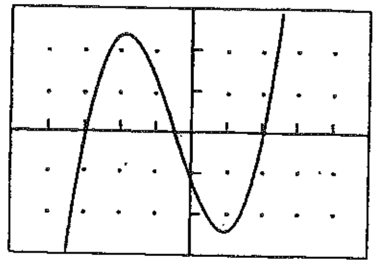


5

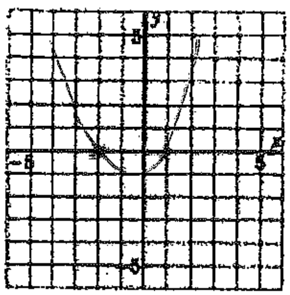
Key

Graphing f' from f Given the graph of the function f below, sketch a graph of the *derivative* of f.

11.



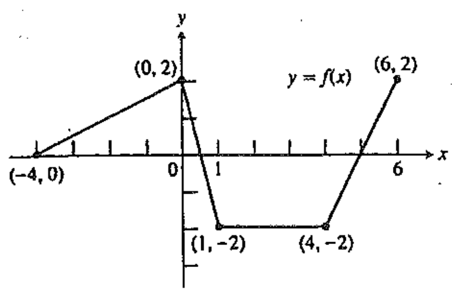
[−5, 5] by [−3, 3]



$f'(x)$

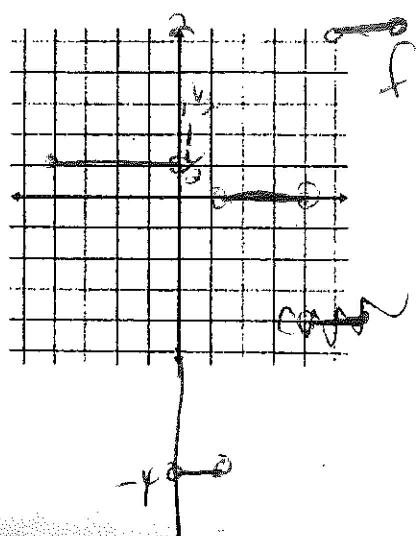
The graph of the function $y = f(x)$ shown here is made of line segments joined end to end.

12.



(a) Graph the function's derivative.

(b) At what values of x between $x = -4$ and $x = 6$ is the function not differentiable?

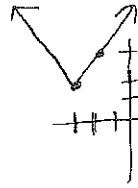


$f'(x)$

pg 6

AP Calculus
3.2 Worksheet

KEY



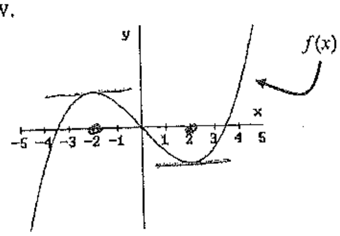
Derivative DNE at $x=-3$

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

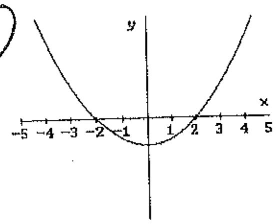
1. If $f(x)=2+|x+3|$ for all values of x, then the value of the derivative $f'(x)$ at $x=3$ is

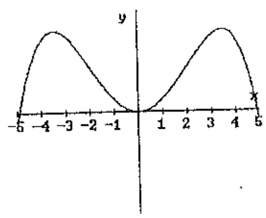
A) -1 B) 0 C) 1 D) 2 E) nonexistent

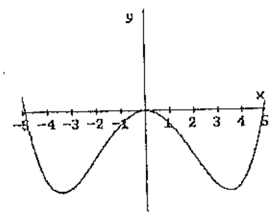
2. The graph of $f(x)$ is shown in the figure below.

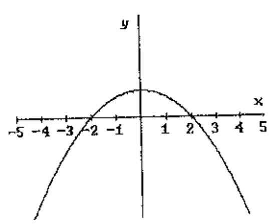


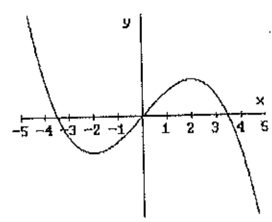
Which of the following could be the graph of $f'(x)$?

A


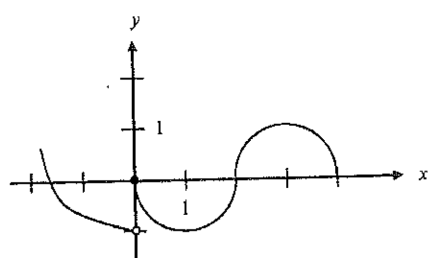
B


C


D


E


3. The graph of the function f shown in the figure below has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, −1) and (3, 1).



$x=0, 2$

For what values of x, $-2<x<4$, is f not differentiable?

A) 0 only B) 0 and 2 only C) 1 and 3 only D) 0, 1, and 3 only E) 0, 1, 2, and 3

$f(x+h) - f(x)$

If f is a function such that $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = 0$, which of the following must be true?

Means $f'(2) = 0$

A) The limit of $f(x)$ as x approaches 2 does not exist.
B) f is not defined at $x = 2$.
C) The derivative of f at $x = 2$ is 0.
D) f is continuous at $x = 0$.
E) $f(2) = 0$

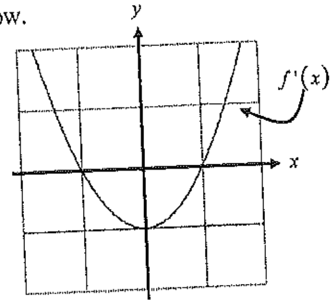
5. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

Means $f'(2)=5$ and if f is differentiable at 2 it must be continuous at 2!

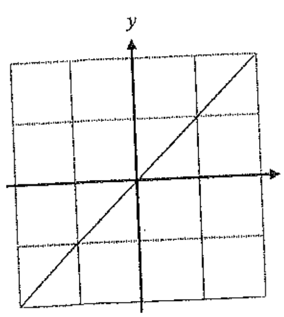
I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$.
III. The derivative of f is continuous at $x = 2$.

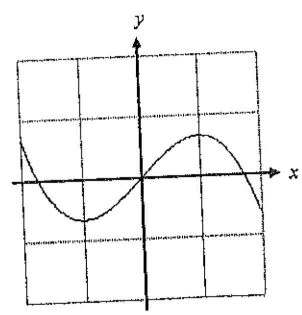
A) I only B) II only C) I and II only D) I and III only E) II and III only

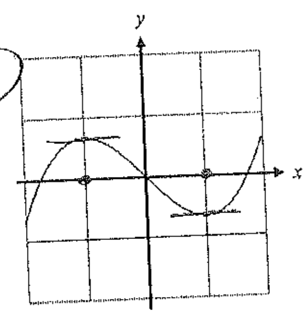
6. The graph of the ***derivative*** of f is shown below.

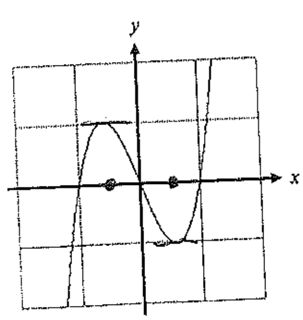


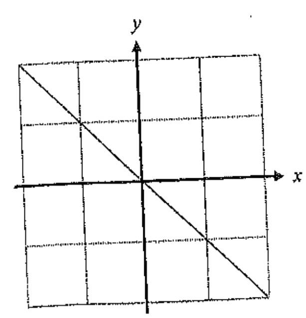
Which of the following could be the graph of f?

A)


B)


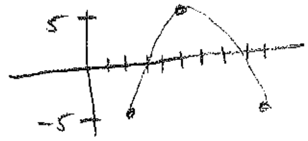
C)


D)


E)


→ No corners, cusps, vertical tangents and continuous!

7. Let f be a function that is differentiable on the open interval $(0, 10)$. If $f(2) = -5$, and $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

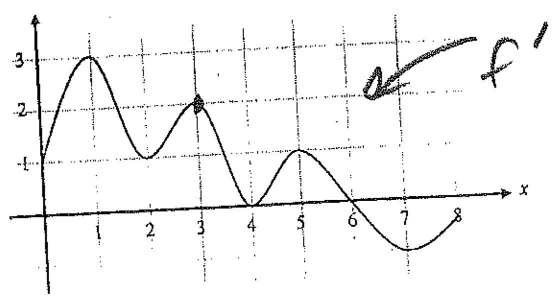


I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent line.
III. For some c, $2 < c < 5$, $f(c) = 3$.

A) none B) I and II only C) I only D) I and III only E) I, II, and III

8. The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown below.

slopes →



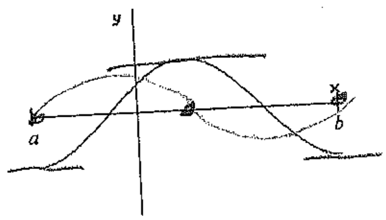
$f'(3) = 2$

$y - 5 = 2(x - 3)$

The point $(3, 5)$ is on the graph of $f(x)$. An equation of the tangent line to the graph of f at $(3, 5)$ is

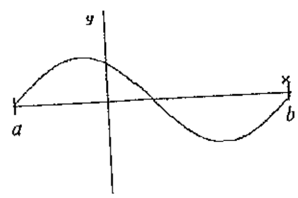
A) $y = 2$ B) $y = 5$ C) $y - 5 = 2(x - 3)$ D) $y + 5 = 2(x - 3)$ E) $y + 5 = 2(x + 3)$

9. The graph of f is shown below.

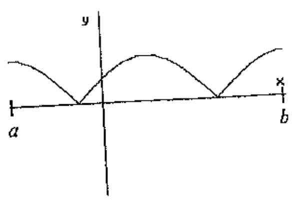


Which of the following could be the graph of the derivative of f?

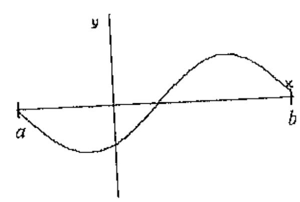
A.



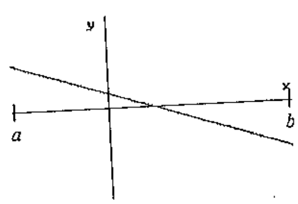
B.



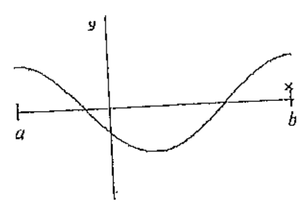
C.



D.



E.



KEY

AP Calculus AB – Estimating the derivative using a table

1. A function $T(x)$ is continuous and differentiable with values given in the table at the right.

x	1.0	1.4	1.8	2.2	2.6
$T(x)$	1.06	2.2	3.2	2.8	3.1

Use the values in the table to estimate the following

A. $T'(1.4) \approx \frac{T(1.8)-T(1)}{1.8-1} = \frac{3.2-1.06}{.8} = \boxed{2.676}$

$T'(2.4) \approx \frac{T(2.6)-T(2.2)}{2.6-2.2} = \boxed{.75}$

B. The average rate of change of $T(x)$ between $x = 1.4$ and 2.2.

$\frac{T(2.2)-T(1.4)}{2.2-1.4} = \frac{2.8-2.2}{.8} = .75$

C. The instantaneous rate of change of $T(x)$ at $x = 1$.

$T'(1) \approx \frac{2.2-1.06}{1.4-1} = 2.85$

D. The equation of the tangent line to $T(x)$ at $x = 1$.

$y - 1.06 = 2.85(x-1)$

$T(x) = 2.85(x-1)+1.06$

2. The following table lists the position s(t) of a particle at time t on the interval 0<t<4.

t (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s (ft)	12.5	26	36.5	44	48.5	50	48.5	44	36.5

a) find the average rate of change (or average velocity) between times t= .5 and t=1.5.

$\frac{44-26}{1.5-.5} = 18$

b) Estimate s'(1.5). Include unit of measure.

$s'(1.5) \approx 12$

c) On what interval(s) does s'(t) appear to be positive?

$0 < t < 2.5$

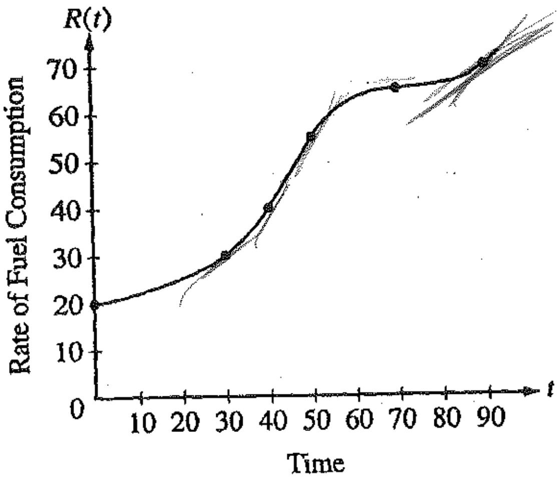
d) On what half second interval is the rate of change of s(t) the greatest?

$(0, .5)$

10

AP CALCULUS AB
AP FREE RESPONSE PROBLEM

APPROX. OF DERIVATIVE



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Give three approximations for the slope of R when $t = 70$ minutes. Indicate units of measure.

   (c) When is the rate of fuel consumption greatest?

   (d) When is the rate of change of the rate of fuel consumption greatest?

(a) $\frac{55-40}{50-40}$ $\frac{15}{10} = 1.5$ gallons/min

(b) $\frac{70-55}{90-50} = \frac{15}{40}$ $3/8$ gallon p min / min $\frac{65-55}{70-50} = \frac{10}{20}$ $1/2$ gallon per min² $\frac{70-65}{90-70} = \frac{5}{20}$ $\frac{1}{4}$ gallons per min²

(c) at 90 minutes

10

(d) From 40-50 min

~~30-0~~ $\frac{30-20}{30-0} = \frac{10}{30}$ .33 $\frac{40-30}{40-30} = \frac{10}{10}$ $\frac{55-40}{50-40}$ $\frac{15}{10}$ 1.5 $\frac{65-55}{70-50} = .50$ $\frac{70-65}{90-70} = .25$