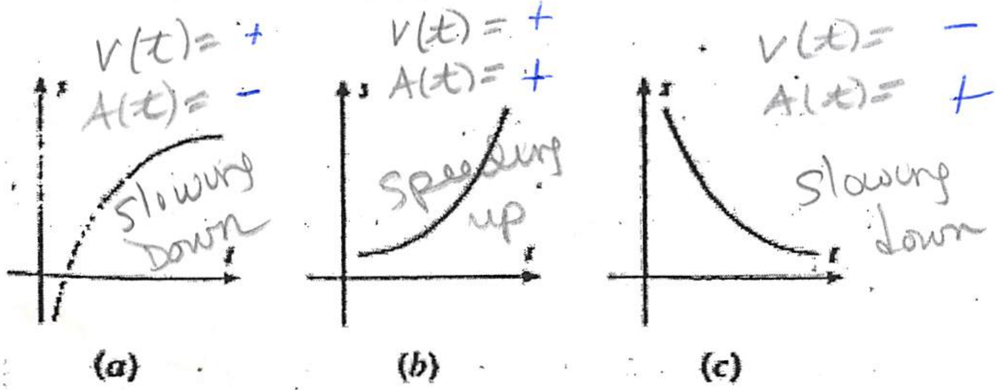
5.3 PVA Practice "METS"

. The graphs of three position functions are shown in the accompanying figure. In each case determine the signs of the velocity and acceleration, then determine whether the particle is speeding up or slowing down.

(a) $v(t) = +$, $A(t) = -$, Slowing Down

(b) $v(t) = +$, $A(t) = +$, Speeding up

(c) $v(t) = -$, $A(t) = +$, Slowing down



2. The position function of a particle moving on a horizontal x-axis is shown in Figure Ex-3 (

(a) Is the particle moving left or right at time t_0? left

(b) Is the acceleration positive or negative at time t_0? −

(c) Is the particle speeding up or slowing down at time t_0?

(d) Is the particle speeding up or slowing down at time t_1?

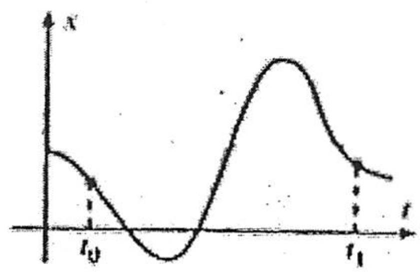


Figure Ex-3

3. For the graphs in the accompanying figure, match the position functions with their corresponding velocity functions.

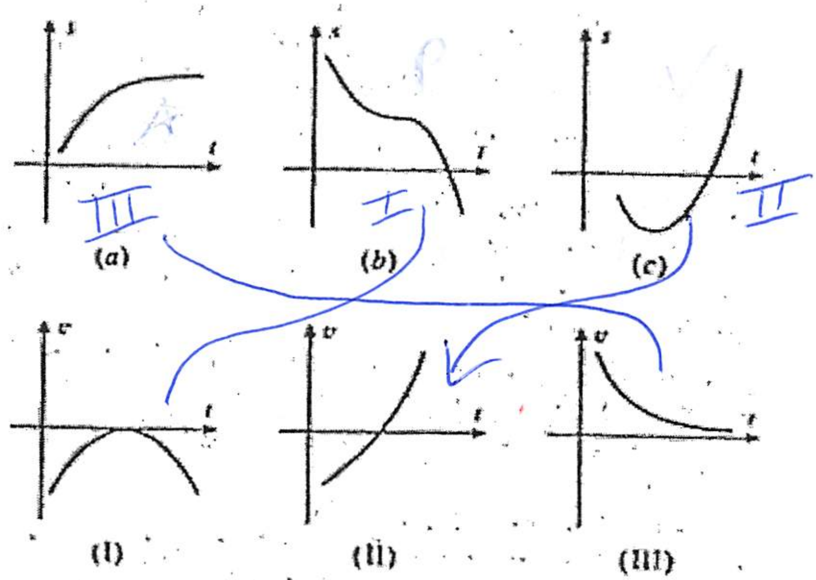
P

V

(a) III

(b) I

(c) II



4. The accompanying figure shows the graph of s versus t for an ant that moves along a narrow vertical pipe (an s-axis with the positive direction up).

(a) When, if ever, is the ant above the origin? $(0,2)\ (4,\infty)$

(b) When, if ever, does the ant have velocity zero? 3

(c) When, if ever, is the ant moving down the pipe? $(0,3)$

5. The accompanying figure shows the graph of velocity versus time for a particle moving along a coordinate line. Make a rough sketch of the graphs of speed versus time and acceleration versus time.

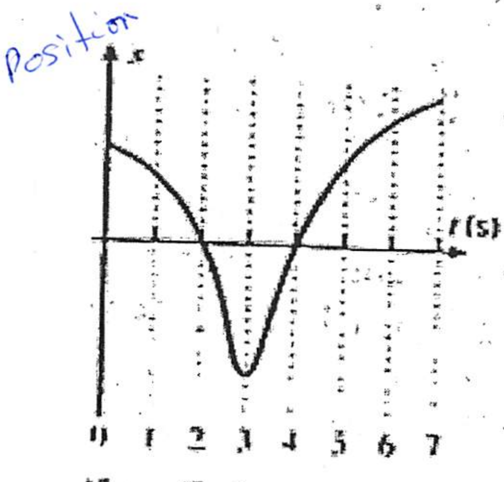


Figure Ex-6

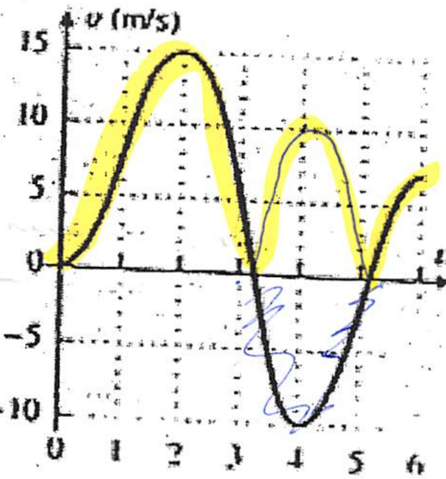
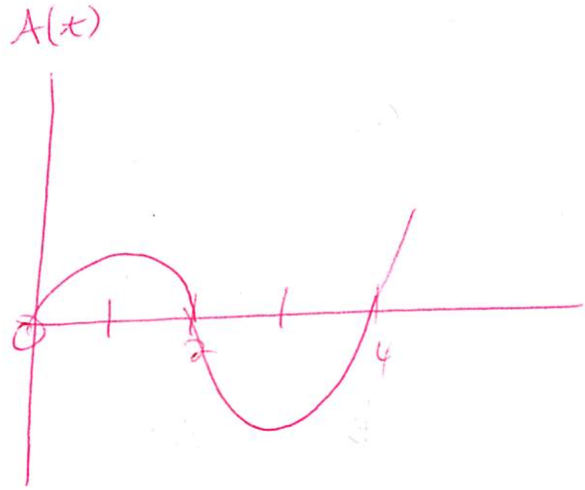


Figure Ex-7

Speed $= |V(t)|$



6. A particle is moving along the x-axis. Its position at time t, is given by the equation:

$$s(t) = \frac{t^4}{4} - \frac{7t^3}{3} + 5t^2 = \frac{1}{4}t^4 - \frac{7}{3}t^3 + 5t^2$$

a. What is the velocity of the particle at $t = 3$? Is the velocity increasing or decreasing at this time? Explain and justify your answer.

$V(t) = t^3 - 7t^2 + 10t$
$t(t^2 - 7t + 10)$
$t(t-5)(t-2)$
$t = 0, 5, 2$

$V(3) = 27 - 63 + 30$
$\boxed{-6}$
$A(3) = -$ decreasing because $A(t)$ is neg

b. At what values of t does the particle change direction? Explain and justify your answer.

$V(t) = t^3 - 7t^2 + 10t$
$t(t^2 - 7t + 10)$
$t(t-5)(t-2)$
$t = 0, 5, 2$

$V(t)$: 0 ++ 2 -- 5 ++

$t = 2, 5$ $V(t) = 0$ and changes sign

c. For which values of t is the position graph concave downwards? For which values is it concave upwards? Explain and justify your answer.

$A(t) = 3t^2 - 14t + 10$

$$\frac{14 \pm \sqrt{(-14)^2 - 4(3)(10)}}{2(3)}$$

.880
3.786

$A(t)$: 0 ++ .880 -- 3.786 ++

concave up because f'' is pos $(0, .880)\ (3.786, \infty)$

concave down $(.880, 3.786)$ b.c. $f''(x)$ is neg

d. For which values of t is the particle speeding up? For which values is it slowing down? Explain and justify your answer.

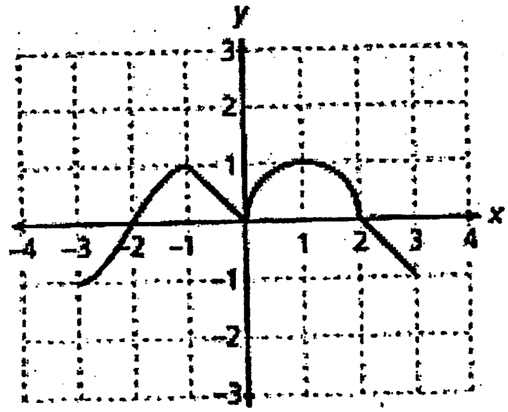
Speeding up - $V(t)$ + $A(t)$ same sign
$(0, .880) \cup (2, 3.786) \cup (5, \infty)$

$V(t) =$ 0 ++ 2 -- 5 ++

$A(t) =$ 0 ++ .880 -- 3.786 ++

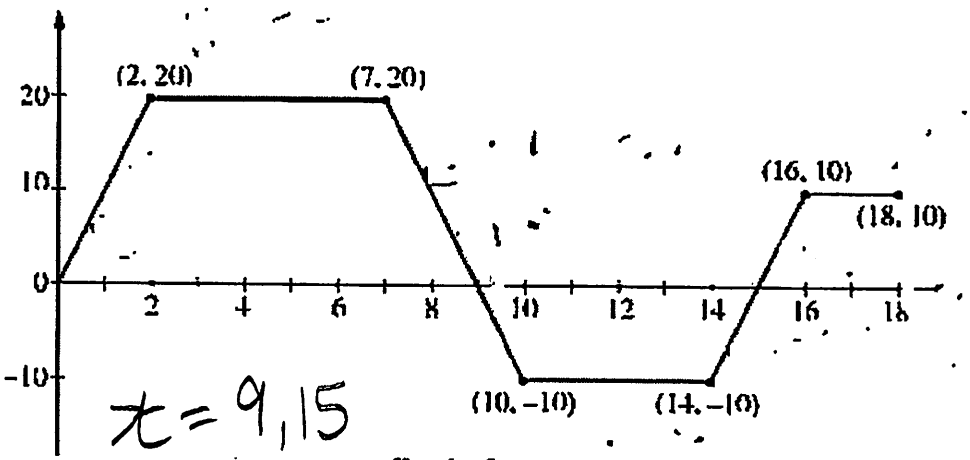
slowing down $V(t)$ + $A(t)$ have opp signs
$(3.786, 5) \cup (.880, 2)$

7. The graph of $f'(x)$ is given below for $x \in [-3,3]$. On which interval(s) is the function $f(x)$ both increasing and concave up?



(A) $(-2,2)$
(B) $(-2,0)\cup(0,2)$
(C) $(-3,-2)$
(D) $(-2,-1)\cup(0,1)$
(E) none of these

8. A graph of $v(t)$ is provided. When does the particle stop and change direction?



$t = 9, 15$

Graph of v

9. Car A has positive velocity $v(t)$ as it travels on a straight road. Where v is a differentiable function of t. The velocity is recorded for selected values over the time interval $0 \le t \le 10$ seconds, as shown in the table below. Use the data from the table to approximate the acceleration of Car A at $t = 8$ seconds. Indicate units of measure.

t (sec)	0	2	5	7	10
$v(t)$ (ft/sec)	0	9	36	61	115

$$\frac{115-61}{10-7} = \frac{54}{3} = 18 \text{ ft/sec}^2$$

10. A particle moves along the y-axis for $0 \le t \le 40$ seconds. The velocity of the particle at time t is given by $v(t) = \sin\left(\frac{\pi}{8}t\right)$ in meters/second. The particle is at position y = 10 meters at time t = 0. Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time t = 10 seconds?

$A(t) = \frac{\pi}{8}\cos\frac{\pi}{8}t$

$\frac{\pi}{8}$ 10

$\cos\frac{5\pi}{4}$

$\frac{\pi}{8} \cdot -\frac{\sqrt{2}}{2}$

$A(10) = \text{neg}$
$V(10) = \text{neg}$

Speeding up
$A(t)$ + $V(t)$
same sign

11. Use $P(t) = t^3 - t^2 - t$

a. Find initial position and Initial motion.

$P(0) = 0$
$V(0) = -1$

$V(t) = 3t^2 - 2t - 1$
$V(0) = -1$

b. When does the particle change direction?

$V(t) = 3t^2 - 2t - 1$
$(3t+1)(t-1)$
$t = -\frac{1}{3}, 1$

$V(t)$ -- | +++ (1)

c. Is the velocity increasing or decreasing at t = 4?

$A(t) = 6t - 2$
$A(4) = 22$

$A(4) = +$
Velocity increasing

d. When is the particle speeding up, slowing down?

$V(t) =$ 0 -- | ++ (1)
$A(t) =$ 0 -- | +++ ($\frac{1}{3}$)

$A(t) = 6t - 2$
$\frac{6t}{6} = \frac{2}{6}$
$t = \frac{1}{3}$

Speeding up $(0, \frac{1}{3}) \cup (1, \infty)$
Slowing down $(\frac{1}{3}, 1)$

For Numbers 12-14: the position function of a particle moving along a coordinate line is given, where s is in feet and t is in seconds. (a) Find the velocity and acceleration functions. (b) Find the position, velocity, speed, and acceleration at time $t = 1$. (c) At what times is the particle stopped? (d) When is the particle speeding up? Slowing down?

12. $s(t) = t^4 - 4t + 2, t \geq 0$

(A) $V(t) = 4t^3 - 4$
$A(t) = 12t^2$

(c) $v(t) = 0$
$4t^3 - 4 = 0$
$t = 1$

(B) $s(1) = -1$
$V(1) = 0$ speed $= 0$
$A(1) = 12$

(d) $v(t) =$ − − | + + (1)
$A(t) =$ + + + | + + + (0)

Speeding up $(1, \infty)$
slowing down $(0, 1)$

13. $s(t) = 3\cos\left(\frac{\pi}{2}t\right), 0 \leq t \leq 5$

$u = \frac{\pi}{2}t$
$du = \frac{\pi}{2}$

S
C
−S
−C

(A) $V(t) = -\frac{3\pi}{2}\sin\frac{\pi}{2}t$

$A(t) = -\frac{3\pi^2}{4}\cos\frac{\pi}{2}t$

(c) $V(t) = 0$
2, 4

(d) $v(t)$ − − − | + + | − − − (0, 2, 4)
$A(t)$ − − | + + | − − | + + (1, 3, 5)

(B) $S(1) = 0$
$V(1) = -\frac{3\pi}{2}$
$A(1) = 0$

Speeding up $(0,1)$ $(2,3)$ $(4,5)$
slowing down ~~(0,1)~~ $(3,4)$
$(1,2)$

14. $s(t) = \frac{t}{t^2+4}, t \geq 0$

For Numbers 12-14: the position function of a particle moving along a coordinate line is given, where s is in feet and t is in seconds. (a) Find the velocity and acceleration functions. (b) Find the position, velocity, speed, and acceleration at time $t = 1$. (c) At what times is the particle stopped? (d) When is the particle speeding up? Slowing down?

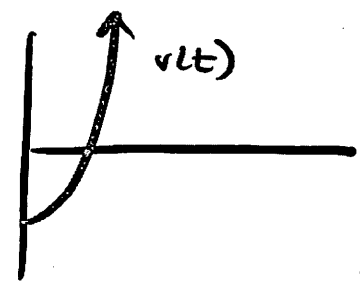
12. $s(t) = t^4 - 4t + 2, t \geq 0$

a. $v(t) = 4t^3 - 4$
$a(t) = 12t^2$

b. $s(1) = -1$
$v(1) = 0$
$a(1) = 12$

c. $0 = 4t^3 - 4$
$0 = 4(t^3 - 1)$
$t = 1$

d. speeding up: $(1, \infty)$
slowing down: $(0, 1)$



13. $s(t) = 3\cos\left(\frac{\pi}{2}t\right), 0 \leq t \leq 5$

a. $v(t) = -3\sin\left(\frac{\pi}{2}t\right)\frac{\pi}{2}$
$= -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right)$

$a(t) = -\frac{3\pi}{2}\cos\left(\frac{\pi}{2}t\right)\cdot\frac{\pi}{2}$
$= -\frac{3\pi^2}{4}\cos\left(\frac{\pi}{2}t\right)$

b. $s(1) = 0$
$v(1) = -\frac{3\pi}{2}$
$a(1) = 0$

c. $-\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right) = 0$
$\sin\frac{\pi}{2}t = 0$
$\frac{\pi}{2}t = \sin^{-1}(0)$
$\frac{2}{\pi}\cdot\frac{\pi}{2}t = 0, \pi$
$\boxed{t = 0, 2}$

d. speeding up: $(0, 1) \cup (2, 3) \cup (4, 5)$
slowing down: $(1, 2) \cup (3, 4)$

14. $s(t) = \frac{t}{t^2+4}, t \geq 0$ $\quad -2t^2$

a. $v(t) = \frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2}$

$v(t) = \frac{-t^2+4}{(t^2+4)^2}$

$a(t) = \frac{(t^2+4)^2(-2t) - (-t^2+4)(2(t^2+4)\cdot 2t)}{(t^2+4)^4}$

b. $s(1) = 1/5$
$v(1) = 3/25$
$a(1) = -2/125$

c. $-t^2 + 4 = 0$
$t = \pm 2$
$\boxed{t = 2}$

d. Speeding up: $(2, 3.46)$
Slowing $\downarrow$: $(0, 2) \cup (3.46, \infty)$

Chapter 5 Part 2 Review Questions

AP® CALCULUS AB
2003 SCORING GUIDELINES

w/ calculator

Question 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(a) $a(t) = -\left[(t+1)\cos\left(\frac{t^2}{2}\right)t + (1)\sin\left(\frac{t^2}{2}\right)\right]$

$-(t+1)\cos\left(\frac{t^2}{2}\right)t - \sin\left(\frac{t^2}{2}\right)$

$-t^2\cos\left(\frac{t^2}{2}\right) - t\cos\left(\frac{t^2}{2}\right) - \sin\left(\frac{t^2}{2}\right)$

$a(2) = -(2)^2\cos\left(\frac{2^2}{2}\right) - 2\cos\left(\frac{2^2}{2}\right) - \sin\left(\frac{2^2}{2}\right)$

$-4\cos(2) - 2\cos(2) - \sin(2)$

$-6\cos 2 - \sin 2 \approx \boxed{1.588}$

$v(2) = -2.728$

speed is decreasing (slowing down) b/c $a(2)$ is + and $v(2)$ is −

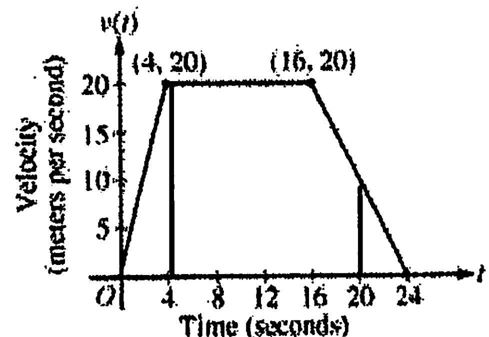
(b.) when the $v(t)$ changes signs. $t = 2.507$, $v(t)$ changes from − to +.

2003 Scoring Rubric Question 2

(a) $a(2) = v'(2) = 1.587$ or 1.588 $v(2) = -3\sin(2) < 0$ Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.	$2: \begin{cases} 1: a(2) \\ 1: \text{speed decreasing} \\ \quad \text{with reason} \end{cases}$
(b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$ $t = \sqrt{2\pi}$ or 2.506 or 2.507 Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.	$2: \begin{cases} 1: t = \sqrt{2\pi} \text{ only} \\ 1: \text{justification} \end{cases}$

AP® CALCULUS AB
2005 SCORING GUIDELINES

Question 5



A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(b) $v'(4)$ d.n.e bc it is not differentiable there.

$v'(20) = \underbrace{\frac{0-20}{24-16}}_{\text{slope}} = \frac{-20}{8} = \frac{-5}{2} \frac{m}{sec^2}$

(c) $a(t) = \begin{cases} 5, & 0 < t < 4 \\ 20, & 4 < t < 16 \\ -\frac{5}{2}, & 16 < t < 24 \end{cases}$

$a(t)$ d.n.e @ $t = 4, 16$

(d) $\frac{v(20) - v(8)}{20-8} = \frac{-5}{6} \frac{m}{sec^2}$.

No the MVT does not apply to v on [8,20] bc v is not diff @ t=16.

2005 Scoring Rubric Question 5

(b) $v'(4)$ does not exist because

$$\lim_{t\to 4^-}\left(\frac{v(t)-v(4)}{t-4}\right) = 5 \neq 0 = \lim_{t\to 4^+}\left(\frac{v(t)-v(4)}{t-4}\right).$$

$$v'(20) = \frac{20-0}{16-24} = -\frac{5}{2}\ \text{m/sec}^2$$

$$3:\begin{cases}1: v'(4) \text{ does not exist, with explanation}\\ 1: v'(20)\\ 1: \text{units}\end{cases}$$

(c)

$$a(t) = \begin{cases}5 & \text{if } 0 < t < 4\\ 0 & \text{if } 4 < t < 16\\ -\frac{5}{2} & \text{if } 16 < t < 24\end{cases}$$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$$2:\begin{cases}1: \text{finds the values } 5, 0, -\frac{5}{2}\\ 1: \text{identifies constants with correct intervals}\end{cases}$$

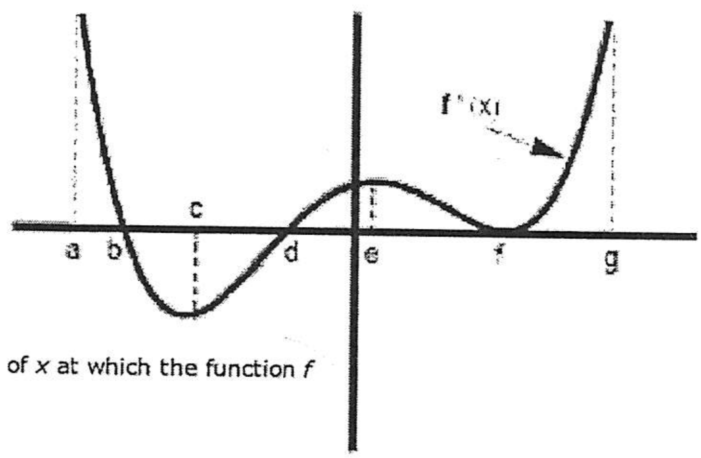
(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20)-v(8)}{20-8} = -\frac{5}{6}\ \text{m/sec}^2.$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$$2:\begin{cases}1: \text{average rate of change of } v \text{ on } [8, 20]\\ 1: \text{answer with explanation}\end{cases}$$

1. Use the graph of $f'(x)$ provided to answer the questions about $f(x)$. Based on the graph of $f'(x)$, find where the graph of $f(x)$ is increasing or decreasing and find the x-values of any relative extrema.



increasing a,b
d,f
f,g

b is rel max
d is rel min

2. Given the graph of f'' in Figure 7.7-2, determine the values of x at which the function f has a point of inflection. (See Figure 7.7-2.)

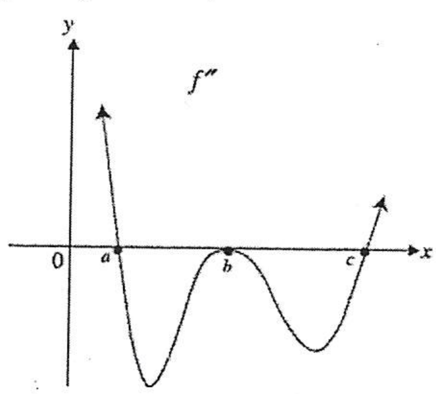


Figure 7.7-2

at a, c
because $f''(x) = 0$
and changes

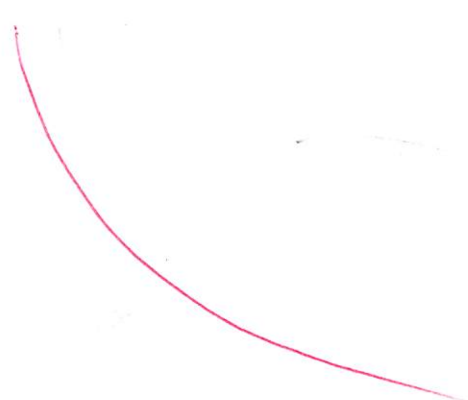
2. Fill in the blanks

a. When the velocity and acceleration of the particle have the same sign, the particle's speed is increasing.

b. When the velocity and acceleration of the particle have opposite signs, the particle's speed is decreasing.

3. Sketch the graph of a function whose first derivative is negative or zero everywhere and whose second derivative starts out negative but becomes positive.

4. Sketch the graph of a position of a particle vs. time if the particle's velocity is negative and its acceleration is positive.



AP® CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

continuous & differen

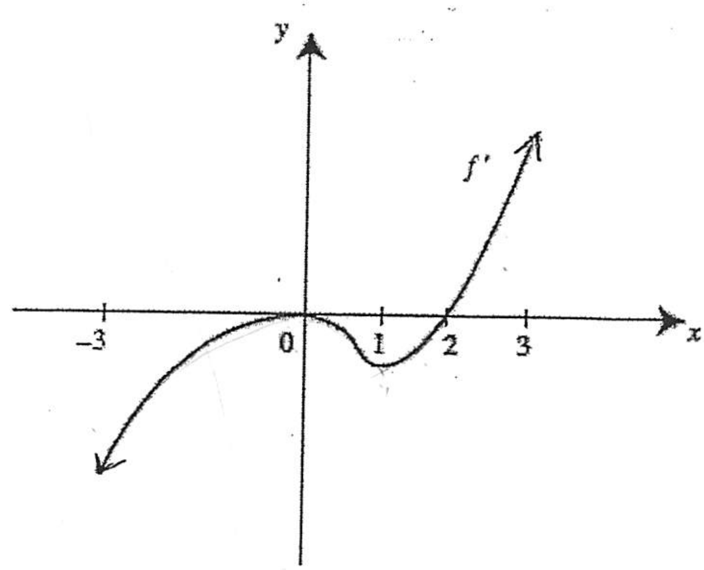
(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

(c) yes, since $v(35) = -10$ and $v(50) = 0$, the IVT guarantees a t in $(35, 50)$ where $v(t) = -5$

(d) yes, since $v(0) = -20$ and $v(25 = -20$, by Rolles thm, there is a t on the interval $(0, 25)$ where $a(t) = 0$

2. Given the graph of the derivative answer the following:



a) Find all x values of the relative extrema of f(x) $x = 2$

b) On what interval(s) is f(x) increasing? $2, \infty$

c) On what interval(s) is f(x) concave down? $(0, 1)$

d) What are the x values of the inflection points? $x = 1, 0$

2006 Scoring Rubric Question 6 Form B

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.	$2 : \begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$
(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.	$2 : \begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$
Units of ft in (a) and ft/sec in (b)	1 : units in (a) and (b)

818 (1989BC). Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

R

a) Find the absolute maximum and minimum values of $f(x)$.

b) Find intervals on which f is increasing.

c) Find the x-coordinate of each point of inflection of the graph of f.

2016 Calculator question

Question 2

R

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer

not yet

~~(c) Find the position of the particle at time $t = 0$.~~

~~(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.~~

2006 Scoring Rubric Question 6 Form B

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.	$2: \begin{cases} 1: v(35) < -5 < v(50) \\ 1: \text{Yes; refers to IVT or hypotheses} \end{cases}$
(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.	$2: \begin{cases} 1: v(0) = v(25) \\ 1: \text{Yes; refers to MVT or hypotheses} \end{cases}$
Units of ft in (a) and ft/sec in (b)	1 : units in (a) and (b)

818 (1989BC). Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

a) Find the absolute maximum and minimum values of $f(x)$.

b) Find intervals on which f is increasing.

c) Find the x-coordinate of each point of inflection of the graph of f.

a) EVT

$f'(x) = -e^x \sin x + e^x \cos x$

$0 = -e^x(\sin x - \cos x)$

$0 = \sin x - \cos x$

$\cos x = \sin x$

$1 = \tan x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	1.55
$\frac{5\pi}{4}$	-35.89
2π	535.49

abs min of -35.89 @ $(\frac{5\pi}{4}, -35.89)$

abs max of 535.49 @ $(2\pi, 535.49)$

b) + − +

$0 \quad \frac{\pi}{4} \quad \frac{5\pi}{4} \quad 2\pi$

$\boxed{\text{increasing on } (0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)}$

c) $f''(x) = -e^x \cos x + -e^x \sin x + (-e^x \sin x + e^x \cos x)$

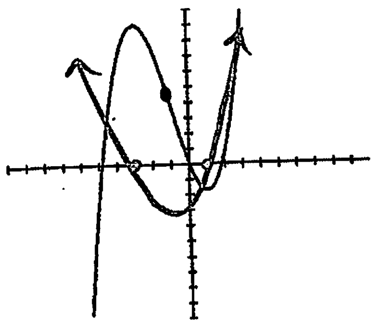
$\cancel{-e^x \cos x} - e^x \sin x - e^x \sin x + \cancel{e^x \cos x}$

$f''(x) - 2e^x \sin x$

$0 = -2e^x \sin x$

$\sin x = 0 \qquad \boxed{x = 0, \pi}$

5. The graph of f is given below.



(a) For what values of x is $f(x)$ zero? Positive? Negative?

$x = -5, 0, 2$ $(-5,0)\cup(2,\infty)$ $(-\infty,-5)\cup(0,2)$

(b) For what values of x is $f'(x)$ zero? Positive? Negative?

$x = -3, 1$ $(-\infty,-3)\cup(1,\infty)$ $(-3,1)$

(c) For what values of x is $f''(x)$ zero? Positive? Negative?

$x = -1$ $(-1,\infty)$ $(-\infty,-1)$

6. Find all critical values for the function on the closed interval $[0, 2\pi]$. Use the Second Derivative to determine all relative min and relative max, Explain and justify each answer.

Express all x-values in radians.
$f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$0 = \cos x - \sin x$

$\sin x = \cos x$

$\tan x = 1$

C.V.'s $\rightarrow$ $\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

because this is where $f'(x)=0$

$f''(x) = -\sin x - \cos x$

$f''(\frac{\pi}{4}) = -\sqrt{2}$ $--$

$f''(\frac{5\pi}{4}) = \sqrt{2}$ $++$

rel max of $\sqrt{2}$ @ $(\frac{\pi}{4}, \sqrt{2})$
bc. $f'' < 0$ & $f' = 0$.

rel min of $-\sqrt{2}$ @ $(\frac{5\pi}{4}, -\sqrt{2})$
bc $f'' > 0$ & $f' = 0$.

Calculus AB
Chapter 3.6 – 4.4 Review

KEY

CONCEPTS:

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?

① Look for where the derivative equals zero or is undefined
② Look at endpoints ③ create a table for a "candidates test"

2. How do you justify relative extrema?

① "candidates test" (see #1) ③ 2^{ND} Derivative Test
② Relative extrema can be justified by finding where the first derivative changes signs
Rel max: f' changes from pos to neg
Rel min: f' changes from neg to pos

3. How do you justify that a function is increasing or decreasing?

f is increasing if $f'>0$
f is decreasing if $f'<0$

4. How do you justify that a function is concave up or concave down?

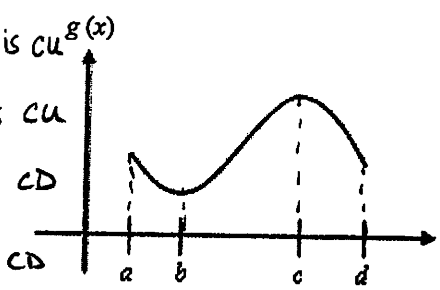
f is concave up if $f''>0$
f is concave down if $f''<0$

5. How do you justify that a function has a point of inflection?

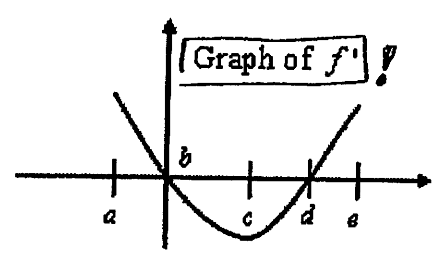
f has a point of inflection if f'' changes signs at that point.

6. Using the graph of $g(x)$ below, determine the signs of $g'(x)$ and $g''(x)$ at each point. Explain your reasoning.

At $x=a$ … $g'(x)<0$ b/c g is dec / $g''(x)>0$ b/c g is cu
At $x=b$ … $g'(x)=0$ [Horizontal Tangent Line] / $g''(x)>0$ b/c g is cu
At $x=c$ … $g'(x)=0$ [Horiz. Tang. Line] / $g''(x)<0$ b/c g is CD
At $x=d$ … $g'(x)<0$ b/c g is dec / $g''(x)<0$ b/c g is CD



7. $\boxed{\text{Given the graph of } f'}$ below answer each of the following questions, and justify your response with a statement that contains the phrase "since f' ____________ …"



a) When is f increasing?
f is increasing on $(a,b)\cup(d,e)$
since $f'>0$

b) When is f decreasing?
f is decreasing on (b,d)
since $f'<0$

c) When is f concave up?
f is concave up on (c,e)
since f' is increasing $[f''>0]$

d) When is f concave down?
f is concave down on (a,c)
since f' is decreasing $[f''<0]$

e) When does f have a relative maximum?
f has a rel. max. at $x=b$
since f' changes signs from pos to neg at $x=b$.

f) When does f have a relative minimum?
f has a rel. min. at $x=d$ since f' changes signs from neg to pos at $x=d$.

g) When does f have a point of inflection?
f has a point of inflection at $x=c$
since f' changes from decreasing to increasing [f'' changed signs]

SKILLS:

8. [Calculator Allowed] If $f(x)$ has an inverse, then $f\left(f^{-1}(x)\right)=x$. Find $\left(f^{-1}\right)'(2)$ if $f(x)=x^3+2x-1$.

$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{3(1)^2+2} = \boxed{\frac{1}{5}}$

First, find $f^{-1}(2)$ by setting $f(x)=2$.

$x^3+2x-1=2$

$x=1$ (on calculator)

Next, find $f'(x)$: $f'(x)=3x^2+2$

9. Find the value of c guaranteed by the MVT for $f(x)=4x^2+5x$ on the interval [–2, 1].

$f'(x) = 8x+5$ $\quad f(-2) = 4(-2)^2+5(-2) = 16-10 = 6$

$f(1) = 4(1)^2+5(1) = 4+5 = 9$

MVT guarantees a value of c between -2 & 1 such that $f'(c) = \frac{f(-2)-f(1)}{-2-1}$

$8c+5 = \frac{6-9}{-3}$

$8c+5=1$

$8c=-4$

$\boxed{c=-1/2}$

10. [Calculator Allowed] Find the value of c guaranteed by the MVT for $f(x)=\sin x$ on the interval [4, 5]. [♫: For those of you doing this problem algebraically, the answer is NOT $c \approx 1.774\ldots$ Why?]

$f'(x) = \cos x$ $\quad f(4) = \sin(4)$, $f(5) = \sin(5)$

MVT: $f'(c) = \frac{f(5)-f(4)}{5-4}$

$\cos c = \sin(5) - \sin(4)$

solve on calculator using graph ...

$\boxed{c \approx 4.509}$

11. Find the following derivatives:

a) $y=\sin^{-1}\left(3x^2\right)$

$\boxed{y' = \frac{6x}{\sqrt{1-9x^4}}}$

b) $y=\tan^{-1}(\sin x)$

$\boxed{y' = \frac{\cos x}{1+\sin^2 x}}$

c) $y=\sec^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x^{-1})$

$\boxed{y' = \frac{-\frac{1}{x^2}}{\left|\frac{1}{x}\right|\sqrt{\frac{1}{x^2}-1}}}$

d) $y=5^{x^{-3}-8}$

$\boxed{y' = 5^{x^{-3}-8} \cdot (\ln 5) \cdot [-3x^{-4}]}$

e) $y=e^{8x}$

$\boxed{y' = e^{8x} \cdot 8}$

f) $y=\log_4\left(\sqrt{9x^3-2}\right)$

$\boxed{y' = \frac{\frac{1}{2}(9x^3-2)^{-1/2} \cdot 27x^2}{\sqrt{9x^3-2}} \cdot \frac{1}{\ln 4}}$

g) $y=\ln\left(7x^2+3\right)$

$\boxed{y' = \frac{14x}{7x^2+3}}$

h) $y=3^{\sec(x)}$

$\boxed{y' = 3^{\sec(x)} \cdot (\ln 3)(\sec x \tan x)}$

i) $y=e^{\ln x} = x$ ← EASY WAY

$\boxed{y'=1}$

OR... $\underbrace{y' = e^{\ln x} \cdot \frac{1}{x}}_{}$

simplifies to $y'=1$

While none of the previous derivative questions included product and quotient rules, you should be able to combine these rules with any rules we have learned before. See your quiz from 3.8 and 3.9 for examples.

12. Suppose that functions f and g and their first derivatives have the following values at $x=-1$ and $x=0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Find the first derivative of the following combinations at the given value of x.

a) $f(g(x))$ at $x=-1$ $\Rightarrow f'(g(-1))\cdot g'(-1)$

$= f'(-1)\cdot g'(-1)$

$= 2\cdot 1$

$= \boxed{2}$

b) $\boxed{f^2(x)g^3(x)}$ at $x=0$ $\to [f(x)]^2\cdot[g(x)]^3$

$[f(0)]^2\cdot 3[g(0)]^2\cdot g'(0) + [g(0)]^3\cdot 2[f(0)]\cdot f'(0)$

$= (-1)^2\cdot 3(-3)^2\cdot(4) + (-3)^3\cdot 2(-1)\cdot(-2) = \boxed{0}$

c) $g(f(x))$ at $x=-1$

$= g'(f(-1))\cdot f'(-1)$

$= g'(0)\cdot f'(-1) = 4\cdot 2 = \boxed{8}$

d) $g(x+f(x))$ at $x=0$

$= g'(0+f(0))\cdot[1+f'(0)]$

$= g'(0+-1)\cdot[1+f'(0)] = g'(-1)\cdot[1+f'(0)]$

$= 1\cdot[1+-2]$

$= \boxed{-1}$

13. Find $\frac{dy}{dx}$ if $x^2y+3y^2=x-2$

$\left[x^2\frac{dy}{dx}+y\cdot 2x\right]+6y\,dy/dx = 1$

$x^2\,dy/dx + 6y\,dy/dx = 1-2xy$

$dy/dx\,(x^2+6y) = 1-2xy$

$\boxed{dy/dx = \frac{1-2xy}{x^2+6y}}$

14. Find $y'''(x)$ if $y=(4x+1)^{10}$

$y' = 10(4x+1)^9\cdot 4 = 40(4x+1)^9$

$y'' = 360(4x+1)^8\cdot 4 = 1440(4x+1)^8$

$y''' = 11520(4x+1)^7\cdot 4 = \boxed{80640(4x+1)^7}$

15. Suppose $y=x^3-3x$. [No Calculator]

a) Find the zeros of the function. $x^3-3x=0$

$x(x^2-3)=0$

$\boxed{x=0 \quad or \quad x=\pm\sqrt{3}}$

b) Determine where y is increasing or decreasing and justify your response.

$y' = 3x^2-3 = 3(x^2-1) = 3(x+1)(x-1)$

y': pos | -1 | neg | 1 | pos

y is increasing on $(-\infty,-1]\cup[1,\infty)$ since $y'>0$

y is decreasing on $[-1,1]$ since $y'<0$

c) Determine all local extrema and justify your response.

There is a rel. max at $x=-1$ since y' changed signs from + to −. $\boxed{\text{Rel max} = 2}$

there is a rel. min at $x=1$ since y' changed signs from − to + $\boxed{\text{Rel min} = -2}$

d) Determine the points where y is concave up or concave down, and find any points of inflection. Justify your responses.

$y''=6x$

y'': neg | 0 | pos

y is concave up on $(0,\infty)$ since $y''>0$

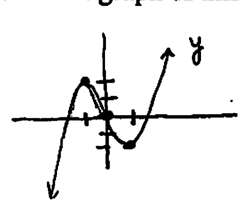
y is concave down on $(-\infty,0)$ since $y''<0$

There is a point of inflection on y when $x=0$ since y'' changed signs at $x=0$.

$\boxed{\text{P.O.I.} = (0,0)}$

e) Use all your information to sketch a graph of this function.

Known Points: $(-1,2)$
$(1,-2)$
$(0,0)$



y'	pos	-1	neg		neg	1	pos
y''	neg		neg	0	pos		pos
y	INC CD		DEC CD		DEC CU		INC CU

16. If $f'(x) = x^2 - 9x + 1$, what does $f(x)$ equal?

Given $f'(x)$ — Go "Backwards"

$$\boxed{f(x) = \frac{1}{3}x^3 - \frac{9}{2}x^2 + x + C}$$

Don't forget the "C"

17. Suppose the acceleration of an object in terms of time is given by $a(t) = 5$.

Derivatives: P, V, A — Undo Derivatives

a) What is the velocity function if $v(2) = 10$?

$v(t) = 5t + C$

if $v = 10$ when $t = 2$

$10 = 5(2) + C$

$0 = C$

$\boxed{\therefore\ v(t) = 5t}$

b) Using your velocity function from part *a*, what is the position function if $s(0) = 5$?

$s(t) = \frac{5}{2}t^2 + C$

if $s = 5$ when $t = 0$, $5 = \frac{5}{2}(0)^2 + C$

$5 = C$

$\boxed{\therefore\ s(t) = \frac{5}{2}t^2 + 5}$

18. Suppose $\boxed{\frac{d^2y}{dx^2} = x^3 - 4x^2}$. Justify each response below.

$y'' = x^2(x-4)$

y'': neg | 0 | neg | 4 | pos

a) Where is y concave up?

y is concave up on $(4, \infty)$ since $y'' > 0$

b) Where is y concave down?

y is concave down on $(-\infty, 0) \cup (0, 4)$ since $y'' < 0$

c) Are there any inflection points on y? If so, where?

Since y'' changes signs at $x = 4$, there is a point of inflection when $x = 4$.

THERE IS NO POI @ $x = 0$ b/c y'' DID NOT CHANGE SIGNS at $x = 0$.

SKILLS AND CONCEPTS APPLIED

19. [Calculator Allowed] The derivative of $h(x)$ is given by $h'(x) = 2\cos\left(x - \frac{\pi}{6}\right) + 1$ on the interval $[-2\pi, 2\pi]$. Justify EVERY response. (graphed below)

a) Where is $h(x)$ increasing?

h is increasing when $h'(x) > 0$

This occurs on $[-2\pi, -3.665] \cup [-1.571, 2.618] \cup [4.712, 2\pi]$

b) Where is $h(x)$ concave down?

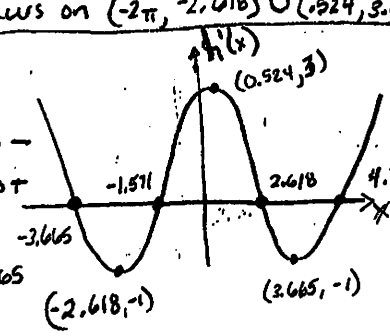
h is concave down when h' is decreasing ($h'' < 0$)

This occurs on $(-2\pi, -2.618) \cup (.524, 3.665)$

c) Find ~~all~~ x-coordinates of all extrema of $h(x)$ on the interval $[-2\pi, 2\pi]$.

Rel max on $h(x)$ at $x = -3.665$ & $x = 2.618$ since h' changes from + to −

Rel min on $h(x)$ at $x = -1.571$ & $x = 4.712$ since h' changes from − to +



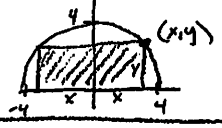
d) Does $h(x)$ have a point(s) of inflection? If so, where?

h has a point of inflection at $x = -2.618$, $x = 0.524$, & $x = 3.665$

h' changes from decreasing to increasing (or vice-versa)

★★ IF YOU DON'T USE CALCULUS FOR #20 & 21 YOU WON'T GET CREDIT ON THE TEST!

20. Find the maximum area of a rectangle inscribed under the curve $f(x)=\sqrt{16-x^2}$.



$A = 2xy = 2x\sqrt{16-x^2}$ Domain: $[0,4]$

$A' = 2x \cdot \left[\tfrac{1}{2}(16-x^2)^{-1/2} \cdot (-2x)\right] + \sqrt{16-x^2} \cdot [2]$

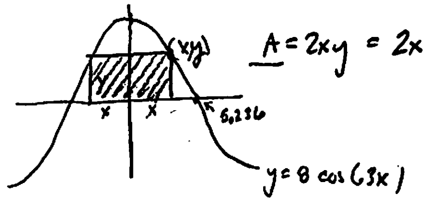
x	0	$2\sqrt{2}$	4
A	0	16	0

MAX AREA = 16

$\boxed{A'} = \dfrac{-2x^2}{\sqrt{16-x^2}} + 2\sqrt{16-x^2} = \dfrac{-2x^2 + 2(16-x^2)}{\sqrt{16-x^2}} = \dfrac{-2x^2+32-2x^2}{\sqrt{16-x^2}} = \boxed{\dfrac{32-4x^2}{\sqrt{16-x^2}}}$

$A'=0$ when $32-4x^2=0$, $\pm 2\sqrt{2}=x$

21. [Calculator Allowed] A rectangle is inscribed under one arch of $y=8\cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?



$A = 2xy = 2x \cdot 8\cos(.3x) = 16x\cos(.3x)$ Domain: $[0, 5.236]$

$A' = 16x \cdot [-\sin(.3x)\cdot(.3)] + \cos(.3x)\cdot[16]$

$A'=0$ at $x \approx 2.8677786...$ (STORE AS B)

x	A
0	0
B	29.925
5.236	0

MAX AREA ≈ 29.925

22. The function f is continuous on $[0,3]$ and satisfies the following:

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

Check endpoints & critical points

x = 0
x = 3

f ' = 0	f ' DNE
x = 1	x = 2

x	0	1	2	3
f(x)	0	-2	0	3

MAX = 3

MIN = -2

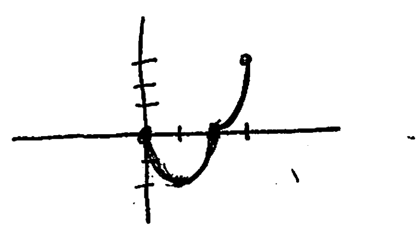
b) Find any points of inflection.

where f" changes signs ... NONE, f" is always POSITIVE

c) Sketch a possible graph of f.

KNOWN POINTS
(0,0)
(1,-2)
(2,0)
(3,3)

f ' : – + + (0, 1, 2, 3)
f" : + + +
f : DEC CU | INC CU | INC CU



NOTE... POINTY PLACE @ x=2 b/c f '(2) DNE

Go back and Review the questions from your assignments in this chapter … especially those in section 4.3.

Understand your notecards!