

HW: Set 3 - MC Qs in Differentiation

11/13/12

$$\begin{aligned}\textcircled{1} \quad y' &= 4(1-x)^3 + 3(1-x)^2(-1)(4x+1) \\ &= (1-x)^2 [4(1-x) + 3(-1)(4x+1)] \\ &= (1-x)^2 (4-4x-3-12x) \\ &= (1-x)^2 (1-16x)\end{aligned}$$

$$\begin{aligned}\textcircled{2} \quad y' &= \frac{(3x+1)(-1) - (2-x)(3)}{(3x+1)^2} = \frac{-3x-1-6+3x}{(3x+1)^2} \\ &= \frac{-7}{(3x+1)^2}\end{aligned}$$

$$\begin{aligned}\textcircled{3} \quad y &= (3-2x)^{1/2} \\ y' &= \frac{1}{2}(3-2x)^{-1/2}(-2) \\ &= \frac{-2}{2\sqrt{3-2x}} = \frac{-1}{\sqrt{3-2x}}\end{aligned}$$

$$\begin{aligned}\textcircled{4} \quad y' &= \frac{(5x+1)^3(0) - 2(3(5x+1)^2(5))}{[(5x+1)^3]^2} \\ &= \frac{-30(5x+1)^2}{(5x+1)^6} = \frac{-30}{(5x+1)^4}\end{aligned}$$

$$\begin{aligned}\textcircled{5} \quad y' &= \frac{2}{3}(3x^{-1/3}) - \frac{1}{2}(4)x^{-1/2} \\ &= \frac{2}{3\sqrt[3]{x}} - \frac{2}{\sqrt{x}}\end{aligned}$$

$$\textcircled{6} A' = f' + 2g'$$

$$A'(3) = 4 + 2(-1) = 2$$

$$\textcircled{7} B' = f'g + g'f$$

$$B'(2) = 3 \cdot 1 + (-2)(5) = 3 - 10 = -7$$

$$\textcircled{8} D' = \frac{g(0) - 1(g')}{g^2} = \frac{-g'}{g^2}$$

$$D'(1) = \frac{-(-3)}{3^2} = \frac{1}{3}$$

$$\textcircled{9} H(x) = (f(x))^{1/2}$$

$$H'(x) = \frac{1}{2}(f(x))^{-1/2}(f'(x)) = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$H'(3) = \frac{f'(3)}{2\sqrt{f(3)}} = \frac{4}{2\sqrt{10}} = \frac{2}{\sqrt{10}}$$

$$\textcircled{10} K(x) = \left(\frac{f}{g}\right)(x)$$

$$K'(x) = \frac{gf' - fg'}{g^2}$$

$$K'(0) = \frac{5(1) - 2(-4)}{5^2} = \frac{5 + 8}{25} = \frac{13}{25}$$

$$\textcircled{11} M(x) = f(g(x))$$

$$M'(x) = f'(g(x))g'(x)$$

$$M'(1) = f'(3)g'(1)$$

$$= 4(-3)$$

$$= -12$$

CHAIN RULE

$$(12) P(x) = f(x^3)$$

$$P'(x) = f'(x^3) \cdot 3x^2$$

$$\begin{aligned} P'(1) &= f'(1) \cdot 3(1)^2 \\ &= 2 \cdot 3 \\ &= 6 \end{aligned}$$

$$(14) y = 2\sqrt{x} - \frac{1}{2\sqrt{x}} = 2x^{1/2} - \frac{1}{2}x^{-1/2}$$

$$\begin{aligned} y' &= \frac{1}{2}(2x^{-1/2}) - (-\frac{1}{2})(\frac{1}{2}x^{-3/2}) \\ &= \frac{1}{\sqrt{x}} + \frac{1}{4\sqrt{x^3}} = \frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}} \end{aligned}$$

$$(15) y = (x^2 + 2x - 1)^{1/2}$$

$$y' = \frac{1}{2}(x^2 + 2x - 1)^{-1/2} (2x + 2)$$

$$= \frac{2x + 2}{2\sqrt{x^2 + 2x - 1}} = \frac{x + 1}{\sqrt{x^2 + 2x - 1}} = \frac{x + 1}{y}$$

(23) f can't have vertical asymptote at $x=a$ since f is linear function with positive slope when $x < a$ and quadratic function when $x > a$.

(24) Horizontal tangent to f at $x=2$

$$(25) f'(1.5) \approx \frac{22-14}{1.6-1.4} = \frac{8}{0.2} = 40$$

$$(27) y = x^2 \sin \frac{1}{x}$$

$$\begin{aligned} \frac{dy}{dx} &= x^2 \left(\cos \frac{1}{x} \right) \left(-\frac{1}{x^2} \right) + 2x \left(\sin \frac{1}{x} \right) \\ &= -\cos \frac{1}{x} + 2x \sin \frac{1}{x} \end{aligned}$$

$$(32) \quad y = \frac{1+x^2}{1-x^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2} \\ &= \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2} \end{aligned}$$

(36) at $x=5$, f is semi-circle with radius = 2 and center at $(4,0)$

$$f = -\sqrt{4 - (x-4)^2} = -(4 - (x-4)^2)^{1/2}$$

$$\begin{aligned} f' &= -\frac{1}{2} (4 - (x-4)^2)^{-1/2} (-2(x-4)) \\ &= \frac{x-4}{\sqrt{4 - (x-4)^2}} \end{aligned}$$

$$f'(5) = \frac{5-4}{\sqrt{4-1^2}} = \frac{1}{\sqrt{3}}$$

$$(43) \quad \frac{d}{dx} (x^3 - xy + y^3) = \frac{d}{dx} (1)$$

$$3x^2 - (x \frac{dy}{dx} + y) + 3y^2 \frac{dy}{dx} = 0$$

$$3x^2 - x \frac{dy}{dx} - y + 3y^2 \frac{dy}{dx} = 0$$

$$(3y^2 - x) \frac{dy}{dx} = y - 3x^2$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

$$(44) \quad \frac{d}{dx} (x + \cos(x+y)) = 0$$

$$1 - [\sin(x+y)](1 + \frac{dy}{dx}) = 0$$

$$(1 + \frac{dy}{dx})(-\sin(x+y)) = -1$$

$$1 + \frac{dy}{dx} = \frac{1}{\sin(x+y)}$$

$$\frac{dy}{dx} = \frac{1}{\sin(x+y)} - 1$$

$$= \csc(x+y) - 1$$

$$(45) \quad \frac{d}{dx} (\sin x - \cos y - 2) = 0$$

$$\cos x + (\sin y) \frac{dy}{dx} = 0$$

$$\sin y \frac{dy}{dx} = -\cos x$$

$$\frac{dy}{dx} = \frac{-\cos x}{\sin y} = -\cos x \cdot \csc y$$

$$(46) \quad \frac{d}{dx} (3x^2 - 2xy + 5y^2) = 1$$

$$6x - 2y - 2x \frac{dy}{dx} + 10y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (10y - 2x) = 2y - 6x$$

$$\frac{dy}{dx} = \frac{2y - 6x}{10y - 2x} = \frac{2(y - 3x)}{2(5y - x)} = \frac{y - 3x}{5y - x}$$

$$(49) \quad f(x) = 16\sqrt{x} = 16x^{1/2}$$

$$f'(x) = \frac{1}{2}(16x^{-1/2}) = 8x^{-1/2}$$

$$f''(x) = -\frac{1}{2}(8x^{-3/2}) = -4x^{-3/2} = \frac{-4}{\sqrt{x^3}}$$

$$f''(4) = \frac{-4}{\sqrt{4^3}} = \frac{-4}{\sqrt{64}} = \frac{-4}{8} = -\frac{1}{2}$$

$$(51) \quad \frac{d}{dx} (x^2 + y^2 = 25)$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-1y - (-x) \frac{dy}{dx}}{y^2} = \frac{-y}{y^2} + \frac{x}{y^2} \cdot \frac{-x}{y}$$

$$= -\frac{1}{y} - \frac{x^2}{y^3}$$

$$\left. \frac{d^2y}{dx^2} \right|_{(0,5)} = -\frac{1}{5} - \frac{0}{5^3} = -\frac{1}{5}$$