Do: 1-13 odd, 19, 21, 23

EXERCISES FOR SECTION 5.6

In Exercises 1–10, solve the differential equation.

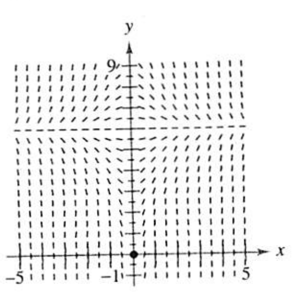
1. $\dfrac{dy}{dx} = x + 2$
2. $\dfrac{dy}{dx} = 4 - x$
3. $\dfrac{dy}{dx} = y + 2$
4. $\dfrac{dy}{dx} = 4 - y$
5. $y' = \dfrac{5x}{y}$
6. $y' = \dfrac{\sqrt{x}}{3y}$
7. $y' = \sqrt{x}\, y$
8. $y' = x(1 + y)$
9. $(1 + x^2)y' - 2xy = 0$
10. $xy + y' = 100x$

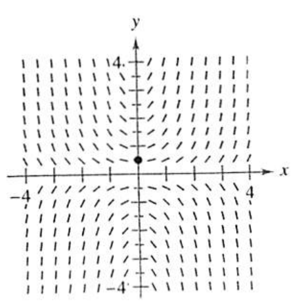
In Exercises 11–14, write and solve the differential equation that models the verbal statement.

11. The rate of change of Q with respect to t is inversely proportional to the square of t.
12. The rate of change of P with respect to t is proportional to $10 - t$.
13. The rate of change of N with respect to s is proportional to $250 - s$.
14. The rate of change of y with respect to x varies jointly as x and $L - y$.

Slope Fields **In Exercises 15 and 16, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketch in part (a). To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

15. $\dfrac{dy}{dx} = x(6 - y), \quad (0, 0)$
16. $\dfrac{dy}{dx} = xy, \quad \left(0, \frac{1}{2}\right)$





In Exercises 17–20, find the function $y = f(t)$ passing through the point (0, 10) with the given first derivative. Use a graphing utility to graph the solution.

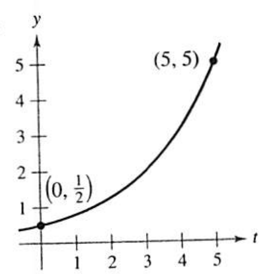
17. $\dfrac{dy}{dt} = \dfrac{1}{2}t$
18. $\dfrac{dy}{dt} = -\dfrac{3}{4}\sqrt{t}$
19. $\dfrac{dy}{dt} = -\dfrac{1}{2}y$
20. $\dfrac{dy}{dt} = \dfrac{3}{4}y$

In Exercises 21–24, write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.

21. The rate of change of y is proportional to y. When $x = 0$, $y = 4$ and when $x = 3$, $y = 10$. What is the value of y when $x = 6$?
22. The rate of change of N is proportional to N. When $t = 0$, $N = 250$ and when $t = 1$, $N = 400$. What is the value of N when $t = 4$?
23. The rate of change of V is proportional to V. When $t = 0$, $V = 20{,}000$ and when $t = 4$, $V = 12{,}500$. What is the value of V when $t = 6$?
24. The rate of change of P is proportional to P. When $t = 0$, $P = 5000$ and when $t = 1$, $P = 4750$. What is the value of P when $t = 5$?

In Exercises 25–28, find the exponential function $y = Ce^{kt}$ that passes through the two given points.

25.



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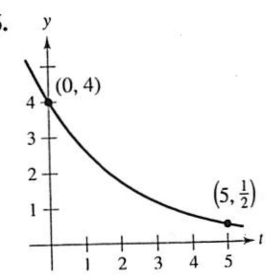

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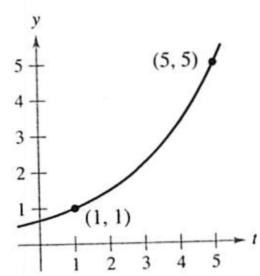

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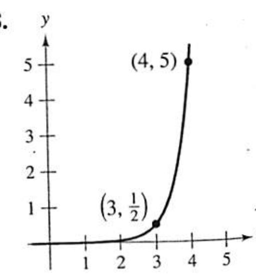


Getting at the Concept

29. In your own words, describe what is meant by a differential equation. Give an example.
30. Give the differential equation that models exponential growth and decay.

In Exercises 31 and 32, determine the quadrants in which the solution of the differential equation is an increasing function. Explain. (Do not solve the differential equation.)

31. $\dfrac{dy}{dx} = \dfrac{1}{2}xy$
32. $\dfrac{dy}{dx} = \dfrac{1}{2}x^2y$

Radioactive
the radioac

Isotope
33. ^{226}Ra
34. ^{226}Ra
35. ^{226}Ra
36. ^{14}C
37. ^{14}C
38. ^{14}C
39. ^{239}Pu
40. ^{239}Pu

41. ***Radioa*** approx remain
42. ***Carbo*** dioxide centuri tree tha amoun ancient carbon tree bu is 5730

Compound a savings a

Initial
Investi
43. \$1000
44. \$20,00
45. \$750
46. \$10,00
47. \$500
48. \$2000

Compounc that must \$500,000 v

49. $r = 7\frac{1}{2}$
51. $r = 8$

Compounc for \$1000 annually,

53. $r = 7$
55. $r = 8.$

107. $\frac{dy}{dt} = \frac{8}{25}y\left(\frac{5}{4} - y\right), y(0) = 1$

$$\frac{dy}{y[(5/4) - y]} = \frac{8}{25}dt \Rightarrow \frac{4}{5}\int\left(\frac{1}{y} + \frac{1}{(5/4) - y}\right)dy = \int\frac{8}{25}dt \Rightarrow$$

$$\ln y - \ln\left(\frac{5}{4} - y\right) = \frac{2}{5}t + C$$

$$\ln\left(\frac{y}{(5/4) - y}\right) = \frac{2}{5}t + C$$

$$\frac{y}{(5/4) - y} = e^{(2/5)t + C} = C_1e^{(2/5)t}$$

$$y(0) = 1 \Rightarrow C_1 = 4 \Rightarrow 4e^{(2/5)t} = \frac{y}{(5/4) - y}$$

$$\Rightarrow 4e^{(2/5)t}\left(\frac{5}{4} - y\right) = y \Rightarrow 5e^{(2/5)t} = 4e^{(2/5)t}y + y = (4e^{(2/5)t} + 1)y$$

$$\Rightarrow y = \frac{5e^{(2/5)t}}{4e^{(2/5)t} + 1} = \frac{5}{4 + e^{-0.4t}} = \frac{1.25}{1 + 0.25e^{-0.4t}}$$

Section 5.6 Differential Equations: Growth and Decay

1. $\frac{dy}{dx} = x + 2$

$$y = \int(x + 2)dx = \frac{x^2}{2} + 2x + C$$

3. $\frac{dy}{dx} = y + 2$

$$\frac{dy}{y + 2} = dx$$

$$\int\frac{1}{y + 2}dy = \int dx$$

$$\ln|y + 2| = x + C_1$$

$$y + 2 = e^{x + C_1} = Ce^x$$

$$y = Ce^x - 2$$

5. $y' = \frac{5x}{y}$

$$yy' = 5x$$

$$\int yy'\,dx = \int 5x\,dx$$

$$\int y\,dy = \int 5x\,dx$$

$$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C_1$$

$$y^2 - 5x^2 = C$$

7. $y' = \sqrt{x}\,y$

$$\frac{y'}{y} = \sqrt{x}$$

$$\int\frac{y'}{y}dx = \int\sqrt{x}\,dx$$

$$\int\frac{dy}{y} = \int\sqrt{x}\,dx$$

$$\ln y = \frac{2}{3}x^{3/2} + C_1$$

$$y = e^{(2/3)x^{3/2} + C_1}$$

$$= e^{C_1}e^{(2/3)x^{3/2}}$$

$$= Ce^{(2/3)x^{3/2}}$$

9. $(1 + x^2)y' - 2xy = 0$

$$y' = \frac{2xy}{1 + x^2}$$

$$\frac{y'}{y} = \frac{2x}{1 + x^2}$$

$$\int \frac{y'}{y}\,dx = \int \frac{2x}{1 + x^2}\,dx$$

$$\int \frac{dy}{y} = \int \frac{2x}{1 + x^2}\,dx$$

$$\ln y = \ln(1 + x^2) + C_1$$

$$\ln y = \ln(1 + x^2) + \ln C$$

$$\ln y = \ln C(1 + x^2)$$

$$y = C(1 + x^2)$$

11. $\dfrac{dQ}{dt} = \dfrac{k}{t^2}$

$$\int \frac{dQ}{dt}\,dt = \int \frac{k}{t^2}\,dt$$

$$\int dQ = -\frac{k}{t} + C$$

$$Q = -\frac{k}{t} + C$$

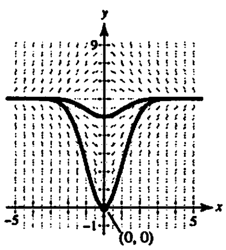
13. $\dfrac{dN}{ds} = k(250 - s)$

$$\int \frac{dN}{ds}\,ds = \int k(250 - s)\,ds$$

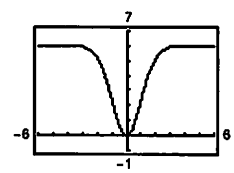
$$\int dN = -\frac{k}{2}(250 - s)^2 + C$$

$$N = -\frac{k}{2}(250 - s)^2 + C$$

15. (a)



(b) $\dfrac{dy}{dx} = x(6 - y),\ (0, 0)$



$$\frac{dy}{y - 6} = -x$$

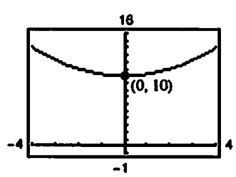
$$\ln|y - 6| = \frac{-x^2}{2} + C$$

$$y - 6 = e^{-x^2/2 + C} = C_1 e^{-x^2/2}$$

$$y = 6 + C_1 e^{-x^2/2}$$

$$(0, 0):\ 0 = 6 + C_1 \Rightarrow C_1 = -6 \Rightarrow y = 6 - 6e^{-x^2/2}$$

17. $\dfrac{dy}{dt} = \dfrac{1}{2}t,\ (0, 10)$



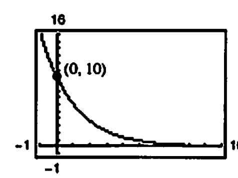
$$\int dy = \int \frac{1}{2}t\,dt$$

$$y = \frac{1}{4}t^2 + C$$

$$10 = \frac{1}{4}(0)^2 + C \Rightarrow C = 10$$

$$y = \frac{1}{4}t^2 + 10$$

19. $\dfrac{dy}{dt} = -\dfrac{1}{2}y,\ (0, 10)$



$$\int \frac{dy}{y} = \int -\frac{1}{2}\,dt$$

$$\ln y = -\frac{1}{2}t + C_1$$

$$y = e^{-(t/2) + C_1} = e^{C_1} e^{-t/2} = Ce^{-t/2}$$

$$10 = Ce^0 \Rightarrow C = 10$$

$$y = 10e^{-t/2}$$

21. $\dfrac{dy}{dx} = ky$

$y = Ce^{kx}$ (Theorem 5.16)

$(0, 4)$: $4 = Ce^0 = C$

$(3, 10)$: $10 = 4e^{3k} \Rightarrow k = \dfrac{1}{3}\ln\left(\dfrac{5}{2}\right)$

When $x = 6$, $y = 4e^{1/3 \ln(5/2)(6)} = 4e^{\ln(5/2)^2}$

$$= 4\left(\frac{5}{2}\right)^2 = 25$$

23. $\dfrac{dV}{dt} = kV$

$V = Ce^{kt}$ (Theorem 5.16)

$(0, 20{,}000)$: $C = 20{,}000$

$(4, 12{,}500)$: $12{,}500 = 20{,}000e^{4k} \Rightarrow k = \dfrac{1}{4}\ln\left(\dfrac{5}{8}\right)$

When $t = 6$, $V = 20{,}000e^{1/4 \ln(5/8)(6)} = 20{,}000e^{\ln(5/8)^{3/2}}$

$$= 20{,}000\left(\frac{5}{8}\right)^{3/2} \approx 9882.118$$