5.1 Estimating with Finite Sums   Calculus

5.1 ESTIMATING WITH FINITE SUMS

Notecards from Section 5.1 & 5.5: LRAM, RRAM, MRAM, Trapezoidal Rule

Example 1: Suppose from the 2nd to 4th hour of your road trip, you travel with the cruise control set to exactly 70 miles per hour for that two hour stretch. How far have you traveled during this time?

Example 2: Sketch a graph modeling the situation in the above example. Geometrically, how can we indicate the total distance traveled?

Example 3: What if the velocity was NOT constant. Say, for instance the velocity in miles per hour is given by the function $v\ t\ = 10t - t^2$, where t is in hours, and we wanted to know the total distance traveled during the first 10 hours. Sketch this graph below. Geometrically speaking, do you think we can find the total distance traveled in the same was as before? Why or why not?

Word of caution to those brave few who are actually reading this ... The following paragraphs are extremely important to the conceptual understanding of what we are about to do in Calculus. However, since you really haven't done anything yet, it might make you a little dizzy at first, so come back and read it again later. If you're reading this for the first time, you might experience that same feeling you get when you've been on the Tilt –A – Whirl one too many times at the fair. (Never been on the Tilt – A – Whirl? ... well, take my word for it, it's not something you want to ride 10 minutes after eating a corn dog and a funnel cake!) Well, I warned you, but you've kept on reading anyway, so here it goes ...

The key to finding the total distance traveled in the last example in a method similar to the first example is to break the time intervals into such short segments, that the velocity over those time segments is almost constant (this will require quite a few intervals). If the velocity is almost constant for each time interval, then we can find the distance traveled for each time interval (which is just the area of an extremely thin rectangle) and add all the areas of all the rectangles together. Sounds simple enough, right? Can you guess what extremely important calculus concept is involved?

We will spend MUCH more time with this later, but it turns out that if we are given the graph of a rate of change (like velocity in miles per hour) we will be able to find the total accumulated change over an interval (like total distance traveled, in miles) by finding the area under the curve.

OK, that last paragraph or two may not have made perfect sense to you ... YET! ... For now, THE GOAL is to figure out a way to find the area under the curve. This chapter actually discusses 5 ways to approximate this area, LRAM, RRAM, MRAM, Trapezoid Rule, and Simpson's Rule, but we're only going to deal with 4 of them. (Simpson's Rule is not in the AP curriculum)

The Area Problem and the Rectangular Approximation Method (RAM)
(a.k.a. Riemann Sums)

Suppose we wanted to know the area of the region bounded by a curve, the x – axis, and the lines $x = a$ and $x = b$, as shown at the right.

The <u>first step</u> is to divide the interval from a to b into subintervals. (The examples below show 4 and 8 subintervals, respectively.)

After dividing the given interval into subintervals, we can then draw rectangles using the width of each subinterval as the base.

The *<u>height of each rectangle is determined by the function value at a point in the specific subinterval</u>*, and can be determined using 3 different methods.

We could use the left endpoint of each subinterval (called LRAM), the right endpoint of each subinterval (RRAM), or the midpoint of each subinterval (MRAM).

Example 4: Which method is shown in the two graphs below?
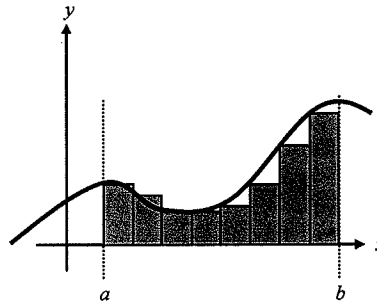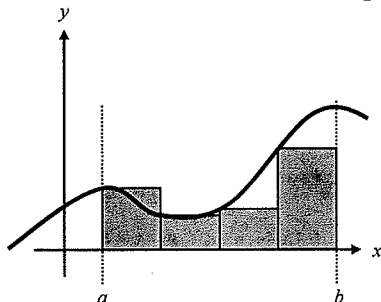
Example 5: The total area under the curve then is approximately equal to the total area of all the rectangles. Which of the graphs above gives a better approximation of the area under the curve? Why? How could it be further improved?

Summary of the Process: A sketch is almost mandatory!

   Step 1: Divide (or Partition) the interval into n subintervals.

        Step 2: Create n rectangles whose base equals the width of each subinterval and whose height is determined by the function value at the left endpoint, the right endpoint, or the midpoint of the subinterval.

        Step 3: Find the area of all n rectangles and add them together.

Example 6: Illustrate the use of RRAM and MRAM on the graphs below. (use 4 rectangles)
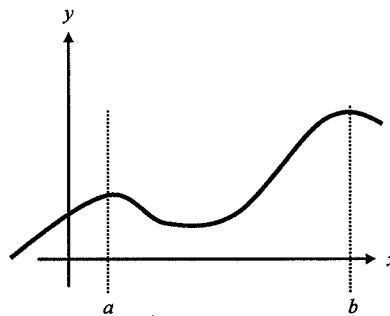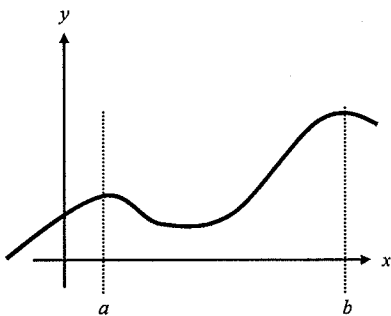
Example 7: Use 4 rectangles to approximate the area under the graph of $y = x^2 - 2x + 2$ from $x = 1$ to $x = 3$. Use LRAM, RRAM, and then MRAM.
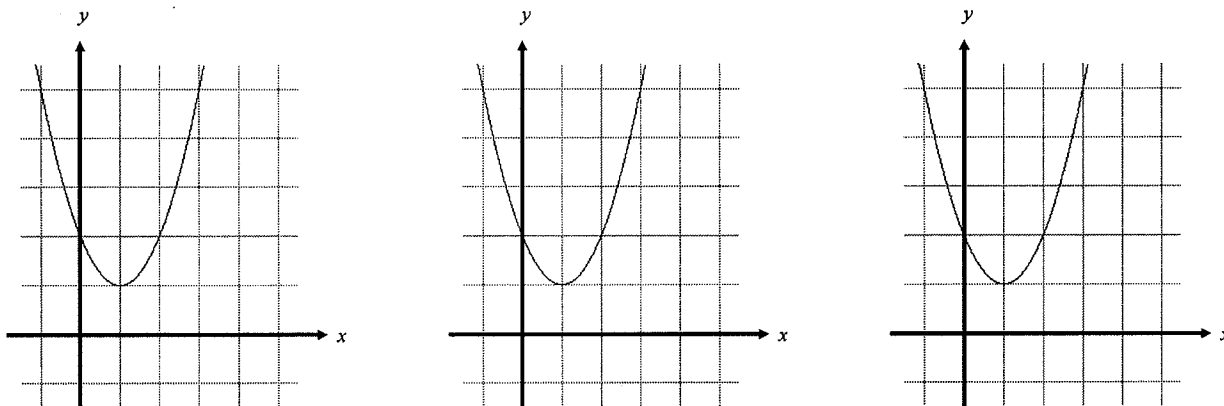
Example 8: Using your rectangles as a guide, find each approximation.

    a) LRAM =

    b) RRAM =

    c) MRAM =

Example 9: It is not necessary to have a graph to estimate the area. Suppose the table below shows the velocity of a model train engine moving along a track for 10 seconds.

Time (sec)	Velocity (in./sec)	Time (sec)	Velocity (in./sec)
0	0	6	11
1	12	7	6
2	22	8	2
3	10	9	6
4	5	10	0
5	13		

    a) Using a left Riemann Sum with 10 subintervals, estimate the distance traveled by the engine in the first 10 seconds.

    b) Using a Midpoint Riemann Sum with 5 subintervals, estimate the distance traveled by the engine in the first 10 seconds.