AP Calculus AB – More Review Chp 4 & Sec. 5.6

1. $xy=10$ Find dy/dt when $x=8$ given that $dx/dt=5$

2. Find the $\frac{d^2y}{dx^2}$ given $1 - xy = x - y$

3. Find dy/dx of the curve $y = \cos^3(x^2)$

4. Find the slope of the curve:
   $y = \sqrt{x^2+1}\,\sin(2x)$

5. Find the velocity of the function if its position is $s(t) = 4x^2\tan(x^3-1)$.

6. Assume x and y are both differentiable functions of time.
   If $3x - 4y^3 = -32$ find dx/dt when $x = 0$ and $dy/dt = 3$.

7. Find the equation of the tangent line to the curve at the point (1,-1)
$x^2y + 3x = y^2 + 1$

8. Find dy/dx for the following curve $x^2y + y^2x = -2$

9. Find the equation of the tangent line to the curve $y = x\sin 4x$ when $x = \pi$

10. Find the slope of the curve $y^2 + yx + 3x - 6y = -3$ at the point(s) when $x=1$

8. Find the derivative of the function: $y = \frac{4}{x^3}$.

(A) $-4x^2$

(B) $-\frac{12}{x^2}$

(C) $\frac{12}{x^2}$

(D) $\frac{12}{x^4}$

(E) $-\frac{12}{x^4}$

9. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(7)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) none of the above

10. Find the second derivative of $f(x)$ if $f(x) = (2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

11. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

(A) $8\sin(3x)$

(B) $24\sin(3x)$

(C) $8\sin(3x)\cos(3x)$

(D) $12\sin(3x)\cos(3x)$

(E) $24\sin(3x)\cos(3x)$

12. The equation of the tangent line to the graph of the function $f(x) = \cos(x)$ at $x = \frac{\pi}{2}$ is:

(a) $y = 0$

(b) $y = x - 1$

(c) $y = -x + \dfrac{\pi}{2}$

(d) $y = -\sin(x)$

(e) $y = x - \dfrac{\pi}{2}$

13. The derivative of the function $\dfrac{\sin(2x)}{1 + x^2}$ is:

(a) $\dfrac{\cos(2x)}{x}$

(b) $\dfrac{\cos(2x)}{(1 + x^2)^2}$

(c) $\dfrac{\cos(2x)}{2x}$

(d) $2\dfrac{\cos(2x)(1 + x^2) - x\sin(2x)}{(1 + x^2)^2}$

(e) $\dfrac{\cos(2x)(1 + x^2) - 2x\sin(2x)}{(1 + x^2)^2}$

14) The slope of the tangent is -1 at the point (0,1) on $x^3-6xy-ky^3=a$, where k and a are constants.

The values of the constant a and k are:

a) k=1, a=-1 b) k=2, a=-2 c) k=3, a=-3 d) k=-2, a=4 e) k=-1, a=3

15) What is the slope of the line tangent to the curve $4x^2+3xy=34$ at the point (2,3)?

a) -16/3 b) -25/6 C) -5 d) -4 e) 7/6

16) Given $x\cos y=x^2 + y^3$, then dy/dx =

a) $\dfrac{2x}{-3y^2-\sin y}$ b) $\dfrac{2x}{3y^2-\sin y}$ c) $\dfrac{2x+\cos y}{-3y^2-\sin y}$

d) $\dfrac{2x+\cos y}{3y^2+x\sin y}$ e) $\dfrac{2x-\cos y}{3y^2-x\sin y}$

AP Calculus AB – More Review Chp 4 & Sec. 5.6

1. xy=10 Find dy/dt when x=8 given that dx/dt =5

Product Rule

$\frac{dx}{dt}y + x\frac{dy}{dt} = 0$

$8y = 10$

$y = \frac{5}{4}$

$5\left(\frac{5}{4}\right) + 8\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-\frac{25}{4}}{8}$

$\frac{dy}{dt} = \frac{-25}{4}\cdot\frac{1}{8} = \frac{-25}{32}$

2. Find the $\frac{d^2y}{dx^2}$ given $\mathbf{1 - xy = x - y}$

$-y - xy' = 1 - y'$

$y' - xy' = 1 + y$

$y' = \frac{1+y}{1-x}$

Quotient Rule

$y'' = \frac{y'(1-x) - (-1)(1+y)}{(1-x)^2}$

$y'' = \frac{\frac{(1+y)\cancel{(1-x)}}{\cancel{(1-x)}} + (1+y)}{(1-x)^2} = \frac{2+2y}{(1-x)^2}$

3. Find dy/dx of the curve $y = \cos^3(x^2)$

Chain Rule

$u = \cos(x^2)$

$\frac{du}{dx} = -\sin(x^2)\cdot(2x)$

$y = u^3$

$y' = 3u^2\cdot\frac{du}{dx} = 3(\cos x^2)^2(-2x\sin(x^2))$

$\boxed{= -6x\cos^2(x^2)\sin(x^2)}$

4. Find the slope of the curve:
 $y = \sqrt{x^2+1}\sin(2x)$ Product Rule & Chain Rule

$y' = \cancel{2}x\cdot\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\sin(2x) + 2\cos(2x)\sqrt{x^2+1}$

$\boxed{y' = \frac{x\sin(2x)}{\sqrt{x^2+1}} + 2\cos(2x)\sqrt{x^2+1}}$

5. Find the velocity of the function if its position is $s(t) = 4x^2\tan(x^3-1)$. Product Rule & Chain Rule

$v(t) = p'(t) = 8x\tan(x^3-1) + 4x^2\sec^2(x^3-1)\cdot 3x^2$

$\boxed{v(t) = 8x\tan(x^3-1) + 12x^4\sec^2(x^3-1)}$

6. Assume x and y are both differentiable functions of time.
 If $3x^2 - 4y^3 = -32$ find dx/dt when x = 0 and dy/dt = 3.

$3(0)^2 - 4y^3 = -32$

$y^3 = 8$

$y = \sqrt[3]{8} = 2$

$(0, 2)$

$6x\frac{dx}{dt} - 12y^2\frac{dy}{dt} = 0$

$6(0)\frac{dx}{dt} - 12(2)^2 3 = 0$

$\frac{dy}{dt}$ = undefined

* To change this prob. to work make it $3x - 4y^3 = -32$

$3\frac{dx}{dt} - 12y^2\frac{dy}{dt} = 0$

$3\frac{dx}{dt} - 12(2)^2(3) = 0$

$\frac{dx}{dt} = \frac{144}{3} = 48$

7. Find the equation of the tangent line to the curve at the point (1,-1)
$x^2y + 3x = y^2 + 1$

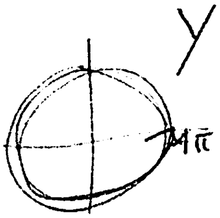
$$2xy + x^2y' + 3 = 2yy'$$
$$x^2y' - 2yy' = -3 - 2xy$$
$$y'(x^2 - 2y) = -3 - 2xy$$
$$\rightarrow y' = \frac{-3-2xy}{x^2-2y}$$
$$y'\big|_{(1,-1)} = \frac{-3-2(1)(-1)}{1^2-2(-1)} = \frac{-1}{3}$$

8. Find dy/dx for the following curve $x^2y + y^2x = -2$

$$2xy + x^2y' + 2yy'x + y^2 = 0$$
$$y'(x^2 + 2xy) = -2xy - y^2$$
$$\boxed{y' = \frac{-2xy - y^2}{x^2 + 2xy}}$$

9. Find the equation of the tangent line to the curve $y = x\sin 4x$ when $x = \pi$

$$y(\pi) = \pi \sin 4\pi = \pi \cdot 0 = 0 \qquad (\pi, 0)$$



$$y' = \sin(4x) + x(4\cos 4x)$$
$$y'(\pi) = \sin(4\pi) + \pi(4\cos 4\pi)$$
$$0 + \pi(4(1)) = 4\pi$$
$$y - 0 = 4\pi(x - \pi)$$
$$\boxed{y = 4\pi(x - \pi)}$$

10. Find the slope of the curve $y^2 + yx + 3x - 6y = -3$ at the point(s) when x=1

$$y^2 + y(1) + 3(1) - 6y = -3$$
$$y^2 - 5y + 6 = 0$$
$$(y-2)(y-3) = 0$$
$$y = 2, 3$$
(1,2) & (1,3)

Two points

Slope

$$2yy' + y'x + y + 3 - 6y' = 0$$
$$y'(2y + x - 6) = -3 - y$$
$$y' = \frac{-3-y}{2y+x-6}$$

$$\rightarrow y'\big|_{(1,2)} = \frac{-3-2}{2(2)+1-6} = \frac{-5}{-1} = 5$$

$$y'\big|_{(1,3)} = \frac{-3-3}{2(3)+1-6} = \frac{-6}{1} = -6$$

Textbook Pg 186 #1-13 odds

8 Find the derivative of the function: $y=\frac{4}{x^3}$. $= 4x^{-3}$

(A) $-4x^2$

(B) $-\frac{12}{x^2}$

(C) $\frac{12}{x^2}$

(D) $\frac{12}{x^4}$

(E) $-\frac{12}{x^4}$

E

$y' = -12x^{-4} = \frac{-12}{x^4}$

9. If the nth derivative of y is denoted as $y^{(n)}$ and $y=-\sin x$, then $y^{(7)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) none of the above

D

$y = -\sin x$

$y' = -\cos x$

$y'' = \sin x$

$y''' = \cos x$

$y^{(4)} = -\sin x$

$y^{(5)} = -\cos x$

$y^{(6)} = \sin x$

$\boxed{y^{(7)} = \cos x}$

10. Find the second derivative of $f(x)$ if $f(x)=(2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

E

Chain Rule

$f'(x) = 4(2x+3)^3 \cdot 2$

$8(2x+3)^3$

$f''(x) = 8\cdot 3(2x+3)^2 \cdot 2$

$f''(x) = 48(2x+3)^2$

11. Find $\frac{dy}{dx}$ for $y=4\sin^2(3x)$. $= 4u^2$

E

(A) $8\sin(3x)$

(B) $24\sin(3x)$

(C) $8\sin(3x)\cos(3x)$

(D) $12\sin(3x)\cos(3x)$

(E) $24\sin(3x)\cos(3x)$

$u = \sin(3x)$

$\frac{du}{dx} = 3\cos(3x)$

$y' = 8u \cdot \frac{du}{dx}$

$y' = 8(\sin(3x)) \cdot 3\cos(3x)$

$24\sin(3x)\cos(3x)$

12 The equation of the tangent line to the graph of the function $f(x) = \cos(x)$ at $x = \frac{\pi}{2}$ is:

C

(a) $y = 0$

(b) $y = x - 1$

(c) $y = -x + \frac{\pi}{2}$

(d) $y = -\sin(x)$

(e) $y = x - \frac{\pi}{2}$

$$f'(x) = -\sin x$$

$$f'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$$

$$f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$y - 0 = -1\left(x - \frac{\pi}{2}\right) = -x + \frac{\pi}{2}$$

13 The derivative of the function $\frac{\sin(2x)}{1+x^2}$ is:

D

(a) $\frac{\cos(2x)}{x}$

(b) $\frac{\cos(2x)}{(1+x^2)^2}$

(c) $\frac{\cos(2x)}{2x}$

factored 2 out

(d) $2\frac{\cos(2x)(1+x^2) - x\sin(2x)}{(1+x^2)^2}$

(e) $\frac{\cos(2x)(1+x^2) - 2x\sin(2x)}{(1+x^2)^2}$

$$y' = \frac{2\cos(2x)(1+x^2) - 2x(\sin(2x))}{(1+x^2)^2}$$

14) The slope of the tangent is -1 at the point (0,1) on x^3-6xy-ky^3=a, where k and a are constants. The values of the constant a and k are:

y'

B

a) k=1, a=-1 b) k=2, a=-2 c) k=3, a=-3 d) k=-2, a=4 e) k=-1, a=3

$$3x^2 - 6y - 6xy' - 3ky^2y' = 0$$

$$3(0)^2 - 6(1) - 6(0)(-1) - 3k(1)^2(-1) = 0$$

$$-6 + 3k = 0$$

$$0^3 - 6(0)(-1) - (2)(1)^3 = a \rightarrow \boxed{-2 = a} \leftarrow \boxed{k = \frac{6}{3} = 2}$$

15) What is the slope of the line tangent to the curve $4x^2+3xy=34$ at the point (2,3)?

B

a) -16/3 b) -25/6 C) -5 d) -4 e) 7/6

$$8x + 3y + 3xy' = 0$$

$$y' = \frac{-8x - 3y}{3x}$$

$$y'\Big|_{(2,3)} = \frac{-8(2) - 3(3)}{3(2)}$$

$$\boxed{\frac{-25}{6}}$$

Product Rule

16) Given xcosy=x^2 + y^3 , then dy/dx =

$$\cos y + x(-\sin y)y' = 2x + 3y^2y'$$

$$-x\sin y)y' - 3y^2y' = 2x - \cos y$$

a) $\frac{2x}{-3y^2 - \sin y}$ b) $\frac{2x}{3y^2 - \sin y}$ c) $\frac{2x+\cos y}{-3y^2 - \sin y}$

d) $\frac{2x+\cos y}{3y^2 + x\sin y}$ e) $\frac{2x-\cos y}{3y^2 - x\sin y}$

No answer option is correct!

$$y'(-x\sin y - 3y^2) = 2x - \cos y$$

$$y' = \frac{2x - \cos y}{-x\sin y - 3y^2} = \frac{-2x + \cos y}{x\sin x + 3y^2}$$

AP Calculus AB REVIEW Sheet for 4.1, 4.2, 4.4 and 5.6 TEST!

In 1-11, Find dy/dx for each:

1. $y = e^{\cos x} + e^{\ln x}$
2. $y = 8^{\sec x}$
3. $y = 4\sin^5(5 - 2x)$
4. $y = \sqrt{1-\tan(3x)}$
5. $y = \ln(\sec x)$
6. $y = \csc(\frac{3}{x^2})$
7. $y=\log_6(3-x^2)$
8. $y=5^{-x}$
9. $y=x^2e^{-x}$
10. $y = \ln\sqrt{\cos x}$
11. $y=\left(\frac{x-3}{2x+1}\right)^3$
12. Find $\frac{dy}{dx}$ if $x^2y+3y^2 = x$.
13. Find $y'''(x)$ if $y=(4x+1)^7$

14. A container has the shape of an open right circular cone. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of -3/10 cm/hr. Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

15. A 14 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the end be moving away from the wall when the top is 6 ft above the ground?

16. An aircraft is climbing at a 45° angle to the horizontal. How fast is the aircraft gaining altitude if its horizontal speed is 400 mi/hr?

17. A pebble is dropped into a still pool and sends out a circular ripple whose radius increases at a constant rate of 4 ft/s. How fast is the area of the region enclosed by the ripple increasing at the end of 8 s?

Review from the book:

Page 168; #50, Page 186; 39, 45, 47, 66 a-d, 79

KEY

AP Calculus AB REVIEW Sheet for 4.1, 4.2, 4.4 and 5.6 TEST!

In 1-11, Find dy/dx for each:

1. $y = e^{\cos x} + e^{\ln x}$ $= e^{\cos x} + x$ $\quad y' = e^{\cos x}(-\sin x) + 1$

2. $y = 8^{\sec x}$ $\quad y' = 8^{\sec x}(\ln 8)(\sec x \tan x)$

3. $y = 4\sin^5(5 - 2x)$ $\quad y' = -40\sin^4(5-2x)\cos(5-2x)$

4. $y = \sqrt{1 - \tan(3x)}$ $= (1-\tan(3x))^{1/2}$ $\quad y' = \frac{1}{2}(1-\tan(3x))^{-1/2}(-\sec^2(3x)\cdot 3)$ $= \dfrac{-3\sec^2(3x)}{2\sqrt{1-\tan 3x}}$

5. $y = \ln(\sec x)$ $\quad y' = \frac{1}{\sec x}\cdot \sec x \tan x = \boxed{\tan x}$

6. $y = \csc(\frac{3}{x^2})$ $\quad y' = -\csc\left(\frac{3}{x^2}\right)\cot\left(\frac{3}{x^2}\right)\cdot -6x^{-3} = \frac{6}{x^3}\csc\left(\frac{3}{x^2}\right)\cot\left(\frac{3}{x^2}\right)$

7. $y=\log_6(3-x^2)$ $= \dfrac{\ln(3-x^2)}{\ln 6} = \dfrac{1}{\ln 6}\cdot\dfrac{1}{(3-x^2)}\cdot -2x = \dfrac{-2x}{\ln 6(3-x^2)}$

8. $y=5^{-x}$

$y' = 5^{-x}\cdot\ln 5(-1) = -5^{-x}\ln 5$

9. $y=x^2e^{-x}$ $\quad y' = 2xe^{-x} - x^2e^{-x}$

10. $y = \ln\sqrt{\cos x}$

$y = \frac{1}{2}\ln(\cos x)$ $\quad y' = \frac{1}{2}\cdot\frac{1}{\cos x}\cdot -\sin x = \frac{-\sin x}{2\cos x} = -\frac{1}{2}\tan x$

11. $y = \left(\dfrac{x-3}{2x+1}\right)^3$ $\quad y' = 3\left(\dfrac{x-3}{2x+1}\right)^2\left(\dfrac{(2x+1)-(x-3)(2)}{(2x+1)^2}\right) = 3\dfrac{(x-3)^2}{(2x+1)^2}\cdot\dfrac{7}{(2x+1)^2}$ $= \boxed{\dfrac{21(x-3)^2}{(2x+1)^4}}$

12. Find $\dfrac{dy}{dx}$ if $x^2y + 3y^2 = x$.

13. Find $y'''(x)$ if $y = (4x+1)^7$

(12) $2xy + x^2\dfrac{dy}{dx} + 6y\dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{1-2xy}{x^2+6y}$

(13) $y' = 7(4x+1)^6\cdot 4 = 28(4x+1)^6$

$y'' = 168(4x+1)^5\cdot 4 = 672(4x+1)^5$

$y''' = 3360(4x+1)^4\cdot 4$

$= \boxed{13{,}440(4x+1)^4}$