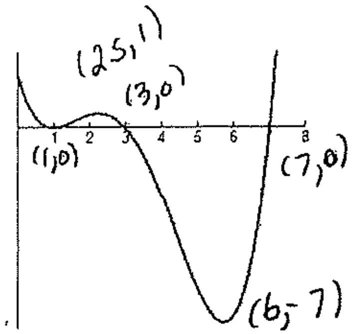
Chapter 3 Review for ch 5 test



graph of $f'(x)$

(FOR #1, #2)

1. The graph of the first derivative of a function is shown above. At what values of x does the function, $f(x)$, have relative extrema? Justify completely.

2. At what values of x does the function, $f(x)$, have points of inflection? Justify completely.

3. Let $f(x)$ be the function defined by $f(x) = k + 12x + 3x^2 - 2x^3$, where k is a constant.

(a) On what interval(s) is the function increasing? Justify your answer.

(b) If the relative maximum value of f is 30, then what is the value of k? [Justify]

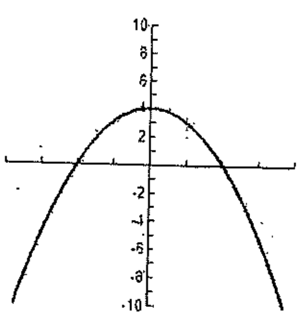
(c) Find the interval where the function f is concave up. [Justify]

4. Let f be a twice-differentiable function such that $f(10)=25$ and $f(25)=10$. And, let $g(x)$ be the function given by $g(x)=f(f(x))$.

(a) Explain why [using Calculus] there must be value c, $10<c<25$, such that $g'(c)=1$.

(b) Show that $g'(10)=g'(25)$. Use this result to show that there must be a value k, $10<k<25$, such that $g''(k)=0$.

5 Compare the values of $f(0)$, $f'(0)$, $f''(0)$



graph of f

6 The table below gives values of the velocity, $v(t)$, of a Inferi [An "inferi" is sort of like a zombie but made by the Dark Lord] at selected times.

t (sec)	0	1	3	6	10	15
$v(t)$ m/sec	5	1	−1	5	10	13

(a) Is there a time during $0<t<15$, that the velocity is equal to 9 m/sec Justify completely.

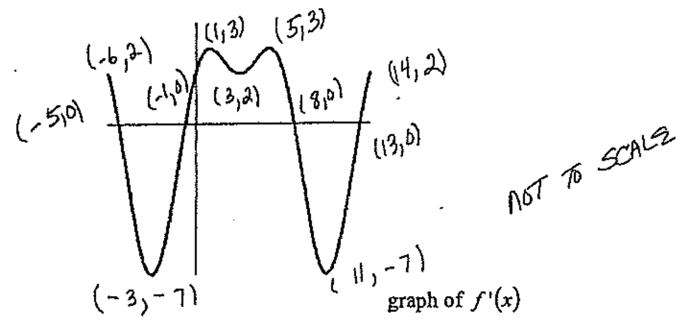
(b) Find an approximation for the acceleration at time $t=2$ and indicate units. [Show all work]

(c) Show that there must be a time interval such that the acceleration, $a(t)$, is equal to zero

5

My Chapter 5 Graph Handout 1

1. Let $f(x)$ be a differentiable function. The graph of $f'(x)$, the first derivative, is shown below.



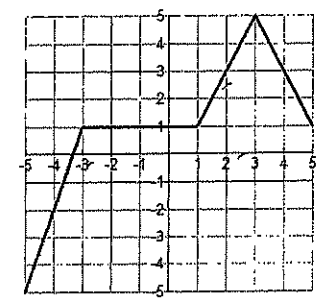
graph of $f'(x)$

(a) Use the graph of $f'(x)$ to locate the x-value(s) of all relative extrema of f

[Justify completely]

(b) Use the graph of $f'(x)$ to locate the x-value(s) of all points of inflection of f [Justify completely]

2. Let $g(x)$ be a differentiable function. The graph of $g'(x)$, the first derivative of $g(x)$ is shown below.



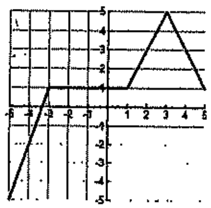
graph of $g'(x)$

(a) On what interval(s), does the graph of g increase? [Justify completely]

(b) Use the graph of $g'(x)$ to find the x-value(s) of any relative extrema of g

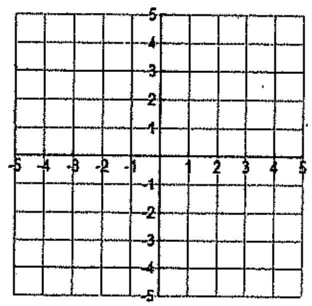
[Justify completely]

Question 2 continued



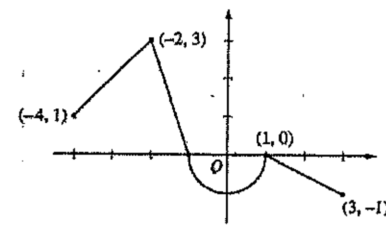
graph of $g'(x)$

(c) On the axes provided below, draw an accurate graph of $g''(x)$



(d) On what interval(s) is the graph of $g(x)$ concave down? [Justify completely]

3. The graph below is the graph of $g'(x)$ which is defined for the closed interval $[-4, 3]$



graph of $g'(x)$

(A) Identify the critical values of g [not $g'(x)$] and classify them as relative minima, maxima, or neither. Justify fully.

(B) Use a limit to prove that the $g''(-2)$ does not exist.

13. Suppose $\frac{d^2y}{dx^2} = x^3 - 4x^2$. Justify each response below.

a) Where is y concave up?

b) Where is y concave down?

c) Are there any inflection points on y? If so, where?

14. [Calculator Allowed] The derivative of $h(x)$ is given by $h'(x) = 2\cos\left(x - \frac{\pi}{6}\right) + 1$ on the interval $[-2\pi, 2\pi]$. Justify EVERY response.

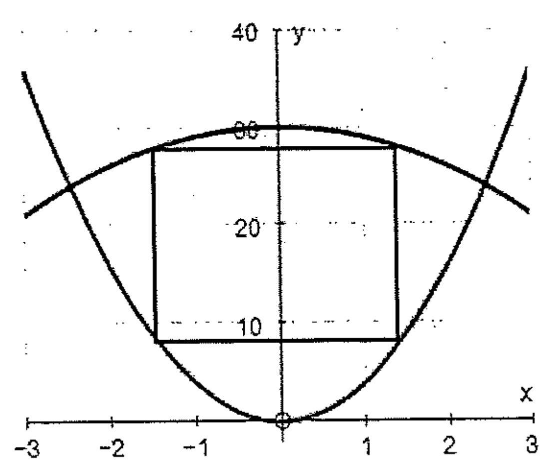
a) Where is $h(x)$ increasing?

b) Where is $h(x)$ concave down?

c) Find the x-coordinates of all extrema of $h(x)$ on the interval $[-2\pi, 2\pi]$.

d) Does $h(x)$ have a point(s) of inflection? If so, where?

15. A rectangle is inscribed between the parabolas $y = 4x^2$ and $y = 30 - x^2$ as shown in the picture. What is the maximum area of such a rectangle? Justify your response using CALCULUS.



16. **USE CALCULUS:** Find the maximum area of a rectangle inscribed under the curve $f(x)=\sqrt{16-x^2}$.

17. [Calculator Allowed] **USE CALCULUS:** A rectangle is inscribed under one arch of $y=8\cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?

18. The function f is continuous on [0, 3] and satisfies the following:

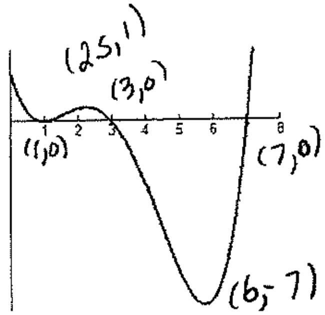
x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

b) Find any points of inflection.

c) Sketch a possible graph of f.

Chapter 3: Review for test ch3



graph of $f'(x)$ (FOR #1, #2)

1. The graph of the first derivative of a function is shown above. At what values of x does the function, $f(x)$, have relative extrema? Justify completely.

Rel max at $x=3$ because $f'(x)$ changed + to −
Rel min at $x=7$ because $f'(x)$ changed − to +

2. At what values of x does the function, $f(x)$, have points of inflection? Justify completely.

$x = 2.5, 1, 6$
because $f''(x) = 0$
and concavity changed

3. Let $f(x)$ be the function defined by $f(x) = k + 12x + 3x^2 - 2x^3$, where k is a constant.

(a) On what interval(s) is the function increasing? Justify your answer.

$f'(x) = -6x^2 + 6x + 12$
$-6(x^2 - x - 2)$
$x = 2, -1$

− − | + + | − −
−1 2

increasing $(-1, 2)$ because $f'(x) =$ ~~negat~~ positive

(b) If the relative maximum value of f is 30, then what is the value of k? [Justify]

$30 = k + 12(2) + 3(4) - 2(8)$

$k = 10$

(c) Find the interval where the function f is concave up. [Justify] $(-\infty, \frac{1}{2})$

concave up $(-\infty, \frac{1}{2})$ because $f''(x)$ is pos

$f''(x) = -12x + 6$
$-6(2x - 1)$
$2x - 1 = 0$
$2x = \frac{1}{2}$

$f''(x)$ + + | − − −
$\frac{1}{2}$

4. Let f be a twice-differentiable function such that $f(10)=25$ and $f(25)=10$. And, let $g(x)$ be the function given by $g(x)=f(f(x))$.

(a) Explain why [using Calculus] there must be value c, $10<c<25$, such that $g'(c)=1$.

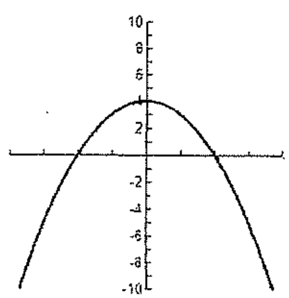
$g(10) = 10$
$g(25) = 25$

since its a continuous function on a closed interval and the slope from end to end is 1 - By M.V.T there must be a c - between 10 and 25 where the slope is 1

$\frac{25-10}{25-10} = \frac{15}{15} = 1$

(b) Show that $g'(10)=g'(25)$. Use this result to show that there must be a value k, $10<k<25$, such that $g''(k)=0$.

5 Compare the values of $f(0)$, $f'(0)$, $f''(0)$



graph of f

$f(0) = 4$
$f'(0) = 0$
$f''(0) = -$

6 The table below gives values of the velocity, $v(t)$, of a Inferi [An "inferi" is sort of like a zombie but made by the Dark Lord] at selected times.

t (sec)	0	1	3	6	10	15
$v(t)$ m/sec	5	1	−1	5	10	13

(a) Is there a time during $0<t<15$, that the velocity is equal to 9 m/sec Justify completely. By IVT - if velocity goes from 5 to 10, it must have passed through 9

(b) Find an approximation for the acceleration at time $t=2$ and indicate units. [Show all work]

$\frac{-1-1}{3-1} = \frac{-2}{2} = -1 \text{ m/sec}^2$

(c) Show that there must be a time interval such that the acceleration, $a(t)$, is equal to zero

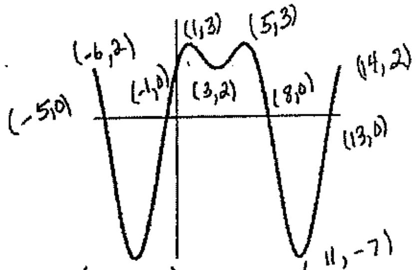
$f(0) = 5$ and $f(6) = 5$

Since its a continuous function and $f(a) = f(b)$ then by Rolles thm there must be a place where $a(t) = 0$

Celine 1/4/12

My Chapter 3 Graph Handout 1

1. Let $f(x)$ be a differentiable function. The graph of $f'(x)$, the first derivative, is shown below.



graph of $f'(x)$

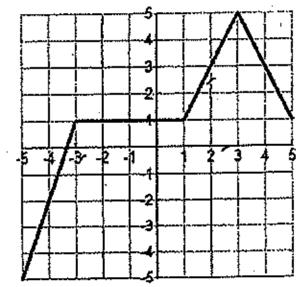
NOT TO SCALE

(a) Use the graph of $f'(x)$ to locate the x–value(s) of all relative extrema of f [Justify completely] $x = -5, -1, 8, 13$ since $f'(x) = 0$ at those values and f' changes signs at those values (the 1st derivative test).

(b) Use the graph of $f'(x)$ to locate the x–value(s) of all points of inflection of f [Justify completely] $x = -3, 1, 3, 5, 11$

Since $f''(x) = 0$ for those values and f'' changes sign at those values (2nd der. test).

2. Let $g(x)$ be a differentiable function. The graph of $g'(x)$, the first derivative of $g(x)$ is shown below.



graph of $g'(x)$

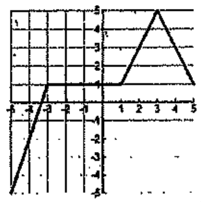
(a) On what interval(s), does the graph of g increase? [Justify completely]

graph of g increases when $g' > 0$, so the intervals are $(-3.5, 5]$

(b) Use the graph of $g'(x)$ to find the x–value(s) of any relative extrema of g [Justify completely]

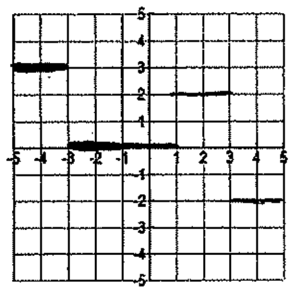
There is a relative minimum when $x = -3.5$, since $g'(-3.5) = 0$ and $g'(x)$ changes from negative to positive values at that x-value (1st der. test).

Question 2 continued



graph of $g'(x)$

(c) On the axes provided below, draw an accurate graph of $g''(x)$



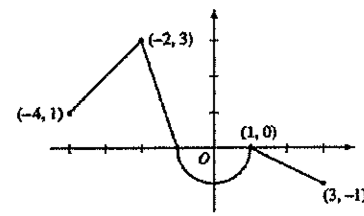
(d) On what interval(s) is the graph of $g(x)$ concave down? [Justify completely]

By the 2nd der test

$g(x)$ is concave down when $x \in (3, 5)$

since $g'' < 0$ for those x-values.

3. The graph below is the graph of $g'(x)$ which is defined for the closed interval $[-4, 3]$



graph of $g'(x)$

(A) Identify the critical values of g [not $g'(x)$] and classify them as relative minima, maxima, or neither. Justify fully.

CV $x = -1, 1$

$x = -1$ Rel max because $g'(x)$ changed pos to neg

$x = 1$ is neither because $g'(x)$ did not change sign

(B) Use a limit to prove that the $g''(-2)$ does not exist.

$$\lim_{x \to -2^-} g'(x) \neq \lim_{x \to -2^+} g'(x)$$

12. Suppose that functions f and g and their first derivatives have the following values at $x=-1$ and $x=0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Find the first derivative of the following combinations at the given value of x.

a) $f(g(x))$ at $x=-1$ $\Rightarrow f'(g(-1))\cdot g'(-1)$

$= f'(-1)\cdot g'(-1)$

$= 2\cdot 1$

$= \boxed{2}$

b) $\boxed{f^2(x)g^3(x)}$ at $x=0$ $\rightarrow [f(x)]^2\cdot[g(x)]^3$

$[f(0)]^2\cdot 3[g(0)]^2\cdot g'(0) + [g(0)]^3\cdot 2[f(0)]\cdot f'(0)$

$= (-1)^2\cdot 3(-3)^2\cdot(4) + (-3)^3\cdot 2(-1)\cdot(-2) = \boxed{0}$

c) $g(f(x))$ at $x=-1$

$= g'(f(-1))\cdot f'(-1)$

$= g'(0)\cdot f'(-1) = 4\cdot 2 = \boxed{8}$

d) $g(x+f(x))$ at $x=0$

$= g'(0+f(0))\cdot[1+f'(0)]$

$= g'(0+-1)\cdot[1+f'(0)] =$

$= g'(-1)\cdot[1+f'(0)]$

$= 1\cdot[1+-2]$

$= \boxed{-1}$

13. Find $\frac{dy}{dx}$ if $x^2y+3y^2=x-2$

$\left[x^2\frac{dy}{dx}+y\cdot 2x\right]+6y\,dy/dx = 1$

$x^2\,dy/dx + 6y\,dy/dx = 1-2xy$

$dy/dx\,(x^2+6y) = 1-2xy$

$\boxed{dy/dx = \frac{1-2xy}{x^2+6y}}$

14. Find $y'''(x)$ if $y=(4x+1)^{10}$

$y' = 10(4x+1)^9\cdot 4 = 40(4x+1)^9$

$y'' = 360(4x+1)^8\cdot 4 = 1440(4x+1)^8$

$y''' = 11520(4x+1)^7\cdot 4 = \boxed{80640(4x+1)^7}$

15. Suppose $y=x^3-3x$. [No Calculator]

a) Find the zeros of the function. $x^3-3x=0$

$x(x^2-3)=0$

$\boxed{x=0 \text{ or } x=\pm\sqrt{3}}$

b) Determine where y is increasing or decreasing and justify your response.

$y' = 3x^2-3 = 3(x^2-1) = 3(x+1)(x-1)$

y': pos | -1 | neg | 1 | pos

y is increasing on $(-\infty,-1]\cup[1,\infty)$ since $y'>0$

y is decreasing on $[-1,1]$ since $y'<0$

c) Determine all local extrema and justify your response.

There is a rel. max at $x=-1$ since y' changed signs from $+$ to $-$. $\boxed{\text{Rel max} = 2}$

there is a rel. min at $x=1$ since y' changed signs from $-$ to $+$ $\boxed{\text{Rel min} = -2}$

d) Determine the points where y is concave up or concave down, and find any points of inflection. Justify your responses.

$y'' = 6x$

y'': neg | 0 | pos

y is concave up on $(0,\infty)$ since $y''>0$

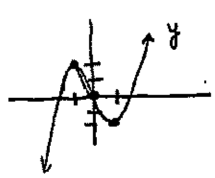
y is concave down on $(-\infty,0)$ since $y''<0$

There is a point of inflection on y when $x=0$ since y'' changed signs at $x=0$.

$\boxed{\text{P.O.I.} = (0,0)}$

e) Use all your information to sketch a graph of this function.

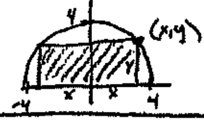
Known Points: $(-1,2)$
$(1,-2)$
$(0,0)$



		-1		0		1	
y'	pos		neg		neg		pos
y''	neg		neg		pos		pos
y	INC CD		DEC CD		DEC CU		INC CU

★★ IF YOU DON'T USE CALCULUS FOR #20 & 21 YOU WON'T GET CREDIT ON THE TEST!

20. Find the maximum area of a rectangle inscribed under the curve $f(x)=\sqrt{16-x^2}$.



$A = 2xy = 2x\sqrt{16-x^2}$ Domain: $[0,4]$

$A' = 2x \cdot \left[\tfrac{1}{2}(16-x^2)^{-1/2} \cdot (-2x)\right] + \sqrt{16-x^2} \cdot [2]$

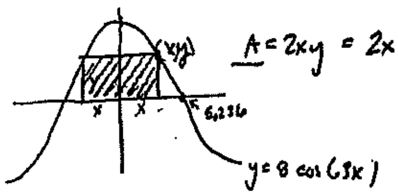
x	0	$2\sqrt{2}$	4
A	0	16	0

MAX AREA = 16

$\boxed{A'} = \dfrac{-2x^2}{\sqrt{16-x^2}} + 2\sqrt{16-x^2} = \dfrac{-2x^2 + 2(16-x^2)}{\sqrt{16-x^2}} = \dfrac{-2x^2 + 32 - 2x^2}{\sqrt{16-x^2}} = \boxed{\dfrac{32-4x^2}{\sqrt{16-x^2}}}$

$A' = 0$ when $32 - 4x^2 = 0$, $\pm 2\sqrt{2} = x$

21. [Calculator Allowed] A rectangle is inscribed under one arch of $y = 8\cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?



$A = 2xy = 2x \cdot 8\cos(.3x) = 16x\cos(.3x)$ Domain: $[0, 5.236]$

$A' = 16x \cdot [-\sin(.3x) \cdot (.3)] + \cos(.3x) \cdot [16]$

$A' = 0$ at $x \approx 2.86777786\ldots$ (STORE AS B)

x	A
0	0
B	29.925
5.236	0

MAX AREA ≈ 29.925

22. The function f is continuous on $[0, 3]$ and satisfies the following:

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

Check endpoints & critical points

endpoints: $x = 0$, $x = 3$

critical points: $f' = 0$: $x = 1$ | f' DNE: $x = 2$

x	0	1	2	3
f(x)	0	-2	0	3

MAX = 3

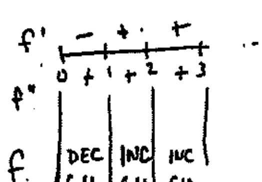
MIN = -2

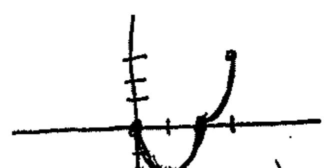
b) Find any points of inflection.

where f'' changes signs ... NONE, f'' is always POSITIVE

c) Sketch a possible graph of f.

KNOWN POINTS
(0,0)
(1,-2)
(2,0)
(3,3)





NOTE... POINTY PLACE @ x=2 b/c f'(2) DNE

Go back and Review the questions from your assignments in this chapter ... especially those in section 4.3.

Understand your notecards!