AP CALCULUS AB - REVIEW #2 -CHP 7 WS (Mr. Leckie)

27. Derive (SHOW EVERY STEP) $y = y_0 e^{kt}$ from $\frac{dy}{dt} = ky$ and $y(0) = y_0$.

28. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$.

b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$

c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

d) Use your solution from part c to find $f(1.2)$

29. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

A $e^{\tan x} + 4$
B $e^{\tan x} + 5$
C $5e^{\tan x}$
D $\tan x + 5$
E $\tan x + 5e^x$

30. [No Calculator] If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

A) $-\frac{1}{2}\ln 2$

B) $-\frac{1}{4}$

C) $\frac{1}{2}\ln 2$

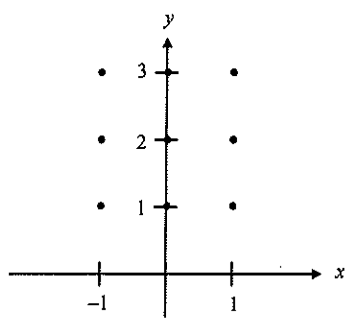
D) $\frac{\sqrt{2}}{2}$

E) $\ln 2$

31. Consider the differential equation given by $\dfrac{dy}{dx} = \dfrac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

b) Draw a particular solution if $f(0) = 3$



c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

32. [Calculator] During a certain epidemic, the number of people that are infected at any time increases at rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

AP Calculus AB **Chapter 7 Review #3**

Topics on the test – Sec. 7.1, 7.2, 7.4: Integration with u-sub, differential equations, exponential growth and decay, and slope fields

Evaluate each integral:

1. $\int \frac{-x}{\sqrt{4-x^2}}dx$

2. $\int \frac{5}{1-x}dx$

3. $\int \sin(6x)e^{\cos(6x)}\,dx$

4. $\int \frac{3x^4-6x^2+2x-3}{x^2}dx$

5. $\int_0^{\frac{\pi}{4}} \tan x dx$

6. $\int_1^e \frac{\ln x}{x}dx$

7. $\int \frac{x^2}{(16-x^3)^2}dx$

8. $\int \tan^4 x \sec^2 x dx$

9. $\int 2x(x^2-1)^4 dx$

10. $\int \frac{dx}{(x+2)^3}$

11. $\int_0^1 xe^{x^2}dx$

12. $\int_0^{\pi/6} \sin(3x)\,dx$

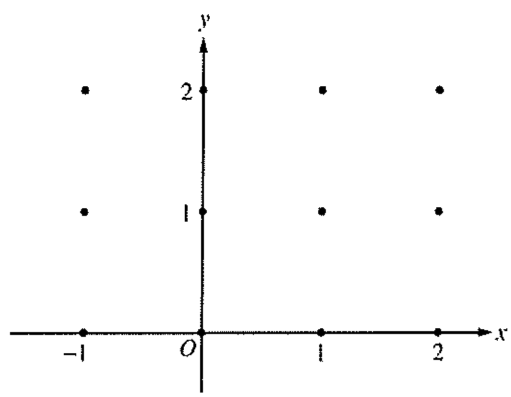
13. Write and solve a differential equation that models the verbal statement:

a) The rate of change of P with respect to t is proportional to 10 – t.

b) N is changing at a rate proportional to N. When t = 0, N = 250 and when t = 1, N = 400. What is the value of N when t = 4?

14. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

1997 AB6/BC6

15.

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

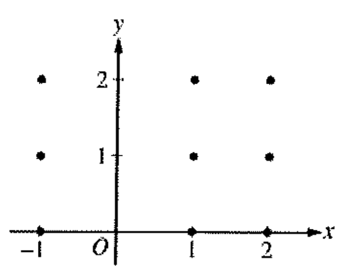
16.

2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{y - 1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

5

Review Mr Leckie + Review for tests 2019

27. Let f be a function with $f(1)=4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$. $\boxed{1/2}$

b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$ $\boxed{4.1}$

c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx}=\frac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

$\boxed{y = \pm\sqrt{x^3+x+14}}$

d) Use your solution from part c to find $f(1.2)$. $\boxed{\approx 4.114}$

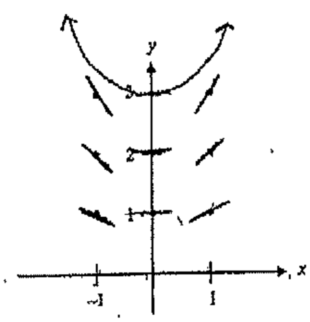
28. If $\frac{dy}{dx} = y\sec^2 x\, dx$ and $y = 5$ when $x = 0$, then $y =$

A	$e^{\tan x}+4$
B	$e^{\tan x}+5$
C	$5e^{\tan x}$
D	$\tan x + 5$
E	$\tan x + 5e^x$

29. Consider the differential equation given by $\frac{dy}{dx}=\frac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

b) Draw a particular solution if $f(0)=3$



c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0)=3$. Use your solution to find $f(0.2)$.

$\boxed{\approx 3.030}$

1

30. [No Calculator] If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

A) $-\frac{1}{2}\ln 2$

B) $-\frac{1}{4}$

C) $\boxed{\frac{1}{2}\ln 2}$

D) $\frac{\sqrt{2}}{2}$

E) $\ln 2$

you can solve this ...

$\int \frac{dy}{y} = \int -2\,dt$

$\ln|y| = -2t + c$

$|y| = e^{-2t+c}$

$|y| = e^{-2t} e^{c}$

$|y| = Ce^{-2t}$

y is on ~~$(-\infty,0)$~~ or $(0,\infty)$ since $y = 1$ is given ⟹ y is only on $(0,\infty)$ ⟹ $|y| = y$

$\therefore y = Ce^{-2t}$

when $t = 0, y = 1$

$1 = Ce^{-2(0)}$

$1 = C$

$\therefore y = e^{-2t}$

when $y = 1/2$, $\frac{1}{2} = e^{-2t}$

$\ln(\frac{1}{2}) = -2t$

$-\frac{1}{2}\ln(\frac{1}{2}) = t$

$-\frac{1}{2}\ln(2^{-1}) = t$

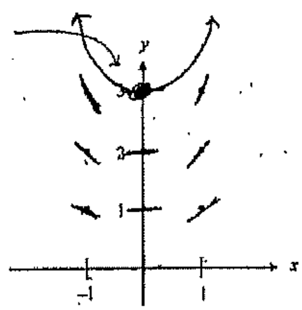
$+\frac{1}{2}\ln(2) = t$

31. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$. ⟹ $\frac{dy}{y} = \frac{1}{2}x\,dx$

y is on $(-\infty,0)$ or $(0,\infty)$

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

b) Draw a particular solution if $f(0) = 3$



$(-1,3)$	$-3/2$
$(-1,2)$	-1
$(-1,1)$	$-1/2$
$(0,3)$	0
$(0,2)$	0
$(0,1)$	0
$(1,3)$	$3/2$
$(1,2)$	1
$(1,1)$	$1/2$

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

$\int \frac{dy}{y} = \int \frac{1}{2}x\,dx$

$\ln|y| = \frac{1}{4}x^2 + C$

$|y| = e^{\frac{1}{4}x^2 + C}$

$|y| = e^{\frac{1}{4}x^2} \cdot e^{C}$

$|y| = Ce^{\frac{1}{4}x^2}$

$y = 3$ means y is only defined on $(0,\infty)$

$\therefore |y| = y$

$y = Ce^{\frac{1}{4}x^2}$ when $x = 0, y = 3$

$3 = Ce^{\frac{1}{4}(0)^2}$

$3 = C$

$\boxed{\therefore y = 3e^{\frac{1}{4}x^2}}$

★ use your calculator to Compare this graph to the one you drew in part (b)

$y = 3e^{\frac{1}{4}(.2)^2}$

$y \approx 3.030$

32. [Calculator] During a certain epidemic, the number of people that are infected at any time increases at rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

Let N = # of people infected

$\frac{dN}{dt} = K \cdot N$

Solution ⟹

$N = N_0 e^{kt}$

$N_0 = 1000$

$N = 1000e^{kt}$

$1200 = 1000e^{k(7)}$

$1.2 = e^{7k}$

$\ln(1.2) = 7k$

$\frac{1}{7}\ln(1.2) = k$

$\therefore N = 1000e^{\frac{1}{7}\ln(1.2)\cdot t}$

when $t = 12$

$N = 1000e^{\frac{1}{7}\ln(1.2)\cdot 12}$

$N \approx 1366.90798$

≈ 1367 people

2

KEY

AP Calculus AB **Chapter 7 Review #3**

Topics on the test – Sec. 7.1, 7.2, 7.4: Integration with u-sub, differential equations, exponential growth and decay, and slope fields

Evaluate each integral:

1. $\int \frac{-x}{\sqrt{4-x^2}}dx$

$u = 4 - x^2$

$\frac{du}{2} = \frac{-2x\,dx}{2}$

$\frac{1}{2}\int \frac{du}{\sqrt{u}} = \cancel{2}\,\frac{1}{\cancel{2}}\,u^{1/2} + C$

$\sqrt{4-x^2} + C$

2. $\int \frac{5}{1-x}dx$

$u = 1-x$

$\frac{du}{-1} = \frac{-dx}{-1}$

$-5\int \frac{du}{u}$

$-5\ln|u| + C$

$-5\ln|1-x| + C$

3. $\int \sin(6x)e^{\cos(6x)}\,dx$

$u = \cos(6x)$

$\frac{du}{-6} = \frac{-6\sin(6x)dx}{-6}$

$-\frac{1}{6}\int e^u du$

$-\frac{1}{6}e^u + C$

$-\frac{1}{6}e^{\cos(6x)} + C$

4. $\int \frac{3x^4 - 6x^2 + 2x - 3}{x^2}dx$

Divide first!

$\int \left(3x^2 - 6 + \frac{2}{x} - \frac{3}{x^2}\right)dx$

$x^3 - 6x + 2\ln|x| + \frac{3}{x} + C$

5. $\int_0^{\frac{\pi}{4}} \tan x\,dx$

$\int \frac{\sin x}{\cos x}dx$

$u = \cos x$

$\frac{du}{-1} = \frac{-\sin x\,dx}{-1}$

$-\int \frac{du}{u} = -\ln|u| \Big|_{\cos 0 = 1}^{\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}} = -\ln\frac{\sqrt{2}}{2} - \cancel{-\ln 1}$ ($\ln 1 = 0$)

OR $-\ln|\cos x|\Big|_0^{\pi/4}$

$-\ln\left(\frac{\sqrt{2}}{2}\right) = \ln\sqrt{2}$

6. $\int_1^e \frac{\ln x}{x}dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int u\,du = \frac{u^2}{2}\Big|_{\ln 1 = 0}^{\ln e = 1}$

OR $\frac{(\ln x)^2}{2}\Big|_1^e$

$\frac{1}{2} - 0 = \frac{1}{2}$

7. $\int \frac{x^2}{(16-x^3)^2}dx$

$u = 16 - x^3$

$\frac{du}{-3} = \frac{-3x^2dx}{-3}$

$-\frac{1}{3}\int \frac{du}{u^2}$

$-\frac{1}{3}\left(-\frac{1}{u}\right) + C$

$\frac{1}{3(16-x^3)} + C$

8. $\int \tan^4 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^4 du$

$\frac{u^5}{5} + C$

$\frac{\tan^5 x}{5} + C$

9. $\int 2x(x^2-1)^4dx$

$u = x^2 - 1$

$du = 2x\,dx$

$\int u^4 du$

$\frac{u^5}{5} + C$

$\frac{(x^2-1)^5}{5} + C$

10. $\int \frac{dx}{(x+2)^3}$

$u = x+2$

$du = dx$

$\int \frac{du}{u^3} = \int u^{-3}du$

$\frac{u^{-2}}{-2} + C$

$\frac{-1}{2(x+2)^2} + C$

11. $\int_0^1 xe^{x^2}dx$

$u = x^2$

$\frac{du}{2} = \frac{2x\,dx}{2}$

$\frac{1}{2}\int e^u du$

$\frac{1}{2}e^u\Big|_{0^2}^{1^2}$

OR

$\frac{1}{2}e^{x^2}\Big|_0^1 = \frac{1}{2}(e^1 - e^0)$

$\frac{e-1}{2}$

12. $\int_0^{\pi/6}\sin(3x)\,dx$

$u = 3x$

$\frac{du}{3} = \frac{3dx}{3}$

$\frac{1}{3}\int \sin u\,du$

$-\frac{1}{3}\cos u\Big|_{3(0) = 0}^{3(\frac{\pi}{6}) = \frac{\pi}{2}}$

OR

$-\frac{1}{3}\cos(3x)\Big|_0^{\pi/6} = \frac{1}{3}$

$-\frac{1}{3}\left(\cos\frac{\pi}{2} - \cos 0\right) = -\frac{1}{3}(0-1)$

$\frac{1}{3}$

13. Write and solve a differential equation that models the verbal statement:

a) The rate of change of P with respect to t is proportional to 10 – t.

$$\frac{dP}{dt} = k(10-t)$$

$$\int dP = \int k(10-t)\,dt$$

$$P = k\left(10t - \frac{t^2}{2}\right) + C$$

b) N is changing at a rate proportional to N. When t = 0, N = 250 and when t = 1, N = 400. What is the value of N when t = 4?

$$\frac{dN}{dt} = kN$$

$$\int \frac{dN}{N} = \int k\,dt$$

$$\ln|N| = (kt + C)$$

$e^\wedge \quad e^\wedge$

$$|N| = Ce^{kt}$$

$$N = \pm Ce^{kt}$$

$$N(0) = Ce^{0} = 250$$

$$C = 250$$

$$N = 250e^{kt}$$

$$\frac{N(1) = 250e^{k(1)}}{250} = \frac{400}{250}$$

$$e^k = \frac{8}{5} \text{ OR } 1.6$$

$$k = \ln(8/5)$$

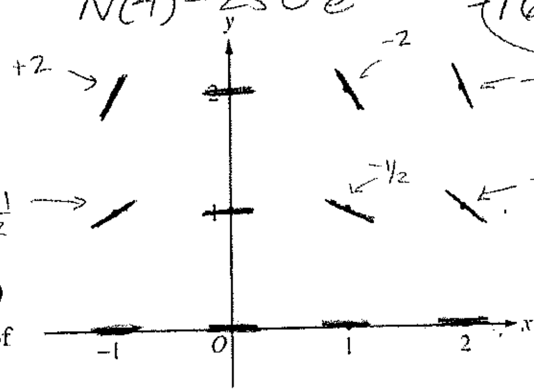
$$N = 250e^{\ln(\frac{8}{5})t}$$

$$N(4) = 250e^{\ln(8/5)4} = 1638.4$$

exact!

14. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

b) $$\left.\frac{dy}{dx}\right|_{(-1,2)} = \frac{-(-1)(2)^2}{2} = \frac{+4}{2} = 2$$

Pt. $(-1,2)$

$$y - 2 = 2(x+1)$$

c) $$\int \frac{dy}{y^2} = \int \frac{-x}{2}\,dx$$

$$\frac{-1}{y} = \frac{-x^2}{4} + C$$

Solve for C w/ $(-1,2)$

$$\frac{-1}{2} = \frac{-(-1)^2}{4} + C$$

$$-\frac{1}{2} = \frac{-1}{4} + C$$

$$\frac{-1}{4} = C$$

$$\frac{-1}{y} = \frac{-x^2}{4} - \frac{1}{4}$$

$$-1 = \left(\frac{-x^2}{4} - \frac{1}{4}\right)y$$

$$y = \frac{-1}{\frac{-x^2}{4} - \frac{1}{4}} \cdot \frac{4}{4}$$

get rid of complex fraction

$$y = \frac{-4}{-x^2-1} = \frac{4}{x^2+1}$$

4

1997 AB6/BC6
Solution

15

(a) $\frac{dv}{dt} = -2v - 32 = -2(v+16)$

$\frac{dv}{v+16} = -2dt$

$\ln|v+16| = -2t + A$

$|v+16| = e^{-2t+A} = e^{A}e^{-2t}$

$v+16 = Ce^{-2t}$

$-50+16 = Ce^{0};\ C = -34$

$v = -34e^{-2t} - 16$

(b) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(-34e^{-2t} - 16\right) = -16$

b/c the exponential part is exp. decay approaching the horizontal asymptote of y = 0.

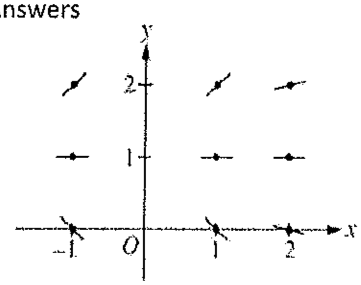
(c) $v(t) = -34e^{-2t} - 16 = -20$

$e^{-2t} = \frac{2}{17};\ t = -\frac{1}{2}\ln\left(\frac{2}{17}\right) = 1.070$

2008 #5 Answers

16

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) $\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^{C}e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^{C}$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)},\ x > 0$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(c) $\lim_{x\to\infty} 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)} = 1 - \sqrt{e}$

b/c as $x \to \infty$ the 1/x approaches 0 and $e^{1/2} = \sqrt{e}$

1 : limit

5