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Limits test review sheet AP Calculus AB

NO CALCULATOR!!

1. If $f(x)=\begin{cases} -x^2-4x-2, x\le -2 \\ x^2+4x+6, x>-2 \end{cases}$ find the $\lim\limits_{x\to -2} f(x)$.

2. Find the intervals on which each of the function is continuous:

   a) $y=\dfrac{1}{(x-2)^2}$

   b) $f(x)=\begin{cases} 5-x, x\le 2 \\ 2x-3, x>2 \end{cases}$

   c) For each function above give the points of discontinuity and the types. Are they removable or non-removable?

3. Find each limit:

a) $\lim\limits_{x\to 4}\sqrt{x^2-3}$

b) $\lim\limits_{x\to\infty} e^{-x}\cos x$

c) $\lim\limits_{x\to 0} e^{x}\sin x$

d) $\lim\limits_{x\to -\infty}\dfrac{2x^2+3}{5x^2+7}$

e) $\lim\limits_{x\to\infty}\dfrac{2x^3+31}{5x^2+7}$

f) $\lim\limits_{x\to\infty}\dfrac{2x^2+9x}{5x^8+7x^4}$

g) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{4x}$

h) $\lim\limits_{x\to 0}\dfrac{1/(x+1)-1}{x}$

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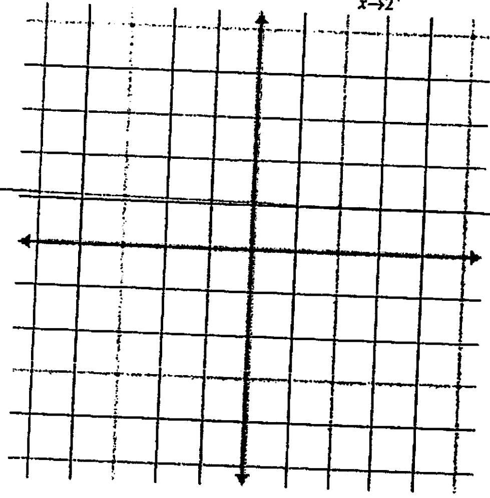
6. Sketch a graph of a function f that satisfies the given conditions.

$\lim_{x\to 2} f(x)$ does not exist, $\lim_{x\to 2^+} f(x) = f(2) = 3$



7. Determine the value of c such that the function is continuous on the entire real line.

$$f(x)=\begin{cases} x+3, x\le 2 \\ cx+6, x>2 \end{cases}$$

8. Write and extended function that would make f(x) = $\frac{5x-5}{x^2-1}$ continuous at x=1.

9. Use the intermediate value theorem to show that f(x) = $2x^3$-3 has a zero in the interval [1,2].

10. Find all of the vertical and horizontal asymptotes for $y=\frac{2x-3}{2x^2-x-3}$

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Name KEY Date

Limits test review sheet AP Calculus AB **NO CALCULATOR!!**

1. If $f(x)=\begin{cases}-x^2-4x-2, x\le -2\\ x^2+4x+6, x>-2\end{cases}$ find the $\lim_{x\to -2} f(x)$.

$\to -4+8-2=2$

$\to 4-8+6=2$

$\lim_{x\to -2} f(x) = \boxed{2}$

2. Find the intervals on which each of the function is continuous:

a) $y=\dfrac{1}{(x-2)^2}$ $(-\infty,2)\cup(2,\infty)$

b) $f(x)=\begin{cases}5-x, x\le 2\\ 2x-3, x>2\end{cases}$ $\to 5-2=3$, $4-3=1$ $(-\infty,2)\cup(2,\infty)$

c) For each function above give the points of discontinuity and the types. Are they removable or non-removable?

a) $x=2$ Infinite

b) $x=2$ jump

* Both Non-Removable

3. Find each limit:

a) $\lim_{x\to 4}\sqrt{x^2-3} = \sqrt{16-3} = \sqrt{13}$

b) $\lim_{x\to\infty} e^{-x}\cos x = \lim_{x\to\infty}\dfrac{\cos x}{e^x} = 0$ (e^x grows very large very quickly & overpowers $\cos x$)

c) $\lim_{x\to 0} e^x \sin x = e^0 \sin 0 = 1\cdot 0 = 0$

d) $\lim_{x\to -\infty}\dfrac{2x^2+3}{5x^2+7} = 2/5$

e) $\lim_{x\to\infty}\dfrac{2x^3+31}{5x^2+7} = \lim_{x\to\infty}\dfrac{2x^3}{5x^2} = \lim_{x\to\infty}\dfrac{2}{5}x = \infty$

f) $\lim_{x\to\infty}\dfrac{2x^2+9x}{5x^8+7x^4} = 0$ (power denom > power num)

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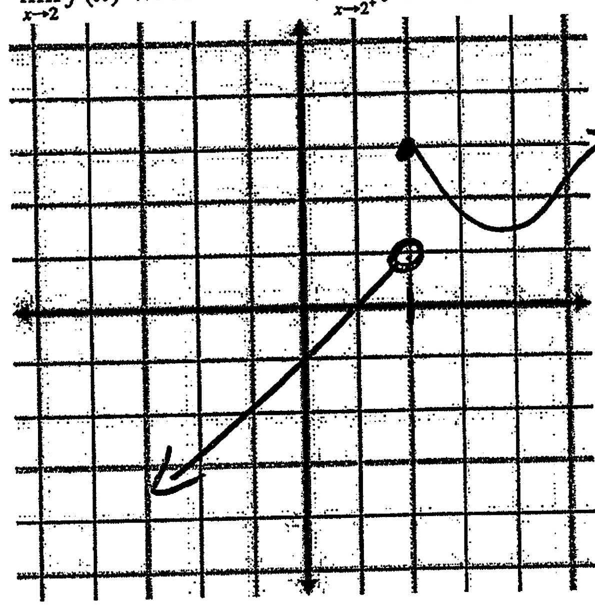
g) $\lim_{x\to 0}\frac{\sin 2x}{4x} = \frac{1}{2}\lim_{x\to 0}\frac{\sin(2x)}{2x} = \frac{1}{2}\cdot 1 = 1/2$

h) $\lim_{x\to 0}\frac{(1/(x+1)-1)}{x}\frac{(x+1)}{(x+1)} = \lim_{x\to 0}\frac{1-(x+1)}{x(x+1)} = \lim_{x\to 0}\frac{\cancel{1}-x-\cancel{1}}{x(x+1)}$

$\lim_{x\to 0}\frac{-\cancel{x}}{\cancel{x}(x+1)} = \frac{-1}{0+1} = -1$

6. Sketch a graph of a function f that satisfies the given conditions.

$\lim_{x\to 2} f(x)$ does not exist, $\lim_{x\to 2^+} f(x) = f(2) = 3$



one possible ANSWER

7. Determine the value of c such that the function is continuous on the entire real line.

$f(x) = \begin{cases} x+3, x \le 2 \\ cx+6, x > 2 \end{cases}$

$2+3 = c(2)+6$

$5 = 2c+6$

$-1 = 2c$

$\boxed{c = -1/2}$

8. Write and extended function that would make f(x) = $\frac{5x-5}{x^2-1}$ at x=1.

$f(x) = \frac{5(x-\cancel{1})}{(x-\cancel{1})(x+1)} = \frac{5}{x+1}$ plug in 1 for x Now $\rightarrow 5/2$

$g(x) = \begin{cases} \frac{5x-5}{x^2-1} & x \ne 1 \\ 5/2, & x = 1 \end{cases}$

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$2-6$

9. Use the intermediate value theorem to show that $f(x) = 2x^3-3$ has a zero in the interval [1,2]. $f(1) = 2-3 = -1$ $f(2) = 13$

Since $f(x)$ is continuous on $[1,2]$ and since $f(1) = -1$ AND $f(2) = 13$ and 0 is in between these 2 values then by IVT there must be a c in $[1,2]$ such that $f(c) = 0$.

10. Find all of the vertical and horizontal asymptotes for $y = \dfrac{2x-3}{2x^2-x-3}$.

H.A. $\lim\limits_{x\to\pm\infty} \dfrac{2x-3}{2x^2-x-3} = 0$ $\boxed{y=0} \leftarrow$ H. A.

V.A $\dfrac{\cancel{2x-3}}{\cancel{(2x-3)}(x+1)} = \dfrac{1}{x+1}$ $\boxed{x=-1} \leftarrow$ V. A.

11. If $f(x) = 1 - x^2$ Find:

a) The slope of the secant line to f(x) from x = -2 to x = 3. -1

b) The slope of the tangent line to f(x) at x = 2. -4

Point $(2,-3)$ c) The equation of the tangent line at x = 2. $y+3 = -4(x-2)$

d) The equation of the normal line at x = 2. $y+3 = \frac{1}{4}(x-2)$

a) $\dfrac{f(3)-f(-2)}{3--2} = \dfrac{-8+3}{5} = \dfrac{-5}{5} = -1$

b) $\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0} \dfrac{1-(2+h)^2-(-3)}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{1}\cancel{-4}-4h-h^2\cancel{+3}}{h}$

$= \lim\limits_{h\to 0} -4-h = -4$

12. If $f(x) = 1/x$ FIND:

a) the instantaneous rate of change of f at x = a. $-1/a^2$

b) The instantaneous rate of change of f at x = 4. $-1/16$

c) The average rate of change of f on the interval [1, 3] $-1/3$

a) $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h} = \lim\limits_{h\to 0} \dfrac{\left(\frac{1}{a+h} - \frac{1}{a}\right) a(a+h)}{h\,(a)(a+h)} = \lim\limits_{h\to 0} \dfrac{a-(a+h)}{ah(a+h)}$

$\lim\limits_{h\to 0} \dfrac{-\cancel{h}}{a\cancel{h}(a+h)} = \dfrac{-1}{a(a+0)} = -1/a^2$

b) $-1/(4)^2 = -1/16$

c) $\dfrac{f(3)-f(1)}{3-1} = \dfrac{1/3-1}{2} = -1/3$

$\frac{1-3}{6}$ $-2/6$