Graphs, Optimization, Rectilinear motion

Optimization occurs at a CV or endpoint- find these (CV-by setting the derivative =0 or undefined) and evaluate to see which gives you the max or min depending on the question. Prove using the first derivative or second derivative test

#1 A rectangular page is to contain 24 in^2 of print. The margins at top and bottom are 1.5 inches and on the left and right side at 1 inch.
What should the dimensions of the page be so that the least amount of paper is used?

#2 200 ft of fencing is used to enclose two adjacent rectangular corrals. What dimensions will enclose the maximum area?

Rectilinear Motion

3. A particle moves along the x-axis so that its velocity v at time t, for the interval [0,5]is given by $V(t) = \ln (t^2-3t+3)$.

A) Find acceleration of the particle at time t=4
~~Find position at time t=1~~

B) Find all times t in the open interval (0,5) at which the particle changes direction. During which time intervals, on the interval [0,5], does the particle travel to the left?

4 Fill in the blank

When the velocity is negative, the particle is moving to the

When the velocity is positive, the particle is moving to the

When the velocity and acceleration of the particle have the same signs, the particles speed is

When the velocity and acceleration of the particle have opposite signs the particles speed is

Name_____ Date_____

1. Given a particles position in feet at time t (in seconds) $s(t) = 2t^2-8t+6$

Find: (SHOW ALL WORK) include units in your answer!!

a) the displacement of the particle in the first 2 seconds. _____

b)The average velocity for the first 2 seconds. _____

c) The velocity at time t and at t=2 seconds. V(t) = _____ v(2)=_____

d) The acceleration of the particle at t = 2 seconds. A(2)_____

e) **Describe** the motion of the particle.

f) When is the particle **speeding up**? _____

Slowing down?_____

g) Give the total distance traveled over the first 6 seconds. _____

AP Calculus AB Optimization & Graph Quiz review sheet

1.A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose and what are its dimensions?

2. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have and what are its dimensions?

3.

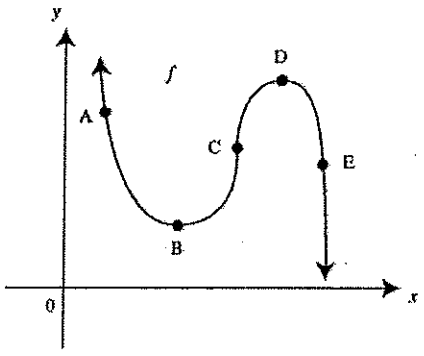
Given the function f in Figure 7.7-1, identify the points where:

a. $f' < 0$ and $f'' > 0$,

b. $f' < 0$ and $f'' < 0$,

c. $f' = 0$,

d. f'' does not exist.



4.

Given the graph of f' in Figure 7.7-7, find where the function f

a. Has its relative extrema.

b. Is increasing or decreasing.

c. Has its point(s) of inflection.

d. Is concave upward or downward.

e. If $f(0)=1$ and $f(6)=5$, draw a sketch of f.

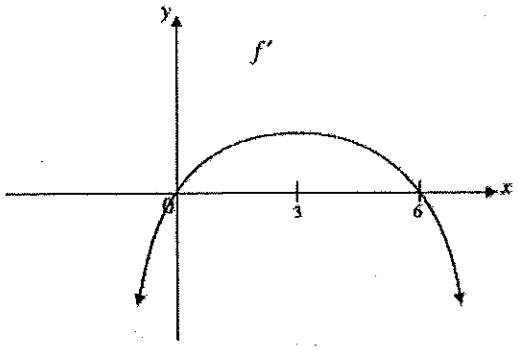


Figure 7.7-7

5.

A function f is continuous on the interval $[-1, 4]$ with $f(-1)=0$ and $f(4)=2$ and the following properties:

INTERVALS	$(-1, 0)$	$x=0$	$(0, 2)$	$x=2$	$(2, 4)$
f'	+	undefined	+	0	−
f''	+	undefined	−	0	−

a. Find the intervals on which f is increasing or decreasing.

b. Find where f has its absolute extrema.

c. Find where f has points of inflection.

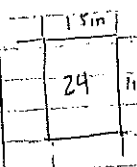
d. Find intervals on which f is concave upward or downward.

e. Sketch a possible graph of f.

6. GIVEN the position function $s(t) = t^3-9t^2+24t+1$ find a) where the particle changes direction and justify your answer b) determine the intervals when $0<t<6$ where the particle is speeding up & slowing down. Justify your anwswer.

Graphs – Optimization, Rectilinear Motion
Review for test

1.



Primary
$A = (x+2)(y+3)$
$A = xy + 3x + 2y + 6$
$A = x(24/x) + 3x + 48/x + 6$
$A = 24 + 3x + 48/x + 6$
$A = 30 + 3x + 48/x$

Secondary:
$A = xy$
$24 = xy$
$\frac{24}{x} = y$

$A' = 3 - 48/x^2$
$0 = 3 - 48/x^2$
$-3 = -48/x^2$
$-3x^2 = -48$
$x^2 = 16$
$x = \pm 4$

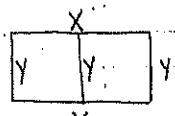
$A'(x)$ = + + + | − − − | + + + min @ $x = 4$
−4 4

$A = xy$
$24 = 4y$
$6 = y$

$A = (x+2)(y+3)$
$A = (4+2)(6+3)$
$A = (6)(9)$
$A = 54$

The dimensions are $x = 6$ $y = 9$

2)

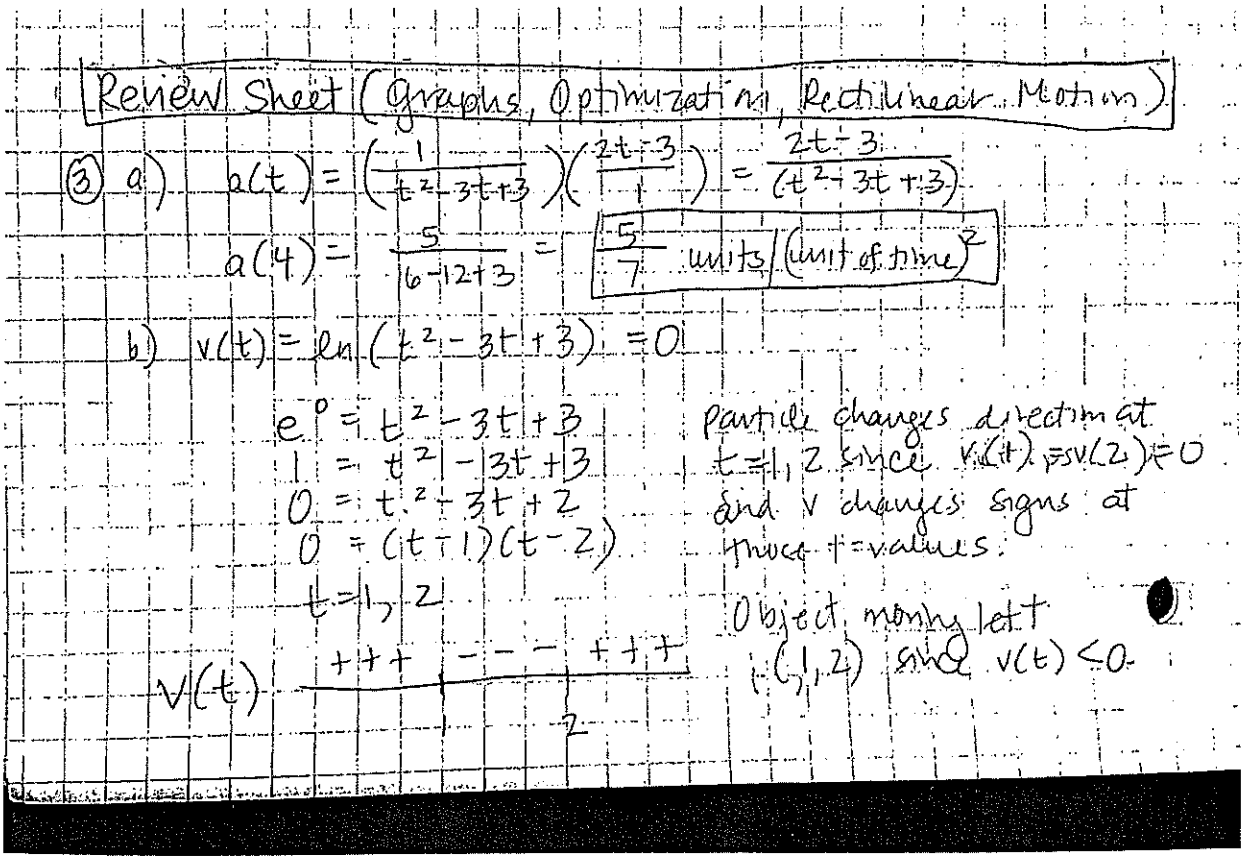


Primary
$A = xy$
$A = x\left(\frac{200 - 2x}{3}\right)$
$A = \frac{200x - 2x^2}{3}$
$A' = \frac{3(200 - 4x)}{9}$
$A' = \frac{200 - 4x}{3} = 0$
$200 - 4x = 0$
$200 = 4x$
$x = 50$

Secondary
$2x + 3y = 200$
$200 - 2x = 3y$
$y = \frac{200 - 2x}{3}$
$100 + 3y = 200$
$3y = 100$
$y = 100/3$

A' + + + + | − − − −
50

dimensions = $50 \times 100/3$



4 Fill in the blank

When the velocity is negative, the particle is moving to the Left or down

When the velocity is positive, the particle is moving to the Right-or-up

When the velocity and acceleration of the particle have the same signs, the particles speed is speeding up

When the velocity and acceleration of the particle have opposite signs the particles speed is slowing down

1. Given a particles position in feet at time t (in seconds) $s(t) = 2t^2 - 8t + 6$

Find: (SHOW ALL WORK) include units in your answer!!

a) the displacement of the particle in the first 2 seconds. -8 ft

$s(0) = 2(0)^2 - 8(0) + 6$ $\quad s(2) = 2(2)^2 - 8(2) + 6$ $\quad \Delta s = s(2) - s(0)$

$s(0) = 6$ ft $\quad = 8 - 16 + 6$ $\quad = -2 - 6$

$= -2$ ft $\quad = -8$ ft

b) The average velocity for the first 2 seconds. -4 ft/s

$s(0) = 6$ ft

$s(2) = -2$ ft $\quad v_{avg} = \frac{s(2) - s(0)}{2 - 0} = \frac{-2-6}{2} = \frac{-8}{2} = -4$ ft/s

c) The velocity at time t and at t=2 seconds. $V(t) = 4t - 8$ $\quad v(2) = 0$ ft/s

$v(t) = 4t - 8$ $\quad v(2) = 4(2) - 8$

$= 8 - 8$

$= 0$ ft/s

d) The acceleration of the particle at t = 2 seconds. $A(2) = 4$ ft/s^2

$a(t) = 4$ $\quad a(2) = 4$ ft/s^2

e) **Describe** the motion of the particle.

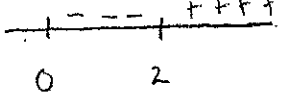
velocity

$v(t) = 4t - 8$

$4t - 8 = 0$

$4t = 8$

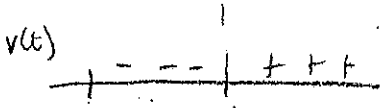
$t = 2$s



The particle is moving to the left from $0 < t < 2$, comes to a stop at $t = 2$, and is moving to right from $t > 2$.

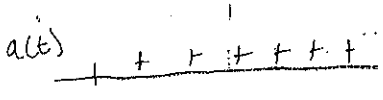
f) When is the particle **speeding up?** $(2, \infty)$

$v(t)$ and $a(t)$ have same signs



Slowing down? $(0, 2)$

$v(t)$ and $a(t)$ have opposite signs



g) Give the total distance traveled over the first 6 seconds. 40 ft

STOPPED: $t = 2$s

$s(6) = 2(6)^2 - 8(6) + 6$

$= 72 - 48 + 6$

$= 30$ ft

$s(0)$ to $s(2)$ $\quad$ $s(2)$ to $s(6)$

6 ft to -2 ft $\quad$ -2 ft to 30 ft

8 ft traveled $\quad$ 32 ft traveled

40 ft traveled TOTAL

AP Calculus AB Optimization & Graph Quiz review sheet

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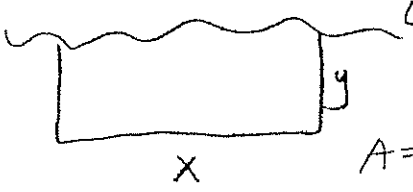


MAXIMIZE $A = xy$

$800 = x + 2y$

$x = 800 - 2y$

$A = (800 - 2y)y$

$A = 800y - 2y^2$

$A' = 800 - 4y = 0$ $\quad y = 200$

$x = 800 - 2(200)$

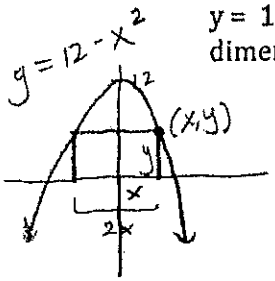
$x = 400$

400 by 200m

$+200 \rightarrow A'$ MAX!

$A = 80{,}000 m^2$

2. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have and what are its dimensions?



$A = 2xy$

$A = 2x(12 - x^2)$

$A = 24x - 2x^3$

$A' = 24 - 6x^2 = 0$

$4 = x^2$

$2 = x$

$y = 12 - 4 = 8$

Dimensions: 4×8

Area = 32

3. MAX ✓ $\quad -2 \quad 2$

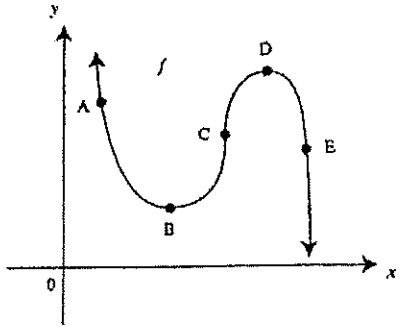
Given the function f in Figure 7.7-1, identify the points where:

a. $f' < 0$ and $f'' > 0$, (A) f is decreasing ($f' < 0$) and concave up ($f'' > 0$) at A

b. $f' < 0$ and $f'' < 0$, (E) f is decreasing & concave down at E

c. $f' = 0$, B, D

d. f'' does not exist. C → vertical tangent line

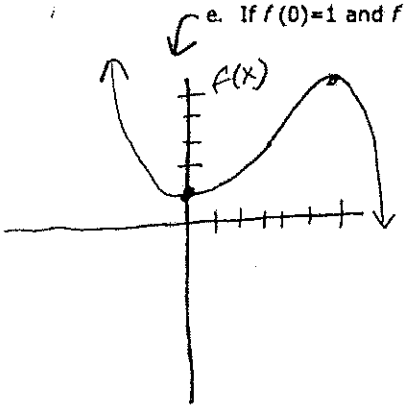


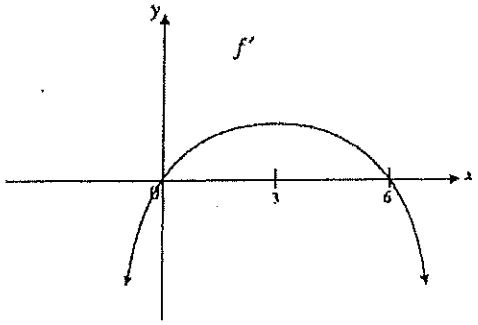
4.

at $x=0$ relative min
at $x=6$ relative MAX

Given the graph of f' in Figure 7.7-7, find where the function f

a. Has its relative extrema. at $x=0$ & 6 since $f'=0$ and changes sign
b. Is increasing or decreasing. Increasing $(0,6)$ Decreasing $(-\infty,0) \cup (6,\infty)$
c. Has its point(s) of inflection. $x=3$
d. Is concave upward or downward. concave up $(-\infty,3)$ concave down $(3,\infty)$
e. If $f(0)=1$ and $f(6)=5$, draw a sketch of f.





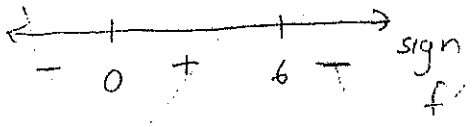
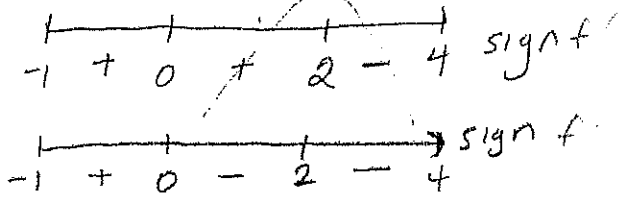


Figure 7.7-7

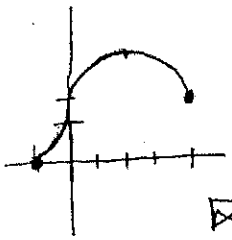
5.

A function f is continuous on the interval $[-1, 4]$ with $f(-1)=0$ and $f(4)=2$ and the following properties:

INTERVALS	$(-1, 0)$	$x=0$	$(0, 2)$	$x=2$	$(2, 4)$
f'	+	undefined	+	0	−
f''	+	undefined	−	0	−

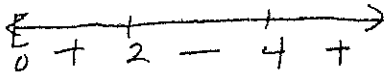


a. Find the intervals on which f is increasing or decreasing. $(-1,0) \cup (0,2)$ $(f'>0)$
b. Find where f has its absolute extrema. at $x=2$, Relative MAX
c. Find where f has points of inflection. $x=0$ since f'' is undefined and changes sign
d. Find intervals on which f is concave upward or downward. concave up $(-1,0)$
e. Sketch a possible graph of f. concave down $(0,2) \cup (2,4)$



6. GIVEN the position function $s(t) = t^3-9t^2+24t+1$ find a) where the particle changes direction and justify your answer b) determine the intervals when $0<t<6$ where the particle is speeding up & slowing down. Justify your anwswer.

a) Find where $v(t)=0$ and changes sign $v(t) = 3t^2-18t+24 = 0$

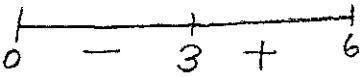


so $t=2,4$

$3(t-6t+8)=0$
$3(t-4)(t-2)=0$
$t=4,2$

b) $a(t) = 6t-18 = 0$

$t=3$



★ speeding up where $v(t)$ & $a(t)$ are same sign $(2,3) \cup (4,6)$
slowing down where $v(t)$ & $a(t)$ are diff signs $(0,2) \cup (3,4)$