4.6 RELATED RATES

When one or more values in n equation change over time, we have related rates. We simply write equations that you might have written prior to this course (with no motion taking place), then differentiate them with respect to time, *t*.

Example 1: Do you remember how we found the derivative of $x^2 + y^2 = 9$?

This derivative was $\frac{dy}{dx}$ … the derivative of y with respect to x.

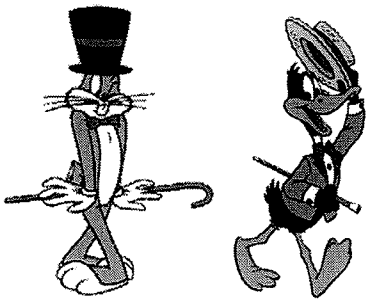
Example 2: How do you suppose we take the derivative of $a^2 + b^2 = c^2$ with respect to *t*.

In related rates problems, you will be asked to solve for one of these rates. In order to do that, you will have to have enough given information to find values for all other variables in the problem.

Example 3: Suppose you are told that the particle moving along the curve $y = x^2$ is moving horizontally at 2 cm/s. Find the rate of change in the particle's vertical position at the exact moment the particle is at (3, 9).

Example 4: Tweety is resting in a bird house 24 feet off the ground. Using a 26 foot ladder which he leaned against the pole holding the bird house, Sylvester tries to steal the small yellow bird. Tweety's bodyguard, Hector the dog, starts pulling the base of the ladder away from the pole at a rate of 2 ft/s. How fast is the ladder falling when it is 10 feet off the ground?

Example 5: Bugs and Daffy finished their final act on the *Bugs and Daffy Show* by dancing off the stage with a spotlight covering their every move. If they are moving off the stage along a straight path at a speed of 4 ft/s, and the spotlight is 20 ft away from this path, what rate is the spotlight rotating when they are 15 feet from the point on the path closest to the spotlight?



Example 6: A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep. The volume of a circular cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.