RELATED RATES- CW-day 1

1. As a ball in the shape of a sphere is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1.5 inches. $V=\frac{4}{3}\pi r^3$

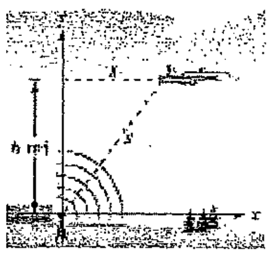
2. A balloon rises at the rate of 8 feet per second from a point on the ground 60 ft from an observer. Find the rate of change of the angle of elevation when the balloon is 25 ft above the ground.

3. A ladder 15 ft long is leaning against a building so that the end is on level ground. The ladder is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second. Find the rate at which the height is changing when the ladder is 9 feet from the building.

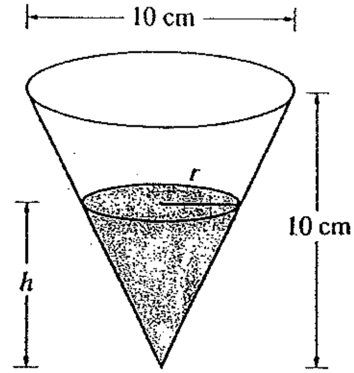
Related Rates- HW- worked out solutions on my webpage

1. A spherical balloon is inflated with gas at a rate of 500 cubic centimeters per minute. What is the rate of change of the radius when the radius is 30 centimeters? $V = \frac{4}{3}\pi r^3$

2 An airplane is flying at an altitude of 6 miles and passes directly over a radar antenna (see figure below).When the plane is 10 miles away (s=10) the radar detects the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane? (you need the Pythagorean thm)



3. A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2m^3$/min, find the rate at which the water level is rising when the water is 3m deep. The volume of a circular cone with radius 4 and height h is given by $V = \frac{1}{3}\pi r^2 h$



5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

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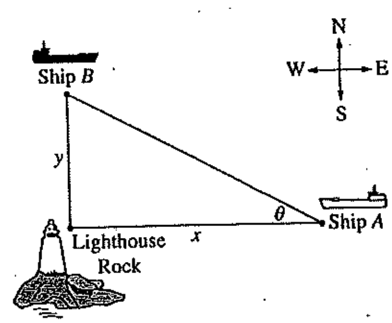
Question 6



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

1. Find dy/dx for the following curve

$$x^2y + y^2x = -2$$

2 Find the $\frac{d^2y}{dx^2}$

1-xy=x-y

3 Joey is perched precariously at the top of a 10 foot ladder leaning against the back wall of an apartment building when it starts to slide down the wall at a rate of 4ft/min. Joey's accomplice, Lou is standing on the ground 6 ft away from the wall. How fast is the base of the ladder moving when it hits lou?

4. Find dy/dx of the curve $y=\cos^3(x^2)$

5. A cone-shaped icicle is dripping from the roof.
The radius of the icicle is decreasing at a rate of 0.2 cm/hour, while the length is increasing at a rate of 0.8 cm/hour.
If the icicle is currently 4 cm in radius and 20 cm long, is the volume of the icicle increasing or decreasing, and at what rate?

Now let's find the equation of the line tangent to the
6 curve $x^2y+3x=y^2+1$ at the point $(1,-1)$

8 Find the derivative of the function: $y=\frac{4}{x^3}$.

(A) $-4x^2$

(B) $-\frac{12}{x^2}$

(C) $\frac{12}{x^2}$

(D) $\frac{12}{x^4}$

(E) $-\frac{12}{x^4}$

9. If the nth derivative of y is denoted as $y^{(n)}$ and $y=-\sin x$, then $y^{(7)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) none of the above

10. Find the second derivative of f(x) if $f(x)=(2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

11. Find $\frac{dy}{dx}$ for $y=4\sin^2(3x)$.

(A) $8\sin(3x)$

(B) $24\sin(3x)$

(C) $8\sin(3x)\cos(3x)$

(D) $12\sin(3x)\cos(3x)$

(E) $24\sin(3x)\cos(3x)$

12. The equation of the tangent line to the graph of the function $f(x) = \cos(x)$ at $x = \frac{\pi}{2}$ is:

(a) $y = 0$

(b) $y = x - 1$

(c) $y = -x + \frac{\pi}{2}$

(d) $y = -\sin(x)$

(e) $y = x - \frac{\pi}{2}$

13. The derivative of the function $\frac{\sin(2x)}{1+x^2}$ is:

(a) $\frac{\cos(2x)}{x}$

(b) $\frac{\cos(2x)}{(1+x^2)^2}$

(c) $\frac{\cos(2x)}{2x}$

(d) $2\frac{\cos(2x)(1+x^2) - x\sin(2x)}{(1+x^2)^2}$

(e) $\frac{\cos(2x)(1+x^2) - 2x\sin(2x)}{(1+x^2)^2}$

14) The slope of the tangent is -1 at the point (0,1) on $x^3-6xy-ky^3=a$, where k and a are constants.

The values of the constant a and k are:

a) k=1, a=-1 b) k=2, a=-2 c) k=3, a=-3 d) k=-2, a=4 e) k=-1, a=3

15) What is the slope of the line tangent to the curve $4x^2+3xy=34$ at the point (2,3)?

a) -16/3 b) -25/6 C) -5 d) -4 e) 7/6

16) Given $x\cos y=x^2+y^3$, then dy/dx =

a) $\frac{2x}{-3y^2-\sin y}$ b) $\frac{2x}{3y^2-\sin y}$ c) $\frac{2x+\cos y}{-3y^2-\sin y}$

d) $\frac{2x+\cos y}{3y^2+x\sin y}$ e) $\frac{2x-\cos y}{-3y^2-x\sin y}$

AP Calculus AB – More Review Chp 4 & Sec. 5.6

1. $xy=10$ Find dy/dt when $x=8$ given that $dx/dt=5$

2. Find the $\frac{d^2y}{dx^2}$ given $\mathbf{1-xy=x-y}$

3. Find dy/dx of the curve $y=\cos^3(x^2)$

4. Find the slope of the curve:
   $y=\sqrt{x^2+1}\,\sin(2x)$

5. Find the velocity of the function if its position is $s(t)=4x^2\tan(x^3-1)$.

6. Assume x and y are both differentiable functions of time.
   If $3x - 4y^3 = -32$ find dx/dt when $x=0$ and $dy/dt=3$.

7. Find the equation of the tangent line to the curve at the point (1,-1)
$x^2y + 3x = y^2 + 1$

8. Find dy/dx for the following curve $x^2y + y^2x = -2$

9. Find the equation of the tangent line to the curve $y = x\sin 4x$ when $x = \pi$

10. Find the slope of the curve $y^2 + yx + 3x - 6y = -3$ at the point(s) when $x=1$