TECHNOLOGY Some graphing utilities, such as *Derive, Maple, Mathcad, Mathematica,* and the *TI-89*, perform symbolic differentiation. Others perform *numerical differentiation* by finding values of derivatives using the formula

$$f'(x) \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

where Δx is a small number such as 0.001. Can you see any problems with this definition? For instance, using this definition, what is the value of the derivative of $f(x) = |x|$ when $x = 0$?

THEOREM 2.1 Differentiability Implies Continuity

If f is differentiable at $x = c$, then f is continuous at $x = c$.

Proof You can prove that f is continuous at $x = c$ by showing that $f(x)$ approaches $f(c)$ as $x \to c$. To do this, use the differentiability of f at $x = c$ and consider the following limit.

$$\begin{aligned}\lim_{x\to c}[f(x) - f(c)] &= \lim_{x\to c}\left[(x-c)\left(\frac{f(x)-f(c)}{x-c}\right)\right] \\ &= \left[\lim_{x\to c}(x-c)\right]\left[\lim_{x\to c}\frac{f(x)-f(c)}{x-c}\right] \\ &= (0)[f'(c)] \\ &= 0\end{aligned}$$

Because the difference $f(x) - f(c)$ approaches zero as $x \to c$, you can conclude that $\lim_{x\to c} f(x) = f(c)$. So, f is continuous at $x = c$.

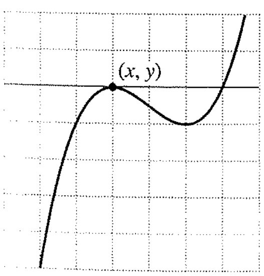
You can summarize the relationship between continuity and differentiability as follows.

1. If a function is differentiable at $x = c$, then it is continuous at $x = c$. So, differentiability implies continuity.
2. It is possible for a function to be continuous at $x = c$ and not be differentiable at $x = c$. So, continuity does not imply differentiability.

EXERCISES FOR SECTION 2.1

In Exercises 1 and 2, estimate the slope of the graph at the point (x, y).

1. (a)



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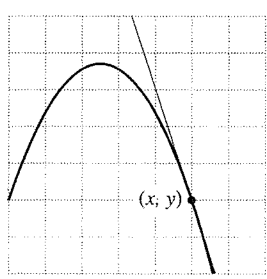
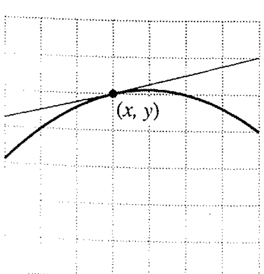


2. (a)



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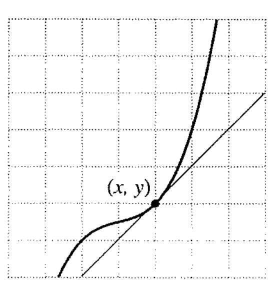
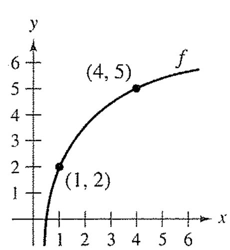


In Exercises 3 and 4, use the graph shown in the figure. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



3. Identify or sketch each of the quantities on the figure.
   (a) $f(1)$ and $f(4)$ (b) $f(4) - f(1)$
   (c) $y = \dfrac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$
4. Insert the proper inequality symbol (< or >) between the given quantities.
   (a) $\dfrac{f(4) - f(1)}{4 - 1}$ $\dfrac{f(4) - f(3)}{4 - 3}$
   (b) $\dfrac{f(4) - f(1)}{4 - 1}$ $f'(1)$

In Exercises 5–10, find the slope of the tangent line to the graph of the function at the specified point.

5. $f(x) = 3 - 2x$, $(-1, 5)$
6. $g(x) = \frac{3}{2}x + 1$, $(-2, -2)$
7. $g(x) = x^2 - 4$, $(1, -3)$
8. $g(x) = 5 - x^2$, $(2, 1)$
9. $f(t) = 3t - t^2$, $(0, 0)$
10. $h(t) = t^2 + 3$, $(-2, 7)$

In Exercises 11–24, find the derivative by the limit process.

11. $f(x) = 3$
12. $g(x) = -5$
13. $f(x) = -5x$
14. $f(x) = 3x + 2$
15. $h(s) = 3 + \frac{2}{3}s$
16. $f(x) = 9 - \frac{1}{2}x$
17. $f(x) = 2x^2 + x - 1$
18. $f(x) = 1 - x^2$
19. $f(x) = x^3 - 12x$
20. $f(x) = x^3 + x^2$
21. $f(x) = \dfrac{1}{x - 1}$
22. $f(x) = \dfrac{1}{x^2}$
23. $f(x) = \sqrt{x + 1}$
24. $f(x) = \dfrac{4}{\sqrt{x}}$

In Exercises 25–32, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

25. $f(x) = x^2 + 1$, $(2, 5)$
26. $f(x) = x^2 + 2x + 1$, $(-3, 4)$
27. $f(x) = x^3$, $(2, 8)$
28. $f(x) = x^3 + 1$, $(1, 2)$
29. $f(x) = \sqrt{x}$, $(1, 1)$
30. $f(x) = \sqrt{x - 1}$, $(5, 2)$
31. $f(x) = x + \dfrac{4}{x}$, $(4, 5)$
32. $f(x) = \dfrac{1}{x + 1}$, $(0, 1)$

In Exercises 33–36, find an equation of the line that is tangent to the graph of f *and* parallel to the given line.

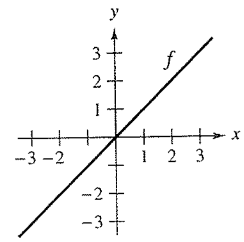
Function	*Line*
33. $f(x) = x^3$	$3x - y + 1 = 0$
34. $f(x) = x^3 + 2$	$3x - y - 4 = 0$
35. $f(x) = \dfrac{1}{\sqrt{x}}$	$x + 2y - 6 = 0$
36. $f(x) = \dfrac{1}{\sqrt{x - 1}}$	$x + 2y + 7 = 0$

37. The tangent line to the graph of $y = g(x)$ at the point $(5, 2)$ passes through the point $(9, 0)$. Find $g(5)$ and $g'(5)$.
38. The tangent line to the graph of $y = h(x)$ at the point $(-1, 4)$ passes through the point $(3, 6)$. Find $h(-1)$ and $h'(-1)$.

Getting at the Concept

In Exercises 39–42, the graph of f is given. Select the graph of f'.

39.



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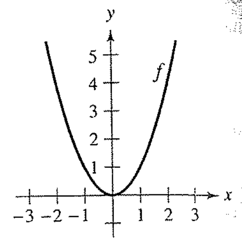

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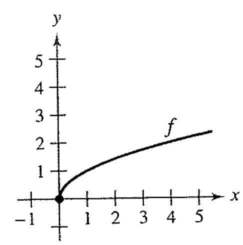

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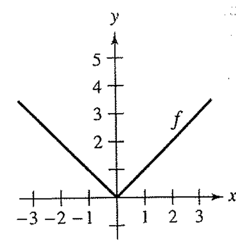
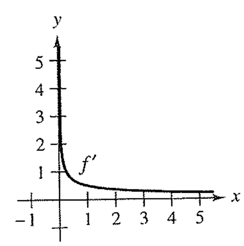


(a)



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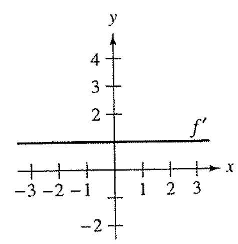
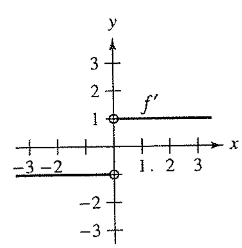
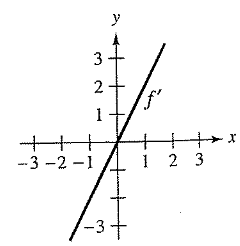


(c)



(d)



43. Sketch a graph of a function whose derivative is always negative.
44. Sketch a graph of a function whose derivative is always positive.
45. Assume that $f'(c) = 3$. Find $f'(-c)$ if (a) f is an odd function and if (b) f is an even function.
46. Determine whether the limit yields the derivative of a differentiable function f. Explain.

(a) $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + 2\Delta x) - f(x)}{2\Delta x}$

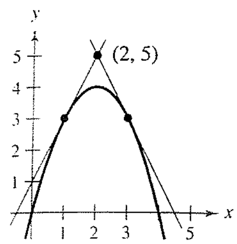
(b) $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + 2) - f(x)}{\Delta x}$

(c) $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$

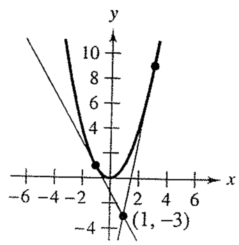
(d) $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

In Exercises 47 and 48, find equations of the two tangent lines to the graph of f that pass through the indicated point.

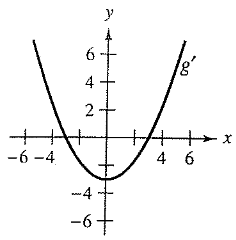
47. $f(x) = 4x - x^2$



48. $f(x) = x^2$



49. *Graphical Reasoning* The figure shows the graph of g'.



(a) $g'(0) =$

(b) $g'(3) =$

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$?

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$?

(e) Is $g(6) - g(4)$ positive or negative? Explain.

(f) Is it possible to find $g(2)$ from the graph? Explain.

50. *Graphical Reasoning* Use a graphing utility to graph each function and its tangent lines when $x = -1$, $x = 0$, and $x = 1$. Based on the results, determine whether the slope of a tangent line to the graph of a function is always distinct for different values of x.

(a) $f(x) = x^2$ (b) $g(x) = x^3$

Graphical, Numerical, and Analytic Analysis **In Exercises 51 and 52, use a graphing utility to graph f on the interval $[-2, 2]$. Complete the table by graphically estimating the slopes of the graph at the indicated points. Then evaluate the slopes analytically and compare your results with those obtained graphically.**

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$									
$f'(x)$									

51. $f(x) = \frac{1}{4}x^3$ **52.** $f(x) = \frac{1}{2}x^2$

Graphical Reasoning **In Exercises 53 and 54, use a graphing utility to graph the functions f and g in the same viewing window where**

$$g(x) = \frac{f(x + 0.01) - f(x)}{0.01}.$$

Label the graphs and describe the relationship between them.

53. $f(x) = 2x - x^2$ **54.** $f(x) = 3\sqrt{x}$

In Exercises 55 and 56, evaluate $f(2)$ and $f(2.1)$ and use the results to approximate $f'(2)$.

55. $f(x) = x(4 - x)$ **56.** $f(x) = \frac{1}{4}x^3$

Graphical Reasoning **In Exercises 57 and 58, use a graphing utility to graph the function and its derivative in the same viewing window. Label the graphs and describe the relationship between them.**

57. $f(x) = \dfrac{1}{\sqrt{x}}$ **58.** $f(x) = \dfrac{x^3}{4} - 3x$

Writing **In Exercises 59 and 60, consider the functions f and $S_{\Delta x}$ where**

$$S_{\Delta x}(x) = \frac{f(2 + \Delta x) - f(2)}{\Delta x}(x - 2) + f(2).$$

(a) Use a graphing utility to graph f and $S_{\Delta x}$ in the same viewing window for $\Delta x = 1, 0.5$, and 0.1.

(b) Give a written description of the graphs of S for the different values of Δx in part (a).

59. $f(x) = 4 - (x - 3)^2$ **60.** $f(x) = x + \dfrac{1}{x}$

In Exercises 61–70, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).

61. $f(x) = x^2 - 1, \quad c = 2$ **62.** $g(x) = x(x - 1), \quad c = 1$

63. $f(x) = x^3 + 2x^2 + 1, \quad c = -2$

64. $f(x) = x^3 + 2x, \quad c = 1$ **65.** $g(x) = \sqrt{|x|}, \quad c = 0$

66. $f(x) = 1/x, \quad c = 3$ **67.** $f(x) = (x - 6)^{2/3}, \quad c = 6$

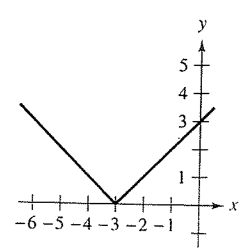
68. $g(x) = (x + 3)^{1/3}, \quad c = -3$

69. $h(x) = |x + 5|, \quad c = -5$

70. $f(x) = |x - 4|, \quad c = 4$

In Exercises 71–80, describe the x-values at which f is differentiable.

71. $f(x) = |x + 3|$



72. $f(x) = |x^2 - 9|$