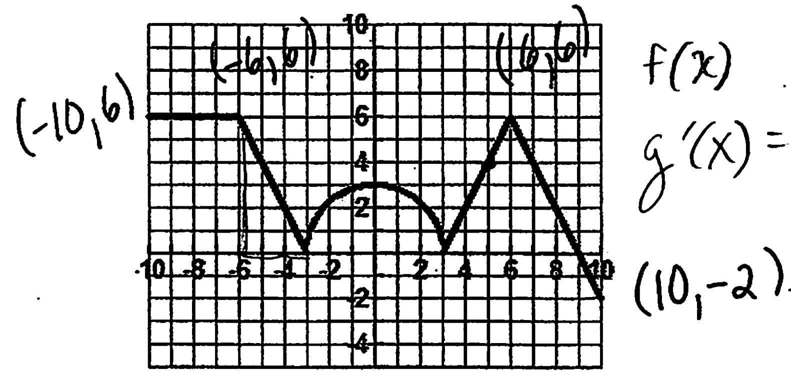
My Big FTC Picture Problem ______Key______

Show all work/steps. Justify fully.



$f(x)$

$g'(x) = f(x)$

Graph of f

A continuous function f is defined on the closed interval $-10 \le x \le 10$. The graph of f consists of a semi-circle and four line segments as shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $\lim_{x \to 5} f(x) = 4$

(b) Find the average rate of change for f on the interval $-10 \le x \le 10$ $\quad \dfrac{f(10)-f(-10)}{10--10} = \dfrac{-2-6}{20}$

$-8/20 = -2/5$

(c) Does the Mean Value Theorem guarantee a value c, $-10 < c < 10$ such that $f'(c)$ will equal the average rate of change from part (b)? No → Not diff on $(-10,10)$

(d) Show [using Calculus] that $f'(6)$ does not exist

The left hand derivative ≠ The right hand derivative

$$\boxed{\lim_{x \to 6^+} f' = -2 \qquad \lim_{x \to 6^-} f' = 2}$$

(e) Find the value of $g(-3) = 0$

(f) Find the value of $g(3)$ $9\pi/2$

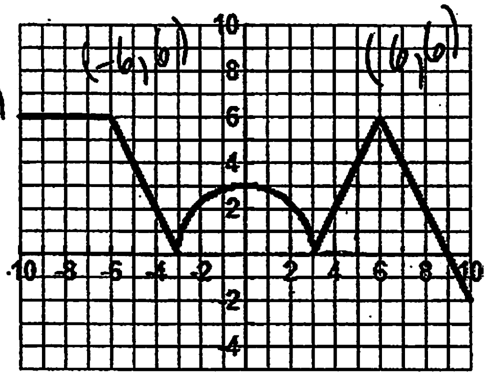
(g) Find the value of $g(-10) = -(9+24) = \boxed{-33}$

(h) Find the value of $g(10)$

$$\int_{-3}^{10} f(t)\,dt = \frac{9\pi}{2} + \frac{1}{2}(6)(6) - \frac{1}{2}(1)(2)$$

$$\frac{9\pi}{2} + 18 - 1 = \boxed{\frac{9\pi}{2} + 17}$$

$$\int_{-3}^{x} f(t)\,dt = g(x)$$



$g'(x) = f(x)$

$g''(x) = f'(x)$

Graph of $f = g'(x)$

(i) Find $g'(x)$ $= f(x)$

(j) Find the x –value(s) of the critical value(s) for the graph of g and classify as relative minimum, relative maximum, or neither $g' = 0$ $-3, 3, 9$

Relative Max at $x = 9$

Neither at $x = -3, 3$

(k) Find all intervals where the graph of g is increasing

where $g' > 0$ $(-10, 9)$

(l) Find all intervals where the graph of g is decreasing

$(9, 10)$

(m) Find the absolute extrema for the graph of g

Abs Min at $x = -10 \to -33$

Abs Max at $x = 9 \to \frac{9\pi}{2} + 18$

(n) Find $g'(5)$ $= f(5) = 4$

$\frac{9\pi}{2} + 4$

(o) Write the equation of the line tangent to the graph of g at $x = 5$

$(5, g(5)) = (5, 9\pi/2 + 4)$ $m_T = 4$

$y - (9\pi/2 + 4) = 4(x - 5)$

$y = 4x - 16 + 9\pi/2$

(p) Use the tangent line from part (o) to estimate $g(5.1)$

$4(5.1) - 16 + 9\pi/2 \approx 18.537$

(q) Does the tangent line from part (o) lie above or below the graph of g?

$g'' > 0$ concave up so below

(r) Is the estimate you found in part (p) an over- or an under-estimate?

underestimate

(s) Find $g'(-4)$ $= 2$

$g'(-4) = f(-4) = 2$

(t) Write the equation of the line tangent to the graph of g at $x = -4$

$(-4, g(-4))$ $m_T = 2$

$(-4, -1)$ $m_T = 2$

$y = 2x + 7$

$g(-4) = = -1$

$y + 1 = 2(x + 4)$

(u) Use the tangent line from part (t) to find an estimate for $g(-4.1)$

$g(-4.1) \approx 2(-4.1) + 7 = -1.2$

(v) Does the tangent line from part (t) lie above or below the graph of g?

$g''(-4) < 0$ concave down

so Above