going to use the notation dx to represent this infinitely tiny distance.

The summation notation of sigma is going to be replaced with an *Integral Sign*, $\int$, which looks somewhat like a giant "S" for sum.

The $f(c_k)$ which represented a different function value for each interval is going to be replaced with $f(x)$ since the x – values are going to be soooooooo close together it's almost as if we are evaluating the function at EVERY x – value in the interval $[a, b]$. Combining all of this we have the following notation:

$$\int_a^b f(x)\,dx$$

We read the notation above as "***The Integral of f of x from a to b***"

Important ♬ (Actually it's a Theorem): IF the function is continuous, THEN the Definite Integral will exist. However, the converse, while true some of the time is NOT ALWAYS true.

Using Definite Integrals as Area

We can define the **area under the curve** $y = f(x)$ **from** a **to** b as an *integral* from a to b ...
... AS LONG AS THE CURVE IS NONNEGATIVE AND INTEGRABLE on the closed interval $[a, b]$.

Drawing a picture and using geometry is still a valid method of finding areas in this class!

Example 1: For each of the following examples, sketch a graph of the function, shade the area you are trying to find, then use geometric formulas to evaluate each integral.

a) $\int_2^9 3\,dx$

b) $\int_{-2}^1 |x|\,dx$.

c) $\int_{-3}^3 \sqrt{9-x^2}\,dx$

So … what happens if the "area" is below the x-axis … as I mentioned before, "area" is inherently positive, but a Riemann sum … and therefore an Integral can have negative values if the curve lies below the x-axis.

Example 2: Consider the function $f(x)=3-x$. Sketch a graph of this function.

a) What is the "AREA" between the curve and the $x-$axis between $x=4$ and $x=8$?

b) Evaluate $\int_4^8 (3-x)\,dx$

Example 3: Given $\int_0^\pi \sin x\,dx=2$, use what you know about a sine function to evaluate the following integrals.

a) $\int_\pi^{2\pi} \sin x\,dx$

b) $\int_0^{2\pi} \sin x\,dx$

c) $\int_0^{\pi/2} \sin x\,dx$

d) $\int_{-\pi}^{\pi} \sin x\,dx$

e) $\int_0^\pi (2+\sin x)\,dx$