AP CALCULUS AB
GRAPHING CALCULATOR WORKSHEET #2

FINDING LIMITS NUMERICALLY AND GRAPHICALLY

Using a graphing calculator find the following limits either graphically or numerically. Show a sketch of the graph or reproduce part of the table as evidence.

1. $\lim_{x \to 0+} \frac{x+1}{x}$

2. $\lim_{x \to -1+} \frac{x}{x+1}$

3. $\lim_{x \to 3+} \frac{x-4}{x-3}$

4. $\lim_{x \to 1} \frac{x-3}{x^2-1}$

5. $\lim_{x \to 0} \frac{|x|}{x}$

6. $\lim_{x \to 1-} \frac{x-3}{x^2-1}$

7. $\lim_{x \to 2} \frac{x-2}{\sqrt{x^2-4}}$

8. $\lim_{x \to 2} f(x)$

if $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & if \cdot x \neq 2 \\ 5 & if \cdot x = 2 \end{cases}$

9. $\lim_{\theta \to 0} \dfrac{\tan \theta}{\theta}$

10. $\lim_{t \to 0} \dfrac{1-\cos t}{t^2}$

11. $\lim_{x \to 0} \dfrac{\sin 2x}{x}$

12. $\lim_{x \to 9} \dfrac{\sqrt{x}-3}{x-9}$

AP CALCULUS AB

GRAPHING CALCULATOR WORKSHEET #2

FINDING LIMITS NUMERICALLY AND GRAPHICALLY

Using a graphing calculator find the following limits either graphically or numerically. Show a sketch of the graph or reproduce part of the table as evidence.

1. $\lim\limits_{x \to 0^+} \dfrac{x+1}{x}$ DNE $+\infty$

2. $\lim\limits_{x \to -1^+} \dfrac{x}{x+1}$ DNE $-\infty$

3. $\lim\limits_{x \to 3^+} \dfrac{x-4}{x-3}$ DNE $-\infty$

4. $\lim\limits_{x \to 1} \dfrac{x-3}{x^2-1}$ DNE $-\infty$

5. $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ DNE left + right not same

6. $\lim\limits_{x \to 1^-} \dfrac{x-3}{x^2-1}$ DNE $+\infty$

7. $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{x^2-4}}$ DNE left + rt not =

8. $\lim_{x \to 2} f(x)$ = 4

if $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & if \cdot x \neq 2 \\ 5 & if \cdot x = 2 \end{cases}$

9. $\lim_{\theta \to 0} \dfrac{\tan \theta}{\theta}$ = 1

10. $\lim_{t \to 0} \dfrac{1 - \cos t}{t^2}$ = .5

11. $\lim_{x \to 0} \dfrac{\sin 2x}{x}$ = 2

12. $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$ = 1/6

AP CALCULUS AB
FINDING LIMITS ANALYTICALLY / Algebraically

Find the following limits WITHOUT using a graphing calculator. SHOW your work.

1. $\lim\limits_{x \to -1} \dfrac{x^2+3x+2}{x^2+1}$

2. $\lim\limits_{x \to 1} \dfrac{3x^3-4x^2-5x+2}{x^2-x-2}$

3. $\lim\limits_{x \to 1} f(x)$ for $f(x) = \begin{cases} x^2+4, & x \neq 1 \\ 2, & x = 1 \end{cases}$

4. $\lim\limits_{x \to \pi} \dfrac{1-\cos x}{x}$

5. $\lim\limits_{x \to \pi} \tan 5x$

6. If $\lim\limits_{x \to c} f(x) = -\dfrac{1}{2}$, $\lim\limits_{x \to c} g(x) = \dfrac{2}{3}$, find $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)}$.

7. $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4}$

8. $\lim\limits_{x \to -9} \dfrac{x^2+6x-27}{x+9}$

9. $\lim_{x \to 2} \frac{x-2}{|x-2|}$

10. $\lim_{x \to 4} \frac{x^2-5x+4}{x-4}$

11. $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x}$

12. $\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$

13. $\lim_{x \to 0} \frac{\sin 5x}{x}$

14. $\lim_{x \to 0} \frac{\sin x^2}{x^2}$

AP CALCULUS AB

FINDING LIMITS ANALYTICALLY

Find the following limits WITHOUT using a graphing calculator. SHOW your work.

1. $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 + 1}$ 0

2. $\lim_{x \to 1} \frac{3x^3 - 4x^2 - 5x + 2}{x^2 - x - 2}$ 2

3. $\lim_{x \to 1} f(x)$ for $f(x) = \begin{cases} x^2 + 4, & x \neq 1 \\ 2, & x = 1 \end{cases}$ 5

4. $\lim_{x \to \pi} \frac{1 - \cos x}{x}$ $\frac{2}{\pi}$

5. $\lim_{x \to \pi} \tan 5x$ 0

6. If $\lim_{x \to c} f(x) = -\frac{1}{2}$, $\lim_{x \to c} g(x) = \frac{2}{3}$, find $\lim_{x \to c} \frac{f(x)}{g(x)}$. $-\frac{3}{4}$

7. $\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$ $\frac{1}{4}$

8. $\lim_{x \to -9} \frac{x^2 + 6x - 27}{x + 9}$ -12

9. $\lim_{x \to 2} \frac{x-2}{|x-2|}$ = DNE left limit ≠ Rt limit

10. $\lim_{x \to 4} \frac{x^2 - 5x + 4}{x - 4}$ = 3

11. $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$ = $\frac{1}{4}$

12. $\lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$ = $\frac{1}{2\sqrt{x}}$

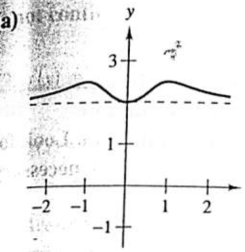
13. $\lim_{x \to 0} \frac{\sin 5x}{x}$ = 5

14. $\lim_{x \to 0} \frac{\sin x^2}{x^2}$ = 1

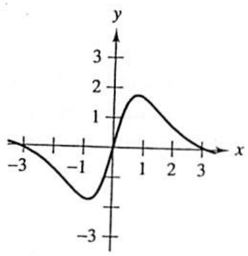
EXERCISES FOR SECTION 3.5

In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.

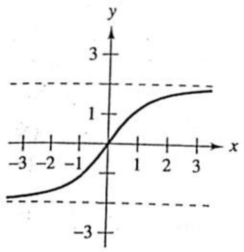
(a)



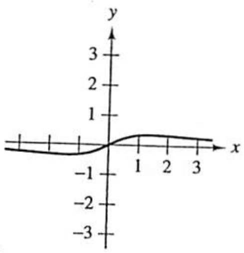
(b)



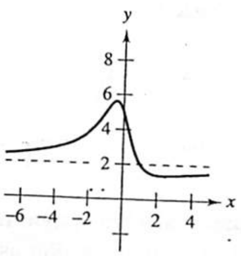
(c)



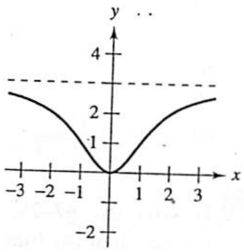
(d)



(e)



(f)



1. $f(x) = \frac{3x^2}{x^2 + 2}$
2. $f(x) = \frac{2x}{\sqrt{x^2 + 2}}$
3. $f(x) = \frac{x}{x^2 + 2}$
4. $f(x) = 2 + \frac{x^2}{x^4 + 1}$
5. $f(x) = \frac{4 \sin x}{x^2 + 1}$
6. $f(x) = \frac{2x^2 - 3x + 5}{x^2 + 1}$

Numerical and Graphical Analysis **In Exercises 7–12, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

7. $f(x) = \frac{4x + 3}{2x - 1}$
8. $f(x) = \frac{2x^2}{x + 1}$
9. $f(x) = \frac{-6x}{\sqrt{4x^2 + 5}}$
10. $f(x) = \frac{8x}{\sqrt{x^2 - 3}}$
11. $f(x) = 5 - \frac{1}{x^2 + 1}$
12. $f(x) = 4 + \frac{3}{x^2 + 2}$

In Exercises 13 and 14, find $\lim_{x \to \infty} h(x)$, if possible.

13. $f(x) = 5x^3 - 3x^2 + 10$

(a) $h(x) = \frac{f(x)}{x^2}$ (b) $h(x) = \frac{f(x)}{x^3}$

(c) $h(x) = \frac{f(x)}{x^4}$

14. $f(x) = 5x^2 - 3x + 7$

(a) $h(x) = \frac{f(x)}{x}$ (b) $h(x) = \frac{f(x)}{x^2}$

(c) $h(x) = \frac{f(x)}{x^3}$

In Exercises 15–18, find each of the limits, if possible.

15. (a) $\lim_{x \to \infty} \frac{x^2 + 2}{x^3 - 1}$

(b) $\lim_{x \to \infty} \frac{x^2 + 2}{x^2 - 1}$

(c) $\lim_{x \to \infty} \frac{x^2 + 2}{x - 1}$

16. (a) $\lim_{x \to \infty} \frac{3 - 2x}{3x^3 - 1}$

(b) $\lim_{x \to \infty} \frac{3 - 2x}{3x - 1}$

(c) $\lim_{x \to \infty} \frac{3 - 2x^2}{3x - 1}$

17. (a) $\lim_{x \to \infty} \frac{5 - 2x^{3/2}}{3x^2 - 4}$

(b) $\lim_{x \to \infty} \frac{5 - 2x^{3/2}}{3x^{3/2} - 4}$

(c) $\lim_{x \to \infty} \frac{5 - 2x^{3/2}}{3x - 4}$

18. (a) $\lim_{x \to \infty} \frac{5x^{3/2}}{4x^2 + 1}$

(b) $\lim_{x \to \infty} \frac{5x^{3/2}}{4x^{3/2} + 1}$

(c) $\lim_{x \to \infty} \frac{5x^{3/2}}{4\sqrt{x} + 1}$

In Exercises 19–32, find the limit.

19. $\lim_{x \to \infty} \frac{2x - 1}{3x + 2}$
20. $\lim_{x \to \infty} \frac{3x^3 + 2}{9x^3 - 2x^2 + 7}$
21. $\lim_{x \to \infty} \frac{x}{x^2 - 1}$
22. $\lim_{x \to \infty} \left(4 + \frac{3}{x}\right)$
23. $\lim_{x \to -\infty} \frac{5x^2}{x + 3}$
24. $\lim_{x \to -\infty} \left(\frac{1}{2}x - \frac{4}{x^2}\right)$
25. $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - x}}$
26. $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 + 1}}$
27. $\lim_{x \to -\infty} \frac{2x + 1}{\sqrt{x^2 - x}}$
28. $\lim_{x \to -\infty} \frac{-3x + 1}{\sqrt{x^2 + x}}$
29. $\lim_{x \to \infty} \frac{\sin 2x}{x}$
30. $\lim_{x \to \infty} \frac{x - \cos x}{x}$
31. $\lim_{x \to \infty} \frac{1}{2x + \sin x}$
32. $\lim_{x \to \infty} \cos \frac{1}{x}$

In Exercises 33 and 34, use a graphing utility to graph the function and verify that it has two horizontal asymptotes.

33. $f(x) = \frac{|x|}{x + 1}$
34. $f(x) = \frac{3x}{\sqrt{x^2 + 2}}$

In Exercises 35 and 36, find the limit. (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$.)

35. $\lim_{x\to\infty} x \sin \frac{1}{x}$

36. $\lim_{x\to\infty} x \tan \frac{1}{x}$

In Exercises 37–40, find the limit. (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.) Use a graphing utility to verify your result.

37. $\lim_{x\to-\infty} \left(x + \sqrt{x^2+3}\right)$

38. $\lim_{x\to\infty} \left(2x - \sqrt{4x^2+1}\right)$

39. $\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right)$

40. $\lim_{x\to-\infty} \left(3x + \sqrt{9x^2-x}\right)$

Numerical, Graphical, and Analytic Analysis **In Exercises 41–44, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically. Finally, find the limit analytically and compare your results with the estimates.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

41. $f(x) = x - \sqrt{x(x-1)}$

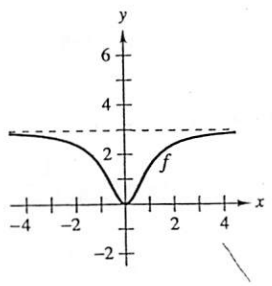
42. $f(x) = x^2 - x\sqrt{x(x-1)}$

43. $f(x) = x \sin \frac{1}{2x}$

44. $f(x) = \frac{x+1}{x\sqrt{x}}$

Getting at the Concept

45. The graph of a function f is shown below. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



(a) Sketch f'.

(b) Use the graphs to estimate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$.

(c) Explain the answers you gave in part (b).

46. Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = 2$ as its only critical number.

$f'(x) < 0$ for $x < 2$

$f'(x) > 0$ for $x > 2$

$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 6$

47. Is it possible to sketch a graph of a function that satisfies the conditions of Exercise 46 and has *no* points of inflection? Explain.

Getting at the Concept *(continued)*

48. If f is a continuous function such that $\lim_{x\to\infty} f(x) = 5$, if possible, $\lim_{x\to-\infty} f(x)$ for each specified condition.

(a) The graph of f is symmetric to the y-axis.

(b) The graph of f is symmetric to the origin.

In Exercises 49–66, sketch the graph of the equation. Look f extrema, intercepts, symmetry, and asymptotes as necessa Use a graphing utility to verify your result.

49. $y = \frac{2+x}{1-x}$

50. $y = \frac{x-3}{x-2}$

51. $y = \frac{x}{x^2-4}$

52. $y = \frac{2x}{9-x^2}$

53. $y = \frac{x^2}{x^2+9}$

54. $y = \frac{x^2}{x^2-9}$

55. $y = \frac{2x^2}{x^2-4}$

56. $y = \frac{2x^2}{x^2+4}$

57. $xy^2 = 4$

58. $x^2y = 4$

59. $y = \frac{2x}{1-x}$

60. $y = \frac{2x}{1-x^2}$

61. $y = 2 - \frac{3}{x^2}$

62. $y = 1 + \frac{1}{x}$

63. $y = 3 + \frac{2}{x}$

64. $y = 4\left(1 - \frac{1}{x^2}\right)$

65. $y = \frac{x^3}{\sqrt{x^2-4}}$

66. $y = \frac{x}{\sqrt{x^2-4}}$

In Exercises 67–76, use a computer algebra system to anal the graph of the function. Label any extrema and/or asympt that exist.

67. $f(x) = 5 - \frac{1}{x^2}$

68. $f(x) = \frac{x^2}{x^2-1}$

69. $f(x) = \frac{x}{x^2-4}$

70. $f(x) = \frac{1}{x^2-x-2}$

71. $f(x) = \frac{x-2}{x^2-4x+3}$

72. $f(x) = \frac{x+1}{x^2+x+1}$

73. $f(x) = \frac{3x}{\sqrt{4x^2+1}}$

74. $g(x) = \frac{2x}{\sqrt{3x^2+1}}$

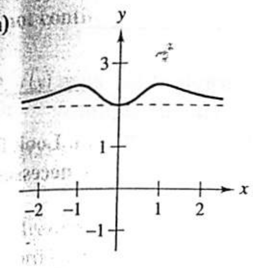
75. $g(x) = \sin\left(\frac{x}{x-2}\right), \quad 3 < x < \infty$

76. $f(x) = \frac{2 \sin 2x}{x}$

EXERCISES FOR SECTION 3.5

In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.

(a)



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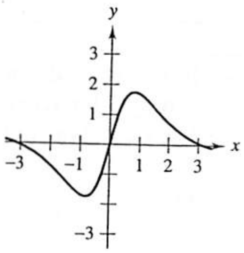

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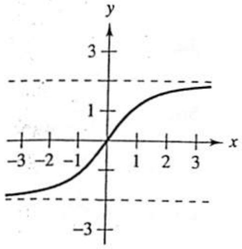

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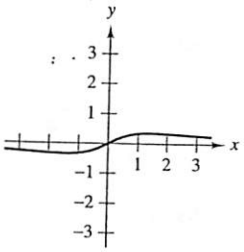

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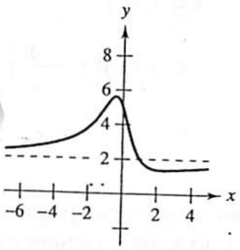
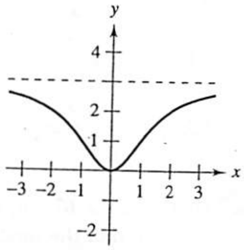


(f)



1. $f(x) = \dfrac{3x^2}{x^2+2}$

2. $f(x) = \dfrac{2x}{\sqrt{x^2+2}}$

3. $f(x) = \dfrac{x}{x^2+2}$

4. $f(x) = 2 + \dfrac{x^2}{x^4+1}$

5. $f(x) = \dfrac{4\sin x}{x^2+1}$

6. $f(x) = \dfrac{2x^2-3x+5}{x^2+1}$

Numerical and Graphical Analysis **In Exercises 7–12, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

7. $f(x) = \dfrac{4x+3}{2x-1}$

8. $f(x) = \dfrac{2x^2}{x+1}$

9. $f(x) = \dfrac{-6x}{\sqrt{4x^2+5}}$

10. $f(x) = \dfrac{8x}{\sqrt{x^2-3}}$

11. $f(x) = 5 - \dfrac{1}{x^2+1}$

12. $f(x) = 4 + \dfrac{3}{x^2+2}$

In Exercises 13 and 14, find $\lim_{x\to\infty} h(x)$, if possible.

13. $f(x) = 5x^3 - 3x^2 + 10$

(a) $h(x) = \dfrac{f(x)}{x^2}$ ∞ (b) $h(x) = \dfrac{f(x)}{x^3}$ 5

(c) $h(x) = \dfrac{f(x)}{x^4}$ 0

14. $f(x) = 5x^2 - 3x + 7$

(a) $h(x) = \dfrac{f(x)}{x}$ (b) $h(x) = \dfrac{f(x)}{x^2}$

(c) $h(x) = \dfrac{f(x)}{x^3}$

In Exercises 15–18, find each of the limits, if possible.

15. (a) $\lim_{x\to\infty} \dfrac{x^2+2}{x^3-1}$

(b) $\lim_{x\to\infty} \dfrac{x^2+2}{x^2-1}$

(c) $\lim_{x\to\infty} \dfrac{x^2+2}{x-1}$

16. (a) $\lim_{x\to\infty} \dfrac{3-2x}{3x^3-1}$

(b) $\lim_{x\to\infty} \dfrac{3-2x}{3x-1}$

(c) $\lim_{x\to\infty} \dfrac{3-2x^2}{3x-1}$

17. (a) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x^2-4}$

(b) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x^{3/2}-4}$

(c) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x-4}$

18. (a) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^2+1}$

(b) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^{3/2}+1}$

(c) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4\sqrt{x}+1}$

In Exercises 19–32, find the limit.

19. $\lim_{x\to\infty} \dfrac{2x-1}{3x+2}$ 2/3

20. $\lim_{x\to\infty} \dfrac{3x^3+2}{9x^3-2x^2+7}$ 1/3

21. $\lim_{x\to\infty} \dfrac{x}{x^2-1}$ 0

22. $\lim_{x\to\infty} \left(4+\dfrac{3}{x}\right)$ 4

23. $\lim_{x\to-\infty} \dfrac{5x^2}{x+3}$ −∞

24. $\lim_{x\to-\infty} \left(\dfrac{1}{2}x - \dfrac{4}{x^2}\right)$

25. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2-x}}$

26. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$

27. $\lim_{x\to-\infty} \dfrac{2x+1}{\sqrt{x^2-x}}$

28. $\lim_{x\to-\infty} \dfrac{-3x+1}{\sqrt{x^2+x}}$

29. $\lim_{x\to\infty} \dfrac{\sin 2x}{x}$ 0

30. $\lim_{x\to\infty} \dfrac{x-\cos x}{x}$

31. $\lim_{x\to\infty} \dfrac{1}{2x+\sin x}$ 0

32. $\lim_{x\to\infty} \cos\dfrac{1}{x}$

In Exercises 33 and 34, use a graphing utility to graph the function and verify that it has two horizontal asymptotes.

33. $f(x) = \dfrac{|x|}{x+1}$

34. $f(x) = \dfrac{3x}{\sqrt{x^2+2}}$

In Exercises 35 and 36, find the limit. (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$.)

35. $\lim_{x\to\infty} x \sin \frac{1}{x}$

36. $\lim_{x\to\infty} x \tan \frac{1}{x}$

In Exercises 37–40, find the limit. (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.) Use a graphing utility to verify your result.

37. $\lim_{x\to-\infty} \left(x + \sqrt{x^2 + 3}\right)$

38. $\lim_{x\to\infty} \left(2x - \sqrt{4x^2 + 1}\right)$

39. $\lim_{x\to\infty} \left(x - \sqrt{x^2 + x}\right)$

40. $\lim_{x\to-\infty} \left(3x + \sqrt{9x^2 - x}\right)$

Numerical, Graphical, and Analytic Analysis **In Exercises 41–44, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically. Finally, find the limit analytically and compare your results with the estimates.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

41. $f(x) = x - \sqrt{x(x - 1)}$

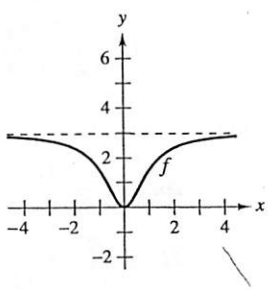
42. $f(x) = x^2 - x\sqrt{x(x - 1)}$

43. $f(x) = x \sin \frac{1}{2x}$

44. $f(x) = \frac{x + 1}{x\sqrt{x}}$

Getting at the Concept

45. The graph of a function f is shown below. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



(a) Sketch f'.

(b) Use the graphs to estimate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$.

(c) Explain the answers you gave in part (b).

46. Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = 2$ as its only critical number.

$f'(x) < 0$ for $x < 2$

$f'(x) > 0$ for $x > 2$

$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 6$

47. Is it possible to sketch a graph of a function that satisfies the conditions of Exercise 46 and has *no* points of inflection? Explain.

Getting at the Concept (*continued*)

48. If f is a continuous function such that $\lim_{x\to\infty} f(x) =$ [illegible]d, if possible, $\lim_{x\to-\infty} f(x)$ for each specified condition.

(a) The graph of f is symmetric to the y-axis.

(b) The graph of f is symmetric to the origin.

In Exercises 49–66, sketch the graph of the equation. Look f extrema, intercepts, symmetry, and asymptotes as necessai Use a graphing utility to verify your result.

49. $y = \frac{2 + x}{1 - x}$

50. $y = \frac{x - 3}{x - 2}$

51. $y = \frac{x}{x^2 - 4}$

52. $y = \frac{2x}{9 - x^2}$

53. $y = \frac{x^2}{x^2 + 9}$

54. $y = \frac{x^2}{x^2 - 9}$

55. $y = \frac{2x^2}{x^2 - 4}$

56. $y = \frac{2x^2}{x^2 + 4}$

57. $xy^2 = 4$

58. $x^2y = 4$

59. $y = \frac{2x}{1 - x}$

60. $y = \frac{2x}{1 - x^2}$

61. $y = 2 - \frac{3}{x^2}$

62. $y = 1 + \frac{1}{x}$

63. $y = 3 + \frac{2}{x}$

64. $y = 4\left(1 - \frac{1}{x^2}\right)$

65. $y = \frac{x^3}{\sqrt{x^2 - 4}}$

66. $y = \frac{x}{\sqrt{x^2 - 4}}$

In Exercises 67–76, use a computer algebra system to anal the graph of the function. Label any extrema and/or asympt that exist.

67. $f(x) = 5 - \frac{1}{x^2}$

68. $f(x) = \frac{x^2}{x^2 - 1}$

69. $f(x) = \frac{x}{x^2 - 4}$

70. $f(x) = \frac{1}{x^2 - x - 2}$

71. $f(x) = \frac{x - 2}{x^2 - 4x + 3}$

72. $f(x) = \frac{x + 1}{x^2 + x + 1}$

73. $f(x) = \frac{3x}{\sqrt{4x^2 + 1}}$

74. $g(x) = \frac{2x}{\sqrt{3x^2 + 1}}$

75. $g(x) = \sin\left(\frac{x}{x - 2}\right), \quad 3 < x < \infty$

76. $f(x) = \frac{2 \sin 2x}{x}$

Mr Leckie

Calculus Limits Review
Chapter

Name: ____________________

There's obviously not enough room for you to work out problems on this paper. This will not be collected, but I highly suggest you understand how to complete each and every problem here! Solutions (along with the worksheet) will be posted on your Assignment page.

For questions 1 – 10, evaluate each limit without using your calculator.

1. $\lim\limits_{x \to ½} \lfloor x \rfloor$

2. $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3}{3x+2}$

3. $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3}{3x^2+2}$

4. $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3}{3x^3+2}$

5. $\lim\limits_{x \to 0} \dfrac{x}{\sin(2x)}$

6. $\lim\limits_{x \to \infty} \dfrac{\sin x}{2x}$

7. $\lim\limits_{x \to 0} \dfrac{\tan(5x)}{\sin(3x)}$

8. $\lim\limits_{x \to \infty} \dfrac{4x^2+5x}{x-3}$

9. $\lim\limits_{x \to \infty} \dfrac{5x-7x^2}{4x^2+1}$

10. $\lim\limits_{x \to -3} \dfrac{|x+3|}{x+3}$

11. Use a table of values to evaluate the following limit: $\lim\limits_{x \to \infty} \left(1+\dfrac{1}{x}\right)^x$
Recognize the number? ... Add this to your notecards under "Limits you should know".

12. Make a table of values (4 of them would work) to evaluate $\lim\limits_{x \to 2} \dfrac{x+3}{x-2}$.

For questions 13 and 14, find ALL asymptotes (vertical, horizontal, and oblique) and justify your response.

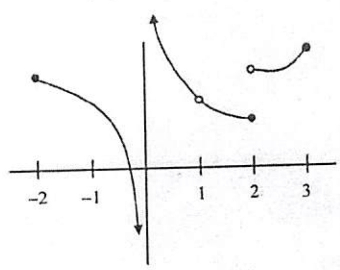
13. $y = \ln x$

14. $f(x) = \dfrac{(x+2)(x-3)}{(x+2)(x-1)}$

15. Let $h(x) = \dfrac{(x-1)(x+3)}{(x+3)(x-2)}$. Identify all values of c where the $\lim\limits_{x \to c} h(x)$ EXISTS.

16. Let $g(x) = \dfrac{x^2+5x+6}{x^2+3x+2}$.
a) Find the domain of $g(x)$.
b) Find the $\lim\limits_{x \to c} g(x)$ for all values of c where $g(x)$ is not defined.
c) Find any horizontal asymptotes and justify your response.
d) Find any vertical asymptotes and justify your response.
e) Write an extension to the function so that $g(x)$ is continuous for all $x < -1$.

17. Using the function below, over what intervals does $\lim\limits_{x \to c} f(x)$ exist?



Calculus
Chapter 2 Review Solutions

I have done my best to make sure all the solutions are correct. Inevitably, there seem to be typos. If you do not agree/understand a solution, email me or find time to ask me about them BEFORE the exam.

1. Use direct substitution. $\lim\limits_{x \to 5/2} \lfloor x \rfloor = \lfloor \frac{5}{2} \rfloor = 2$

2. The numerator has a higher degree than the denominator, so $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3}{3x+2}$ does not exist (DNE).

3. The numerator and denominator have the same degree, so $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3}{3x^2+2} = \dfrac{1}{3}$

4. The numerator has a lower degree than the denominator, so $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3}{3x^3+2} = 0$

5. Multiply the denominator by $\dfrac{2x}{2x}$ … $\lim\limits_{x \to 0} \dfrac{x}{\sin(2x) \cdot \frac{2x}{2x}} = \lim\limits_{x \to 0} \dfrac{\cancel{x}}{\frac{\sin(2x)}{2x} \cdot 2\cancel{x}} = \lim\limits_{x \to 0} \dfrac{1}{\frac{\sin(2x)}{2x} \cdot 2} = \dfrac{1}{1 \cdot 2} = \dfrac{1}{2}$

6. Take out the $\dfrac{1}{2}$ to get $\lim\limits_{x \to \infty} \dfrac{\sin x}{2x} = \dfrac{1}{2} \lim\limits_{x \to \infty} \dfrac{\sin x}{x} = \dfrac{1}{2} \cdot 0 = 0$ … or realize that as $x \to \infty$, the denominator gets really large while the numerator stays between 1 and -1.

7. Rewrite tan (5*x*) in terms of sine and cosine and multiply by 1/sin(3*x*) instead of dividing …

$\lim\limits_{x \to 0} \dfrac{\tan(5x)}{\sin(3x)} = \lim\limits_{x \to 0} \dfrac{\sin(5x)}{\cos(5x)} \cdot \dfrac{1}{\sin(3x)}$ … Multiply the numerator by $\frac{5x}{5x}$ … and the denominator by $\frac{3x}{3x}$

$\lim\limits_{x \to 0} \dfrac{\sin(5x) \cdot \frac{5x}{5x}}{\cos(5x)} \cdot \dfrac{1}{\sin(3x) \cdot \frac{3x}{3x}} = \lim\limits_{x \to 0} \dfrac{\frac{\sin(5x)}{5x} \cdot 5\cancel{x}}{\cos(5x)} \cdot \dfrac{1}{\frac{\sin(3x)}{3x} \cdot 3\cancel{x}}$

Once the *x* is cancelled above, you can evaluate the limit … $\lim\limits_{x \to 0} \dfrac{\frac{\sin(5x)}{5x} \cdot 5}{\cos(5x)} \cdot \dfrac{1}{\frac{\sin(3x)}{3x} \cdot 3} = \dfrac{1 \cdot 5}{1} \cdot \dfrac{1}{1 \cdot 3} = \dfrac{5}{3}$

8. The numerator grows faster than the denominator, so $\lim\limits_{x \to \infty} \dfrac{4x^2+5x}{x-3}$ does not exist (DNE).

9. The numerator and denominator grow at the same rate, so $\lim\limits_{x \to \infty} \dfrac{5x-7x^2}{4x^2+1} = \dfrac{-7}{4}$

10. A graph of this (make a table if necessary) would show that the $\lim\limits_{x \to -3^+} \dfrac{|x+3|}{x+3} = 1$ and $\lim\limits_{x \to -3^-} \dfrac{|x+3|}{x+3} = -1$ meaning $\lim\limits_{x \to -3} \dfrac{|x+3|}{x+3}$ does not exist (DNE).

11.

x	1000	10000	100000	1000000	10000000	100000000
$\left(1+\frac{1}{x}\right)^x$	2.716923932	2.718145927	2.718268237	2.718280469	2.718281694	2.718281786

Recognize the number? It's *e* … Add this to your notecards under "Limits you should know". … yeah … those things you have to hand in before your test! ☺

12. Since you're approaching 2 from both sides, pick two numbers close to 2 ***on both sides***.

x	$\frac{x+3}{x-2}$
1.99	–499
1.999	–4999
2.001	5001
2.01	501

Based on the table, $\lim_{x\to 2^-}\frac{x+3}{x-2}$ DNE, and as $x\to 2^-$, $\frac{x+3}{x-2}\to-\infty$.

Also, as $x\to 2^+$, $\frac{x+3}{x-2}\to\infty$.

For questions 13 and 14, find ALL asymptotes (vertical, horizontal, and oblique) and justify your response.

13. This function has a vertical asymptote at $x = 0$ because as $x\to 0^+$, $\ln x\to-\infty$. (A parent function you should know)

14. Since $f(x)=\frac{\cancel{(x+2)}(x-3)}{\cancel{(x+2)}(x-1)}=\frac{x-3}{x-1}$ there is NOT a vertical asymptote at $x = -2$ (there's a hole), but there IS a vertical asymptote at $x = 1$, since $\lim_{x\to 1}\frac{x-3}{x-1}$ DNE because as $x\to 1^-$, $\frac{x-3}{x-1}\to\infty$ and as $x\to 1^+$, $\frac{x-3}{x-1}\to-\infty$ Also, since $\lim_{x\to\infty}\frac{(x+2)(x-3)}{(x+2)(x-1)}=1$, there is a horizontal asymptote at $y = 1$.

15. Since $h(x)=\frac{(x-1)\cancel{(x+3)}}{\cancel{(x+3)}(x-2)}=\frac{x-1}{x-2}$, there is a vertical asymptote at $x = 2$, but there is a hole when $x = -3$. Therefore, $\lim_{x\to c}h(x)$ exists for all real numbers except for $x = 2$.

16.

a) Since $g(x)=\frac{x^2+5x+6}{x^2+3x+2}=\frac{\cancel{(x+2)}(x+3)}{(x+1)\cancel{(x+2)}}=\frac{x+3}{x+1}$, the domain is $x\neq-1$ and $x\neq-2$

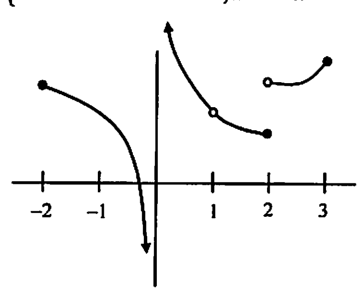
b) Since there is a removable discontinuity (a hole) at $x = -2$, the limit as x approaches –2 exists, but since there is a vertical asymptote at $x = -1$, the limit does not exist as x approaches –1.

c) Since $\lim_{x\to\infty}\frac{x^2+5x+6}{x^2+3x+2}=1$, there is a horizontal asymptote at $y = 1$.

d) There is a vertical asymptote at $x = -1$, since $\lim_{x\to-1^+}\frac{x^2+5x+6}{x^2+3x+2}=\lim_{x\to-1^+}\frac{x+3}{x+1}$ which does not exist (DNE) and as $x\to-1^+$, $\frac{x+3}{x+1}\to\infty$.

e) The only value of x less than –1 where $g(x)$ is not continuous is $x = -2$.
Since there is a removable discontinuity at $x = -2$, we just need to define the value of $g(-2)$ to fill the hole. Using the simplified version of $g(x)$, we see that $g(-2)=\frac{-2+3}{-2+1}=-1$. Thus we write $g(x)=\begin{cases}\frac{x^2+5x+6}{x^2+3x+2} & ;x\neq-1 \text{ and } x\neq-2\\ -1 & ;x=-2\end{cases}$

17. Using the function below, over what intervals does $\lim_{x\to c}f(x)$ exist?



The limit fails to exist at $x = 0$ and at $x = 2$ (both are non-removable discontinuities) .
The limit does exist at $x = 1$, even thought there is a hole (removable discontinuity).

So, the intervals where $\lim_{x\to c}f(x)$ exists are $(-2,0)\cup(0,2)\cup(2,3)$.

♪: The $\lim_{x\to-2^+}f(x)$ also exists, as does $\lim_{x\to 3^-}f(x)$.