Day 1- assign books, review summer assignment
Do Now

Simplify the following expressions:

4. $\ln e^8 + \ln e + \ln 1$

5. $\tan^{-1}(1) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

6. $\sin\left(\frac{3\pi}{2}\right) + \cos\left(\frac{\pi}{3}\right)$

7. $\cos^2(\pi) + \sin^2(\pi)$

8. $\sqrt{x}\left(x^7 - x^{\frac{11}{2}} + \sqrt[3]{x}\right)$

9. $\dfrac{x^4 + 2x^2 + 1 + \sqrt{x}}{\sqrt[3]{x}}$

Simplify and state the domain of the following expression:

10. $\dfrac{x^3 - 64x}{x^2 + 7x - 8}$

Day 2- summer assignment test

Contents

Lesson 1

website btcalculove.weebly.com
For Chapter 2 assignments,worksheets and solutions

Finding Limits graphically
CW- p. 67 #43,44
HW p. 66 #15,16 p. 67 45-50 p. 68 #57,58

,
Definition of a Limit- The (y) value that a function **approaches** as x approaches some number, **(not** what the actual value is at that point)

Limit f(x) look in both directions
X→C

WHEN LIMITS FAIL … 3 WAYS

The $\lim_{x \to c} f(x)$ does not exist when there is no number satisfying the definition.

1. $f(x)$ approaches a different numbers from the right and left.

   Example: $\lim_{x \to 0} \frac{|x|}{x}$

2. $f(x)$ increases or decreases without bound as x approaches c.

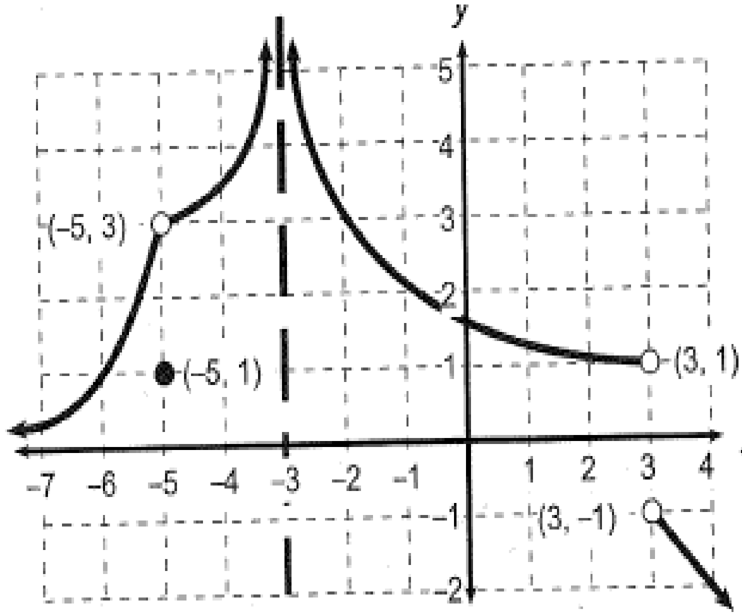
   Example: $\lim_{x \to 1} \frac{1}{(x-1)^2}$

3. $f(x)$ oscillates between two fixed values as x approaches c.

   Example: $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

1. Find the limit of f(x) as x approaches -5
2. Find the limit of f(x) as x approaches -3
3. Find the limit of f(x) as x approaches 3

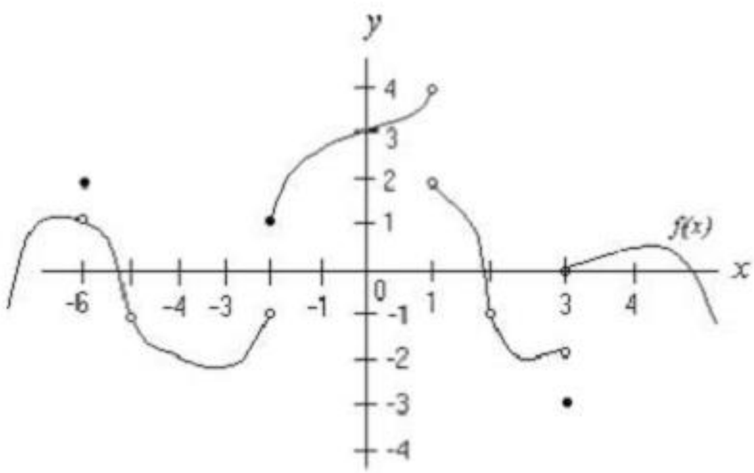
Find
4. f(-5)
5. f(-3)
6. f(3)



7. $\lim_{x \to 0} f(x)$

8. $\lim_{x \to -6} f(x)$

9. $\lim_{x \to 1} f(x)$



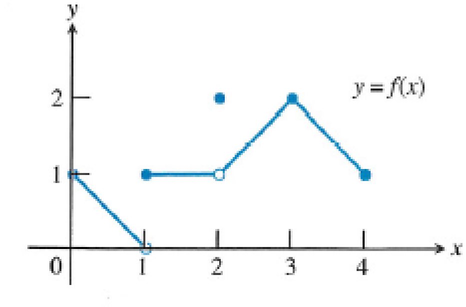
Using your graphing calculator- (use table, table set and value button to help with homework

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$$

Lesson 2

p. 68 #60 (no need to graph)

HW WS selected problems (4,5,7,8,9,10,11)



$$\lim_{x\to 2} f(x)$$

$$f(x) = \begin{cases} -x+1, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ x-1, & 2 < x \le 3 \\ -x+5, & 3 < x \le 4. \end{cases}$$

$$\lim_{x\to 3} f(x)$$
$$\lim_{x\to 1} f(x)$$
$$\lim_{x\to 2} f(x)$$

Use your calculator to find- then we must memorize

$$\underset{x\to 0}{\text{limit}} \frac{|x|}{X}$$

$$\lim_{x\to 0} \frac{1-cosx}{x}$$

$$\underset{x\to 0}{\text{limit}} \frac{\sin x}{x}$$

<u>using the above- find the limits</u>

$$\lim_{x\to 0} \frac{tanx}{x}$$

$$\lim_{x \to 0} \frac{sin4x}{x}$$

$$\lim_{x \to 0} \frac{1 - \cos 3x}{6x}$$

Lesson 3

Go over Homework

Limits Analytically /algebraically

HW- Worksheet finding limits analytically

HOW TO EVALUATE A LIMIT …

#1: Graphically: Graph the function and see where it goes

#2: Numerically: Make a table of values very close to the value you want to evaluate … (be sure to use numbers to the right AND to the left whenever possible)

#3: Substitution: Just plug in the value where you want to evaluate the limit.
If you get $\frac{0}{0}$ … DO SOMETHING ELSE …!

#4: Algebraically:
- Factor the numerator and denominator and cancel any like factors.
- Multiply by a factor of 1 (Includes rationalizing the numerator)
- Simplify the equation use algebraic properties and/or trigonometric identities.

Use Direct Substitution

put the limit (c) into the equation

If #/# it is the limit

If 0/0 it often has a limit, you must do some work-
Factor, conjugates…

If #/0 limit never exists DNE (because you can't divide by 0)

If 0/# limit is 0

Find limit without using a calculator

1. $\lim\limits_{x \to 1} \dfrac{x+1}{x-1}$

2. $\lim\limits_{x \to 2} \dfrac{x^2-3x+2}{x^2-4}$

3. $\lim\limits_{x \to 1} \dfrac{x-1}{x^2-1}$

4. $\lim\limits_{x \to 0} \dfrac{5x^3+8x^2}{3x^4-16x^2}$

5. $\lim\limits_{x \to -3} \dfrac{x^2+4x+3}{x^2-3}$

6. $\lim\limits_{x \to e} \quad ln1 + lnx$

7. $\lim\limits_{x \to \frac{\pi}{2}} \quad xsinx$

8. $\lim\limits_{x \to 9} \dfrac{\sqrt{x}-3}{x-9}$

9. $\lim\limits_{x \to 0} \dfrac{\frac{3}{4+x} - \frac{3}{4}}{x}$

THEOREM 1.2 PROPERTIES OF LIMITS

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Lesson 4

Limits as you go to infinity- answers should say DNE ±∞
CW –WS 3.5 #13,19-26,29,31, HW- book p. 76 -77 #27,29, 35-38 , 61-63

Only applies to limits x→ ±∞
-Highest degree in denominator getting closer to 0, limit =0

-Equal degree in numerator and denominator-limit is the leading coefficients

-Highest degree in numerator; limit DNE because it is going +∞, or -∞ (look at end behavior)

1. $\lim_{x\to\infty} x^3$

2. $\lim_{x\to-\infty} x^3$

3. $\lim_{x\to\infty} \frac{2x^2-4x}{x+1}$

4. $\lim_{x\to-\infty} \frac{2x^2-4x}{x+1}$

5. Limit $\frac{2x}{x^2+5}$
   x→∞

6. Limit $\frac{x^3+5x+2}{x-1}$
   x→∞

7. Limit $\frac{3x^3+2x^2}{5x^3+1}$
   x→∞

8. Limit $\frac{3x^3+2x^2}{5x^3+1}$
   x→-∞

9. Find $\lim_{x\to\infty} \frac{5x+\sin x}{x}$.

Use this guideline, these limits seem reasonable when you consider that for large values of x, the highest power term is the most influential in determining the limit.

When x goes to $+\infty$, then $x > 0$, which implies that $|x| = x$. Hence

$$\lim_{x \to +\infty} f(x) = \frac{\sqrt{4+0}}{3+0} = \frac{2}{3} .$$

When x goes to $-\infty$, then $x < 0$, which implies that $|x| = -x$. Hence

$$\lim_{x \to -\infty} f(x) = -\frac{\sqrt{4+0}}{3+0} = -\frac{2}{3} .$$

$$\lim_{x \to \infty} \left(\frac{3x - 2}{\sqrt{2x^2 + 1}} \right)$$

$$\lim_{x \to -\infty} \left(\frac{3x - 2}{\sqrt{2x^2 + 1}} \right)$$

EXAMPLE 2 Finding a Limit as x Approaches ∞

Find $\lim_{x \to \infty} f(x)$ for $f(x) = \frac{\sin x}{x}$.

EXAMPLE 3 Using Theorem 5

Find $\lim_{x \to \infty} \frac{5x + \sin x}{x}$.

SOLUTION

Notice that

$$\frac{5x + \sin x}{x} = \frac{5x}{x} + \frac{\sin x}{x} = 5 + \frac{\sin x}{x}.$$

So,

$$\lim_{x \to \infty} \frac{5x + \sin x}{x} = \lim_{x \to \infty} 5 + \lim_{x \to \infty} \frac{\sin x}{x} \quad \text{Sum Rule}$$

$$= 5 + 0 = 5. \quad \text{Known values}$$

Now Try Exercise 25.

Lesson 5

Go over HW- (p. 76#35 with calc and end behavior)
CW Leckies review sheet
HW- reverse classroom continuity

Find the limit

$$\lim_{x\to\infty} e^{x}$$

$$\lim_{x\to\infty} \frac{X+e^{-x}}{X}$$

Draw a graph which has the following attributes:

(a) $\lim_{x\to -2} f(x) = 3$

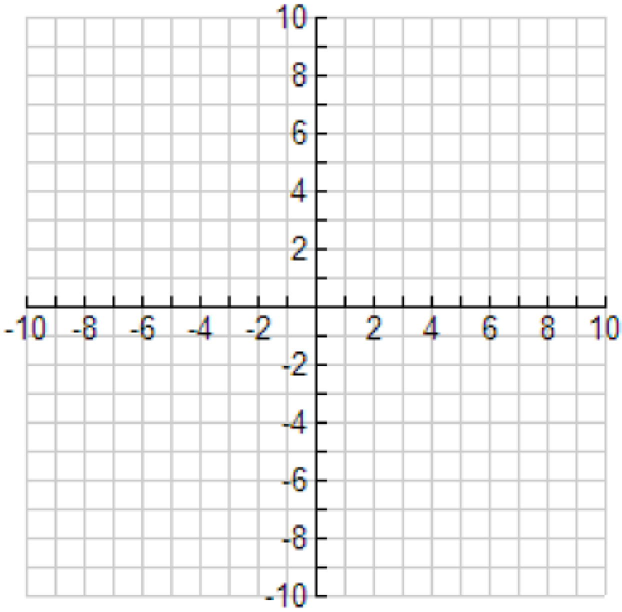
(b) $f(-2) = 1$

(c) $\lim_{x\to -4} f(x) = \infty$

(f) $\lim_{x\to 5^-} f(x) = 5$

(g) $\lim_{x\to 5^+} f(x) = 1$

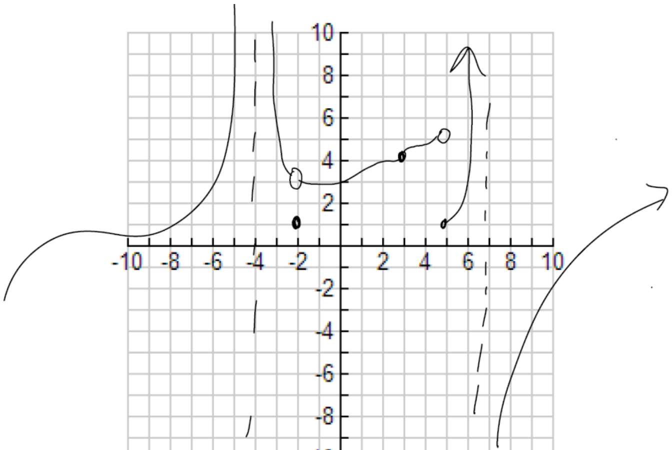
(h) $f(5) = 1$



$= \infty$

$: -\infty$

Rules for Horizontal Asymptotes:



1: The numerator and the denominator have the same degree

- HA = $\frac{\text{Leading Coefficient}}{\text{Leading Coefficient}}$
- Example: $f(x) = \frac{3x}{2x + 5}$

  HA = $y = \frac{3}{2}$

2: The degree of the numerator is less than the degree of the denominator

- HA = 0
- Example: $f(x) = \frac{3x^3 + 7x}{-2x^5 + 18x^4 - 3x}$ HA = y = 0

3: The degree of the numerator is greater the degree of the denominator

- HA = ∞ or $-\infty$
- Example: $f(x) = \frac{18x^3}{2x^2 + 7}$ HA = y = DNE
- $\lim_{x \to \infty} f(x) = \infty$
- $\lim_{x \to -\infty} f(x) = -\infty$

lesson 6

Continuous Functions (reverse classroom the night before)

p. 84 #1-5, 11-18, p. 85 #19-22, 47,48 p. 86# 56-59

<u>Definition of Continuity</u>: Functions are continuous at c if

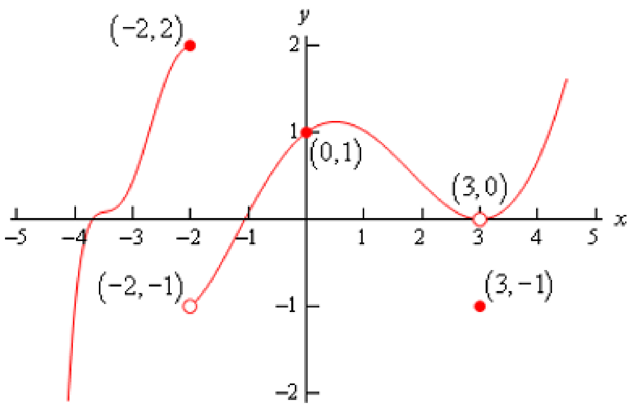
1 f(c) is defined

2 $\lim_{x \to c} f(x) \quad exists$

3 $\lim_{x \to c} f(x) = f(c)$

Example 1 - Given the graph of *f(x)*, shown below, determine if *f(x)* is continuous at $x = -2$ $x = 0$ & $x = 3$

if not give the reason why it's not



f(-2) f(0) f(3)

Find the x values at which the function is not continuous. Is it removable or non removable

1. $f(x) \ = \frac{1}{x}$

2. g(x)=$\frac{x^2-1}{x-1}$

can be rewritten as g(x)= $\frac{(x+1)(x-1)}{x-1}$

it is discontinuous at x=1 it is removable discontinuity

Lesson 7

Making functions continuous, IVT worksheet
CW/HW - WS 2.3 p. 85 #47-50

$$\lim_{x\to 1} \frac{x}{\ln x}$$

What value of B would make this function continuous?

$$f(x) = \begin{cases} 3x + B, & x \le 5 \\ x^2 - 1, & x > 5 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \ne 3 \\ b, & x = 3 \end{cases}$$

Let a and b represent real numbers. Define

$$f(x) = \begin{cases} ax^2 + x - b, & x \le 2 \\ ax + b, & 2 < x < 5. \\ 2ax - 7, & x \ge 5 \end{cases}$$

(a) Find the values of a and b such that f is continuous on the entire real number line.

(b) Evaluate $\lim_{x\to 3} f(x)$.

(c) Let $g(x) = \dfrac{f(x)}{x-1}$. Evaluate $\lim_{x\to 1} g(x)$.

An example of the intermediate value theorem

If you are 5 ft on your 13th birthday and on your 14th birthday you are 5'6" at some time between 13yrs and 14yrs you had to be 5'4" (or anything else between 5 ft and 5'6"

Intermediate Value Theorem-Is an existence theorem- it will not provide a solution. It just tells us of the existence of a solution

EX. f(x) =x^3-x-1. Use IVT to show that there is at least one zero (root) on the interval [1,2]

This is our answer:
Since f(x) is continuous on the interval [1,2] and f(1)= $^-1$ and f(2)=5, IVT guarantees there is a c between 1 and 2 where f(c)=0

Explain why the function has a zero in the specified interval

1. $f(x) = \frac{1}{16}x^4 - x^3 + 3$ interval [1,2]

2 . $f(x) = x^3 + 3x - 2$ interval [0,1]

Verify that IVT applies to the indicated interval and find the value of c guaranteed by the theorem

3. $f(x) = x^2 + x - 1$ $interval$ [0,5] $f(c) = 11$

4. $f(x) = x^2 - 6x + 8$ $interval$ [0,3] $f(c) = 0$

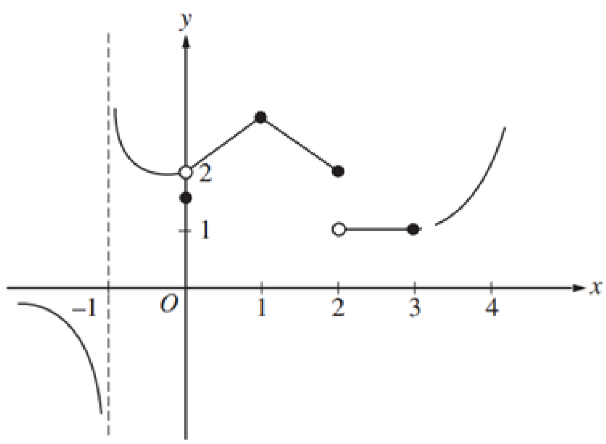
lesson 8

CW multiple choice problems

Go over hw- **AP style test on Tuesday**

1. **Use the Intermediate Value theorem to explain why the function $f(x) = x^2+2x-1$ must have at least one root on the interval [-1,1]**

2.



The graph of a function f is shown above. If $\lim_{x\to b} f(x)$ exists and f is not continuous at b, then $b =$

3. What is the value of k to make this a continuous function?

$$h(x) = \begin{cases} \dfrac{k}{x^2}, & x < -2 \\ 9 - x^2, & x \geq -2 \end{cases}$$

Lesson 9

Go over last night's homework
Cw- multiple choice
HW-review worksheet

1

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on [0, 2] and has values given in the table above. The equation $f(x) = \frac{1}{2}$ has at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1

(D) 2 (E) 3

2. Find the value of a and b that makes this function continuous

$$f(x) = \begin{cases} x^2 - 5x + 3, & x < -1 \\ ax + b, & -1 \le x \le 4 \\ 11 - 3x, & x > 4 \end{cases}$$

3

If f is continuous on $[-2, 4]$ and $f(-2) = 5$, $f(0) = -3$, *and* $f(4) = 711$, then according to the Intermediate Value Theorem, how many zeroes are guaranteed on the closed interval $[-2, 4]$

(A) none
(B) one
(C) two
(D) three
(E) four

Functions are continuous at c if

1 f(c) is defined

2 $\lim_{x \to c} f(x) \quad exists$

3 $\lim_{x \to c} f(x) \; = \; f(c)$

Lesson 10

Review problems p. 96-97 or from Mellina website additional review- mellinamathclass.com

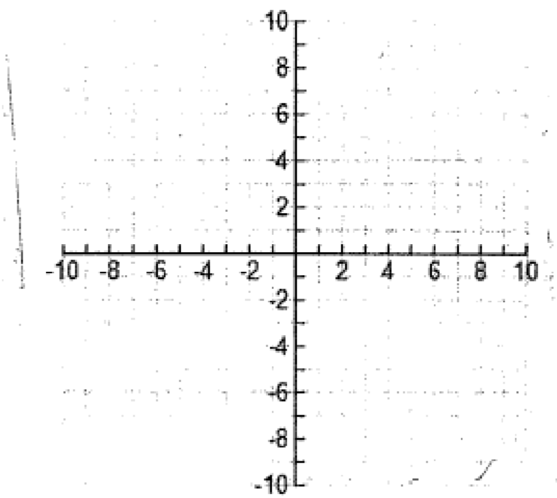
AP Style test tomorrow- MC and FRQ cw- problems below- and review of homework

FR1. On the axes provided below, sketch a graph of a function that has all of the following attributes listed below:

I. $\lim_{x \to 3} f(x) = 4$

II. $f(3) = -2$

III. $\lim_{x \to -3} f(x) = \infty$



FR2. $f(x)$ and $g(x)$ are continuous functions for all $x \in$ Reals. The table below has values for the functions for selected values of x. The function $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	-10
5	7	5
7	11	25

Explain why there must be a value c for $1 < c < 5$ such that $h(c) = 0$.

FR3. Find a such that the function $f(x) = \begin{cases} \dfrac{4\sin x}{x}, & x < 0 \\ a + 15x, & x \ge 0 \end{cases}$ is continuous for all real numbers.

point.

FR1. On the axes provided below, sketch a graph of a function that has all of the following attributes listed below:

I. $\lim_{x \to 3} f(x) = 4$ } REMOVABLE DISC.

II. $f(3) = -2$

III. $\lim_{x \to -3} f(x) = \infty$ VERTICAL ASYMPTOTE

FR2. $f(x)$ and $g(x)$ are continuous functions for all $x \in$ Reals. The table below has values for the functions for selected values of x. The function $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	-10
5	7	5
7	11	25

$h(1) = g(f(1)) + 2$
$= g(3) + 2$
$= -10 + 2$
$h(5) = g(f(5)) + 2$
$= g(7) + 2$
$= 25 + 2$

Explain why there must be a value c for $1 < c < 5$ such that $h(c) = 0$.

By IVT there is a c, $1 < c < 5$, such that $h(1) < h(c) < h(5)$ which means that there is a c, $1 < c < 5$, such that $-8 < h(c) < 27$. Hence, there is an $h(c) = 0$ for $1 < c < 5$

FR3. Find a such that the function $f(x) = \begin{cases} \frac{4\sin x}{x}, & x < 0 \\ a + 15x, & x \geq 0 \end{cases}$ is continuous for all real numbers.

need $f(0) = \lim_{x \to 0} f(x)$

$\lim_{x \to 0^-} f(x) = 4$
$\lim_{x \to 0^+} f(x) = a$

Hence, $a = 4$

How do you find the limits- analytically, graphically, and table

Direct Substitution
put the limit (c) into the equation

If #/# it is the limit

If 0/0 it often has a limit, you must do some work- Factor, conjugates…

If #/0 limit never exists DNE
If 0/# limit is 0

Vertical asymptotes occur where the denominator=0
ie- set the denominator = 0, solve for x

Horizontal asymptotes occur-
Highest degree in denominator getting closer to 0, horizontal asymptote at y=0

Equal degree in numerator and denominator-horizontal asymptote is the leading coefficients

Highest degree in numerator-no horizontal asymptote

Whenever there is a vertical asymptote you have a limit of $+\infty$ or $-\infty$

Test CH 1