

In my calc class, I can't forget
 The stuff we learned before
 In my calc class, I can't forget
 And then I'll learn some more
 Derivatives and integrals
 Limits won't go away
 We'll learn a lot of theorems yet
 In my calc class today

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LESSON 1

the night before assign midnight tutor – soup to nuts Riemann sums <http://www.youtube.com/watch?v=YHYT3HIL1uA>
<http://mathworld.wolfram.com/RiemannSum.html>

RRAM, LRAM and MRAM- use Leckies worksheet-

Hw p. 274-275 #4, 5, 6, 16,

Example 1: Suppose from the 2nd to 4th hour of your road trip, you travel with the cruise control set to exactly 70 miles per hour for that two hour stretch.
How far have you traveled during this time?

Example 2: Sketch a graph modeling the situation in the above example.
Geometrically, how can we indicate the total distance traveled?

The goal is to find the area under the curve- (function)
We will use 4 methods to approximate this area

Left rectangular approximation method LRAM

Right rectangular approximation method RRAM

Midpoint rectangular approximation method MRAM

Trapezoidal approximation method

Use wolfram website

show $y=x^2 + 2$ show left, right with 5 partitions going from 0 to 5
you can type in any equation and get all the Riemann estimates

show with $y=x^2 + 3$ visually and graphically then move on to -

FROM a Chart

Riemann sum, left and right 6 sub intervals

Riemann sum, midpoint- 3 sub intervals

Time (hr)	0	2	4	6	8	10	12
Velocity(mph)	20	30	38	44	48	50	51

Use a LRAM with 6 intervals to Estimate the distance traveled by car

Graph of velocity v time- distance is area under curve - label boxes

Left end point $2 \cdot 20$ $2 \cdot 30$ $2 \cdot 38$ $2 \cdot 44$ $2 \cdot 48$ $2 \cdot 50$

right end pt $2 \cdot 30$ $2 \cdot 38$ $2 \cdot 44$ $2 \cdot 48$ $2 \cdot 50$ $2 \cdot 51$

midpoint $4 \cdot 30$ $4 \cdot 44$ $4 \cdot 50$

When given a table we can only use what is in the table- midpoint make interval

show picture

<http://math.fullerton.edu/mathews/a2001/Animations/Quadrature/Quadrature.html>

LESSON 2

Trapezoidal method

Must show them the integral sign so they can do the homework-
show how to get exact answer with calculator.

HW- p. 316 #3,5,7,

CW- 2003 AP problem parts A,B C, 2006,2005 a,b,c

<http://www.slu.edu/classes/maymk/Riemann/Riemann.html>

Warm-UP

(use a table from your calculator to help move you along)

Estimate the area under the curve $f(x) = x^2$ on the interval $[0,2]$ use 4 subintervals-

Use -LRAM – is this an over or under estimate- justify

Use 2 subintervals for MRAM

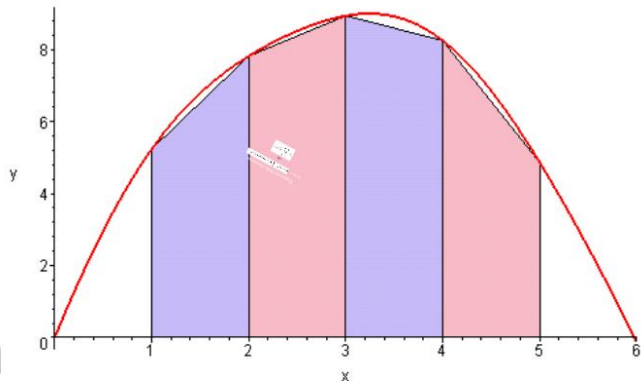
X	0	.5	1	1.5	2
F(x)	0	.25	1	2.25	4

USE CALC $\int_0^2 x^2 dx$

Left	$.5(0)+.5(.25)+.5(1)+.5(2.25)=1.75$
Right	$.5(.25)+.5(1)+.5(2.25)+.5(4)=3.75$
Midpoint	$1(.25) +1(2.25) = 2.5$

Then show trapezoidal method-(sideways trap)
area of trap is $A=\frac{1}{2} h(b_1 + b_2)$

Trapezoidal $\frac{1}{2} (.5) (0+.25) + \frac{1}{2} (.5) (.25+1) + \frac{1}{2} (.5) (1+2.25) + \frac{1}{2} (.5)(2.25+4)=2.75$



trapezoidal area of trapezoid $\frac{1}{2} h (b_1 + b_2)$

4.

x	-5	-3	0	1	5
$f(x)$	10	7	5	8	11

Given the values for $f(x)$ on the table above, approximate the area under the graph of $f(x)$ from $x = -5$ to $x = 5$ using four subintervals and a **Trapezoidal approximation**.

4.

x	-5	-3	0	1	5
$f(x)$	10	7	5	8	11

$$\int_{-5}^5 f(x) dx \approx \frac{1}{2} \cdot 2(10+7) + \frac{1}{2} \cdot 3(7+5) + \frac{1}{2} \cdot 1(5+8) + \frac{1}{2} \cdot 4(8+11) = 79.5$$

Why does the left ep < right ep because slope is pos
 Left ep > right ep neg slope

When given a table we can only use what is in the table- midpoint make interval

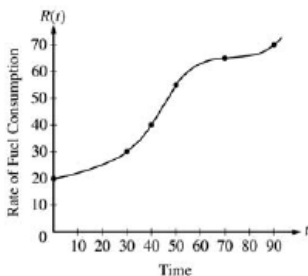
When slope is positive- and you are doing a right Riemann sum- are you overestimating or underestimating?

When slope is negative- and you are doing a right Riemann sum, are you over or under estimating?

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2003 SCORING GUIDELINES

Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- (c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- (d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a)
$$R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10}$$

$$= 1.5 \text{ gal/min}^2$$

- 1 : a difference quotient using numbers from table and interval that contains 45
2 :
1 : 1.5 gal/min²

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

- 1 : $R''(45) = 0$
2 :
1 : reason

(c)
$$\int_0^{90} R(t) dt \approx (30)(20) + (10)(30) + (10)(40)$$

$$+ (20)(55) + (20)(65) = 3700$$

- 1 : value of left Riemann sum
2 :
1 : "less" with reason

Yes, this approximation is less because the graph of R is increasing on the interval.

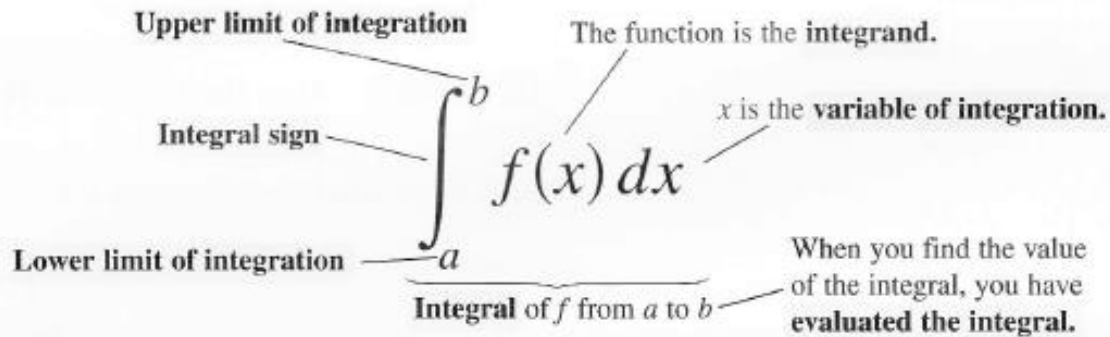
- (d) $\int_0^b R(t) dt$ is the total amount of fuel in gallons consumed for the first b minutes.
 $\frac{1}{b} \int_0^b R(t) dt$ is the average value of the rate of fuel consumption in gallons/min during the

- 2 : meanings
1 : meaning of $\int_0^b R(t) dt$
3 :
1 : meaning of $\frac{1}{b} \int_0^b R(t) dt$
< - 1 > if no reference to time b

LESSON 3-

Cw- complete AP problems from packet-

Hw- reverse classroom mr leckie- follow along with worksheet

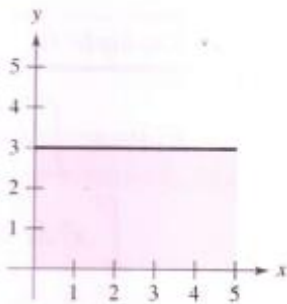


The value of the definite integral of a function over any particular interval depends on the function and not on the letter we choose to represent its independent variable. If we decide to use t or u instead of x , we simply write the integral as

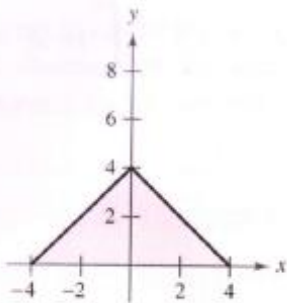
$$\int_a^b f(t) dt \quad \text{or} \quad \int_a^b f(u) du \quad \text{instead of} \quad \int_a^b f(x) dx.$$

Set up the definite integral that yields the area of the region-then use your calculator to get exact area

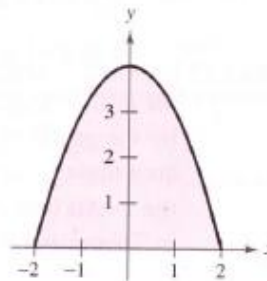
13. $f(x) = 3$



15. $f(x) = 4 - |x|$



17. $f(x) = 4 - x^2$



The rate at which water flows into a tank, in gallons per hour, is given by a positive continuous function R of time t . The table below shows the rate at selected values of t for a 12-hour period.

t (hrs)	0	2	4	6	8	10	12
$R(t)$ (gal/hr)	12.5	13.4	13.9	14.3	14.6	14.8	14.7

1. Use a midpoint Riemann sum with three subintervals to approximate:

$$\int_0^{12} R(t) dt,$$

Particle A moves along a horizontal line with a velocity $v_A(t)$, where $v_A(t)$ is a positive continuous function of t . The time t is measured in seconds, and the velocity is measured in cm/sec. The velocity $v_A(t)$ of the particle at selected times is given in the table below.

t (sec)	0	2	5	7	10
$v_A(t)$ (cm/sec)	1.7	6.8	7.4	15.6	24.9

1. Use data from the table to approximate the distance traveled by particle A over the interval $0 \leq t \leq 10$ seconds by using a right Riemann sum with four subintervals. Show the computations that lead to your answer, and indicate units of measure.
-

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x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.
- Estimate $f'(4)$. Show the work that leads to your answer.
 - Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
 - Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
 - Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.
-

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t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

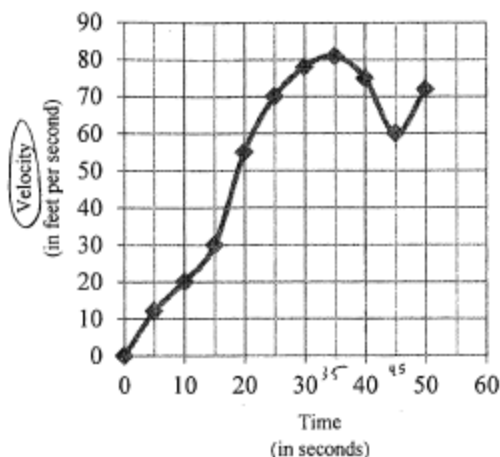
2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$.
- (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)
- (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

- | | |
|---|---|
| <p>(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.</p> <p>(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$
$\left.\frac{dV}{dt}\right _{t=5} = 4\pi(30)^2 2 = 7200\pi$ ft³/min</p> <p>(c) $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
$= 19.3$ ft
$\int_0^{12} r'(t) dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.</p> <p>(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t) dt$.</p> <p>Units of ft³/min in part (b) and ft in part (c)</p> | <p>2: $\begin{cases} 1: \text{estimate} \\ 1: \text{conclusion with reason} \end{cases}$</p> <p>3: $\begin{cases} 2: \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$</p> <p>2: $\begin{cases} 1: \text{approximation} \\ 1: \text{explanation} \end{cases}$</p> <p>1: conclusion with reason</p> <p>1: units in (b) and (c)</p> |
|---|---|

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.

- a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

$$a(t) = v'(t) \quad \text{so when } v(t) \text{ is increasing, } a(t) > 0$$

$$a(t) > 0 \quad \text{on } (0, 35) \cup (45, 50)$$

- b) Find the average acceleration of the car, in ft/sec^2 , over the interval $0 \leq t \leq 50$.

average (rate of change in velocity)

$$\frac{v(50) - v(0)}{50 - 0} = \frac{72 - 0}{50} = 14.4 \text{ ft/sec}^2$$

- c) Approximate $\int_0^{50} v(t) dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

$$\int_0^{50} v(t) dt \approx 10(12) + 10(30) + 10(70) + 10(81) + 10(60)$$

$$= 120 + 300 + 700 + 810 + 600$$

$$= 2530$$

This car has traveled 2530 feet from time = 0 to time = 50 seconds.

Lesson 4

Integral as an area –geometrically

Rules of definite integrals

CW from packet Pages 3,4

HW- p. 294 (1-3)

Section 6.3

Area and Definite Integrals

THEOREM 1 The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

DEFINITION Area Under a Curve (as a Definite Integral)

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ from a to b** is the integral of f from a to b ,

$$A = \int_a^b f(x) dx.$$

$$\int_a^b f(x) dx = -(\text{the area}) \quad \text{if} \quad f(x) \leq 0.$$

$$\int_a^b f(x) dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

Examples: Evaluate each definite integral –(use your knowledge of area)

1. $\int_{-2}^1 2x dx$

2. $\int_0^9 (-|x| + 3) dx$

3. $\int_{-1}^4 4 dx$

Rules for Definite Integrals

1. Order of Integration: $\int_a^b f(x) dx = -\int_b^a f(x) dx$

If you reverse the *order* of integration you get the opposite answer.

2. Zero: $\int_a^a f(x) dx = 0$

This should make sense if you think about the "area" of a rectangle with no width.

3. Constant Multiple: If k is any constant, then $\int_a^b k \cdot f(x) dx = k \cdot \int_a^b f(x) dx$

Taking the constant out of the integral many times makes it simpler to integrate.

4. Sum and Difference: $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

This allows you to integrate functions that are added or subtracted separately. Notice, there are NO rules here for two functions that are multiplied or divided ... that comes later!

5. Additivity: $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

Pay close attention to the limits of integration ... this comes in handy when dealing with total area or other functions where we need to break them into smaller parts.

Example 1: Given $\int_2^6 f(x) dx = 10$ and $\int_2^6 g(x) dx = -2$, find the following:

a) $\int_2^6 [f(x) + g(x)] dx$

b) $\int_2^6 [g(x) - f(x)] dx$

c) $\int_2^6 3f(x) dx$

d) $\int_2^6 (f(x) + 2) dx$

LESSON #5

Review for 30 pt test-

Cw- basic integration WS (1,3)

HW- p. 294 (4-6)

EXAMPLE 1 Using the Rules for Definite Integrals

Suppose

$$\int_{-1}^1 f(x) dx = 5, \quad \int_1^4 f(x) dx = -2, \quad \text{and} \quad \int_{-1}^1 h(x) dx = 7.$$

Find each of the following integrals, if possible.

(a) $\int_4^1 f(x) dx$

(b) $\int_{-1}^4 f(x) dx$

(c) $\int_{-1}^1 [2f(x) + 3h(x)] dx$

(d) $\int_0^1 f(x) dx$

(e) $\int_{-2}^2 h(x) dx$

(f) $\int_{-1}^4 [f(x) + h(x)] dx$

Lesson 6-

review for test-CW - book p. 277 #35, p. 318 31,34,36

test is calculator RRAM-LRAM-MRAM -Trapezoid and no calc-integrals using areas

t (min)	1	2	3	4	5	6	7
$V(t)$ (ft/min)	20	30	35	40	50	45	35

1. Estimate the total distance travelled- using data from the table

a left Riemann sum with 6 sub intervals

a midpoint Riemann with 3 sub intervals

Explain the meaning of your answer

2. Estimate $\int_1^3 (x \sin x) dx$ by using the trapezoidal method with 4 equal subintervals

(Use your table feature of the calculator)

Then use your graphing calculator to evaluate the integral- accurate to 3 decimal places

3.

[No Calculator] Suppose $\int_1^2 f(x) dx = 3$, $\int_1^5 f(x) dx = -13$, and $\int_1^5 g(x) dx = 7$. Find each of the following:

a) $\int_3^5 g(x) dx$

b) $\int_5^1 f(x) dx$

c) $\int_1^5 [g(x) - f(x)] dx$

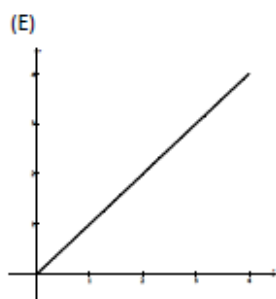
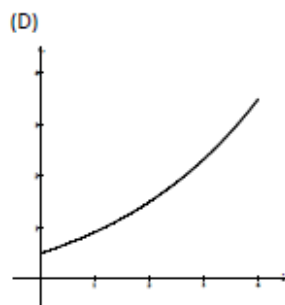
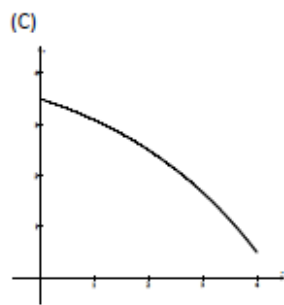
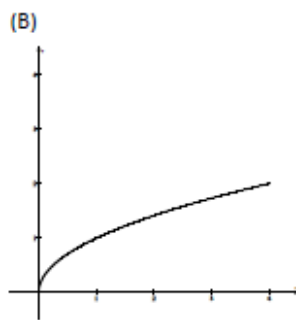
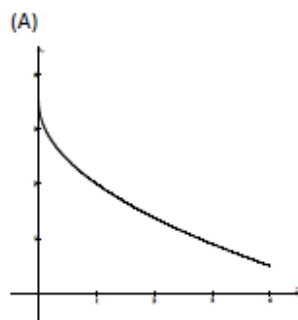
d) $\int_2^5 f(x) dx$

e) $\int_1^5 [3f(x) - g(x)] dx$

f) $\int_1^5 \frac{g(x)}{4} dx$

Part B. Graphing Calculator Allowed.

4. If a trapezoidal sum overapproximates $\int_0^4 f(x) dx$, and a right Riemann sum underapproximates $\int_0^4 f(x) dx$, which of the following could be the graph of $y = f(x)$?



Lesson 7

HW worksheet p.249 #15-39

$\int f(x) dx$ is called an indefinite integral.

In general, $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$

$$\int x^7 dx = \frac{x^8}{8} + C \quad \text{or} \quad \frac{1}{8} x^8 + C$$

$$\frac{d}{dx} \left[\frac{1}{8} x^8 + C \right] = x^7$$

$$\int x^{-3} dx = \frac{x^{-2}}{-2} + C \quad \text{or} \quad -\frac{1}{2} x^{-2} + C$$

original

rewrite

integrate

simplify

1. $\int \frac{1}{x^3} dx$

2. $\int \sqrt{x} dx$

3. $\int 2 \sin x dx$

4. $\int dx$

5. $\int (x+2) dx$

6. $\int (3x^4 - 5x^2 + x) dx$

7. $\int \frac{x+1}{\sqrt{x}} dx$

8. $\int x(x^3 + x) dx$

9. A particle moves along the x -axis so that its acceleration at any time t is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$ its position is $x(1) = 20$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time t .

(b) For what values of t is the particle at rest?

(c) Write an expression for the position $x(t)$ of the particle at any time t .

15. $\int (x + 3) dx$

17. $\int (2x - 3x^2) dx$

19. $\int (x^3 + 2) dx$

21. $\int (x^{3/2} + 2x + 1) dx$

23. $\int \sqrt[3]{x^2} dx$

25. $\int \frac{1}{x^3} dx$

27. $\int \frac{x^2 + x + 1}{\sqrt{x}} dx$

29. $\int (x + 1)(3x - 2) dx$

31. $\int y^2 \sqrt{y} dy$

33. $\int dx$

16. $\int (5 - x) dx$

18. $\int (4x^3 + 6x^2 - 1) dx$

20. $\int (x^3 - 4x + 2) dx$

22. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}} \right) dx$

24. $\int (\sqrt[4]{x^3} + 1) dx$

26. $\int \frac{1}{x^4} dx$

28. $\int \frac{x^2 + 2x - 3}{x^4} dx$

30. $\int (2t^2 - 1)^2 dt$

32. $\int (1 + 3t)t^2 dt$

34. $\int 3 dt$

In Exercises 35–42, find the indefinite integral and check the result by differentiation.

35. $\int (2 \sin x + 3 \cos x) dx$

37. $\int (1 - \csc t \cot t) dt$

39. $\int (\sec^2 \theta - \sin \theta) d\theta$

36. $\int (t^2 - \sin t) dt$

38. $\int (\theta^2 + \sec^2 \theta) d\theta$

40. $\int \sec y (\tan y - \sec y) dy$

Lesson 8 solving for c and FTC

Definite integral-HW- p.295 # 19-26 p. 307 #27-39 odd,

The Fundamental Theorem of Calculus (The Evaluation Part)

We no longer need to evaluate integrals graphically!

If f is continuous at every point of $[a, b]$,

$$\int_a^b f(x) dx =$$

Where $F(x)$ is an _____ of $f(x)$.

Now an integral is the difference of antiderivative of $f(x)$ evaluated at upper bound and then lower bound...

If F is continuous on $[a, b]$

$$\int_a^b f(x) dx = F(b) - F(a)$$

EVALUATE WITHOUT A CALCULATOR- then confirm with your calculator

$$\int_0^2 x^2 dx$$

$$\int_{-1}^2 (3x^2 + 2) dx$$

$$\int_1^e \left(\frac{1}{x} + 2\right) dx$$

$$\int_1^4 3\sqrt{x} dx$$

$$\int_0^1 e^x dx$$

$$\int_0^{\pi/4} \sec^2 x dx$$

Multiple Choice

$$\int_{\frac{1}{4}}^2 \left(\frac{1}{x} + 4\right) dx$$

- a. $\ln \frac{7}{4} + 7$ b. $\ln 8 + 7$ c. $\ln \frac{7}{4} + 9$ d. $\ln 8 + 9$

Lesson 9

Rectilinear -distance and displacement

CW /HW complete - PVA worksheets-

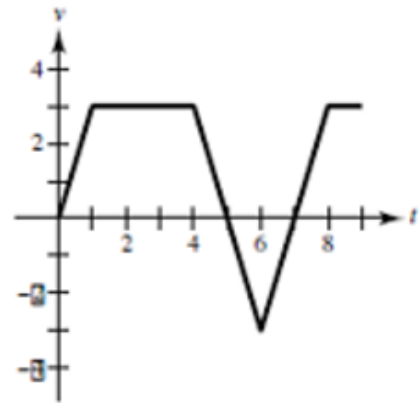
1

$$\int \frac{15x^9 + 4x^2 + 16}{x^5} dx$$

2. A woman is walking a straight path beginning at point P. A graph of the walker's velocity is given below in miles per hour.

- What is the total distance traveled by the woman after 9 hours of walking?
- After nine hours, how far away is she from her starting point P

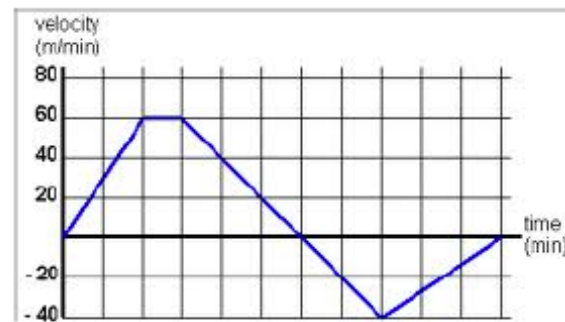
Write an integral expression that describes each (a) and (b)



3. Kevin got on the NJ Turnpike at the 3 mile marker traveling North on cruise control from time 0 to time 2 hours at 70 MPH. What mile marker was he at after the 2 hours? **Set this up as an integral expression.**

4. Kevin left school at time 0, got on the tpke at the 3 mile marker traveling north for $\frac{1}{2}$ an hour at 70 MPH when he realized he forgot his cell phone. He went back to school got his phone and returned to heading home at 70 mph for 2 hours. What was his total distance traveled? **Set this up as an integral expression.**

1. The velocity of a particle is given on the right.
The position of the particle at $t=0$ is 3.



- What is the total distance traveled from $t=0$ to $t=6$?
- What is the total distance traveled from $t=0$ to $t=8$?
- What is the total distance traveled from $t=0$ to $t=11$?
- What is the displacement of the particle from $t=0$ to $t=6$?
- What is the displacement of the particle from $t=0$ to $t=8$?
- What is the displacement of the particle from $t=0$ to $t=11$?
- Where is the particle at $t=6$? At $t=11$?

2. Given $v(t) = 4 - t^2$ and $s(0) = -4$ units

- Find the total distance the particle traveled from $t=0$ to 3 seconds.
- Find the displacement of the particle from $t=0$ to 3 seconds.
- Where is the particle at $t=3$ sec?

NOTES:

Displacement of the particle on $[a,b] = \int_a^b v(t)dt$

Total distance traveled of the particle on $[a,b] = \int_a^b |v(t)|dt$

Position of the particle at $t = a$: initial position + displacement = $s(0) + \int_0^a v(t)dt$

If given a position other than at $t = 0$ just set up a definite integral of $v(t)$ with the position you are given and the one you are looking for and use FTC and algebra to solve for the position you are looking for.

Ex. If given $s(3) = 8$ and you want to find $s(10)$:

$$\int_3^{10} v(t)dt = s(10) - s(3) \quad \text{so then} \quad s(10) = s(3) + \int_3^{10} v(t)dt = 8 + \int_3^{10} v(t)dt$$

If instead you want to find $s(1)$ still knowing that $s(3) = 8$:

$$\int_1^3 v(t)dt = s(3) - s(1) \quad \text{so then} \quad s(1) = s(3) - \int_1^3 v(t)dt = 8 - \int_1^3 v(t)dt$$

Calculator Question:

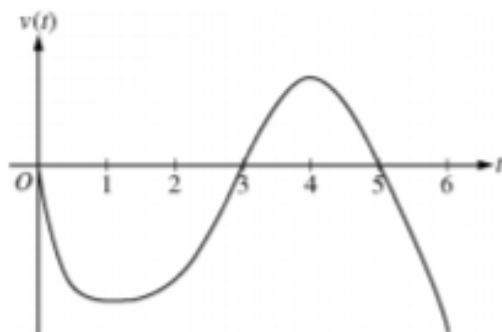
A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

- When is the particle at rest?
- For what values of t is the particle moving to the right?
- Find the acceleration of the particle at time $t = 5$.
- When is the particle's acceleration zero?
- Find the total distance traveled by the particle from $t = 1$ to $t = 8$.
- Find the displacement of the particle from $t = 1$ to $t = 8$.
- Find the position of the particle at $t = 5$.
- Find the position of the particle at $t = 0.3$.
- For what values of t is the particle slowing down? Speeding up?

Lesson 10

Cw/hw FRQ's below and p. 307 #30-40 even and p. 295 #29,30

2008 #4- no calc



Graph of v

A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

- For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

2016 #2 calculator

For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = 1 + 2 \sin\left(\frac{t^2}{2}\right). \text{ The particle is at position } x = 2 \text{ at time } t = 4.$$

- At time $t = 4$, is the particle speeding up or slowing down?
- Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.
- Find the position of the particle at time $t = 0$.
- Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

2012 #6 No calc

For $0 \leq t \leq 12$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = \cos\left(\frac{\pi}{6}t\right). \text{ The particle is at position } x = -2 \text{ at time } t = 0.$$

- For $0 \leq t \leq 12$, when is the particle moving to the left?
- Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.
- Find the acceleration of the particle at time t . Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
- Find the position of the particle at time $t = 4$.

Lesson #11

Fundamental Theorem of Calculus- Power point worksheet

p. 306 1-19 odd p. 307 45-46 set up the integrals and evaluate-using FTC

<http://patrickjmt.com/fundamental-theorem-of-calculus-part-1/>

If F is continuous on $[a, b]$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Second Fundamental Theorem:

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

1. Derivative of an integral.
2. Derivative matches upper limit of integration.
3. Lower limit of integration is a constant.

examples to open up with 1-3 then change

add d/dx of each-next change the limits of integration to something other than an x

$$\int_{\pi}^x \cos t \, dt$$

$$\int_0^x 2t - 5 \, dt$$

$$\int_3^x t^5 \, dt$$

$$\int_3^{x^2} t^5 \, dt$$

Practice with short-cut- take the derivative of each

$$\int_1^x \sqrt[3]{t} \, dt$$

$$\int_x^4 2t \, dt$$

$$\int_5^{x^3} \cos t \, dt$$

$$\int_1^{\cos x} 2t \, dt$$

$$\int_x^{x^2} \frac{t^3}{t^4 + 1} dt$$

Lesson 12

Review p. 320-321 #9,13,14, 15-23 odd 26, 27, 31, 39-42, 45(with calc)

Find $F'(x)$

$$F(x) = \int_2^{3x} \cos(t^3) dt$$

Lesson 13

The FTC Graphically Cw- p. 307 #57, FRQ 2005 (packet page 1)
HW- from packet p. 5 #5,6,7 p. 6 #39,40,41, #10 packet p.1 bottom problem

1. Find $f'(x)$

$$f(x) = \int_2^{3x} \cos(t^2) dt$$

2. $f(x) = 3x^3 - 9x^2 + 1$ on the interval $[-1,4]$

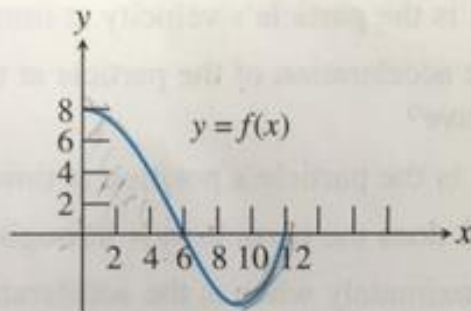
Where is $f(x)$ decreasing? Justify

On what interval is $f(x)$ concave up? Justify

What is the x - value of the inflection point? Justify

$$H(x) = \int_0^x f(t) dt,$$

where f is the continuous function with domain $[0, 12]$ graphed here.

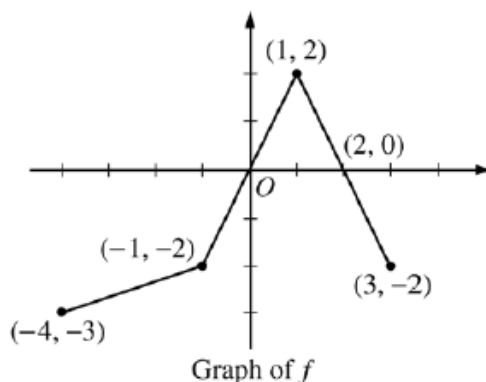


- Find $H(0)$.
- On what interval is H increasing? Explain.
- On what interval is the graph of H concave up? Explain.
- Is $H(12)$ positive or negative? Explain.
- Where does H achieve its maximum value? Explain.
- Where does H achieve its minimum value? Explain.

AP[®] CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 4

The graph of the function f above consists of three line segments.



(a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

(a) $g(-1) = \int_{-4}^{-1} f(t) dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$
 $g'(-1) = f(-1) = -2$
 $g''(-1)$ does not exist because f is not differentiable at $x = -1$.

3 : $\begin{cases} 1 : g(-1) \\ 1 : g'(-1) \\ 1 : g''(-1) \end{cases}$

(b) $x = 1$
 $g' = f$ changes from increasing to decreasing at $x = 1$.

2 : $\begin{cases} 1 : x = 1 \text{ (only)} \\ 1 : \text{reason} \end{cases}$

(c) $x = -1, 1, 3$

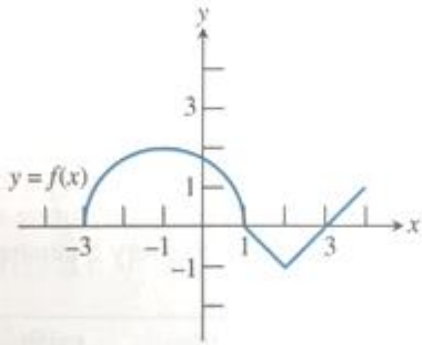
2 : correct values
 <-1> each missing or extra value

(d) h is decreasing on $[0, 2]$
 $h' = -f < 0$ when $f > 0$

2 : $\begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

Solution to last night's homework-

54. The graph of a function f consists of a semicircle and two line segments as shown below. p. 322 #54



Let $g(x) = \int_1^x f(t) dt$.

- (a) Find $g(1)$. 0
- (b) Find $g(3)$. -1
- (c) Find $g(-1)$. $-\pi$
- (d) Find all values of x on the open interval $(-3, 4)$ at which g has a relative maximum. $x = 1$
- (e) Write an equation for the line tangent to the graph of g at $x = -1$. $y = 2x + 2 - \pi$

#5 C $(-1+0+2)$

#6 B

#39 E

#40 A

#41 D

#10 D

f. $x = -1, 2$

g. $[-2\pi, 0]$

http://www.chaoticgolf.com/vodcasts/calc/lesson5_4day2/lesson5_4day2.html

9 minutes on

Lesson 15

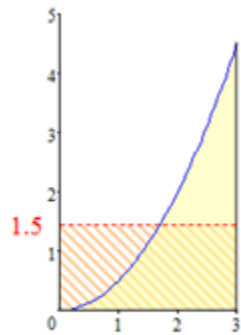
Go over HW questions

Average value- explain the difference between average rate of change and average value-

Open with -p. 323 # 58 use part c to segue into average value)

Hw p. 295 #31-36 skip 34- complete worksheet

The average value of a function is the value that would give the same area if the function was a constant:



$$y = \frac{1}{2}x^2$$

$$A = \int_0^3 \frac{1}{2}x^2 dx$$

$$= \frac{1}{6}x^3 \Big|_0^3 = \frac{27}{6} = \frac{9}{2} = 4.5$$

$$\text{Average Value} = \frac{4.5}{3} = 1.5$$

$$\text{Average Value} = \frac{\text{Area}}{\text{Width}} = \frac{1}{b-a} \int_a^b f(x) dx$$

→

1 Find the average value of $y = \sin x$ on the interval $[0, \pi]$

2 Let $f(x) = 1 - x^2$ on $[-2, 2]$. Find the average value.

3

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (ft/second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above. Find the average acceleration of Rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.

Lesson #16- Test Tuesday

Cw. page 3,4 of packet. My big FTC problem-
HW/ Mr leckie review #1-4 (on ch 6 homework page)

1 $A(t) = 12t - 18$, at $t = 1$ the velocity is $v(1) = 0$

a. Find the velocity expression

b. Set up, but do not evaluate the integral expression that will find the displacement of the particle on the interval $[0,3]$

c. Set up, but do not evaluate the integral expression that will find the total distance travelled by the particle on the interval $[0,3]$

d. Set up, but do not evaluate the integral expression that will give you the average velocity of the particle. $[0,3]$

2016 FRQ

2 The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by

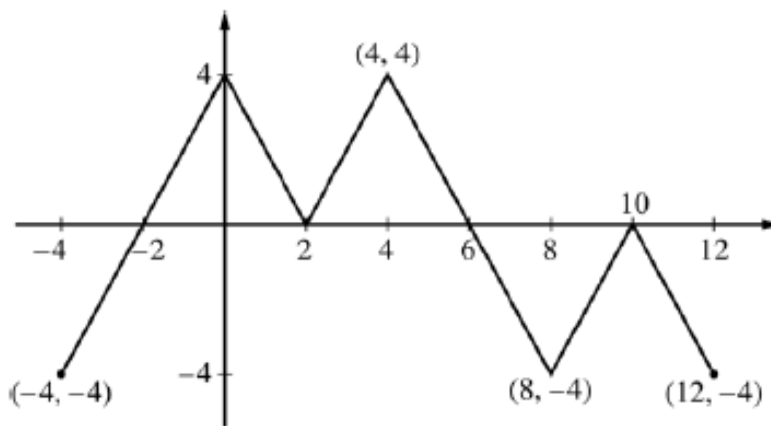
$$g(x) = \int_2^x f(t) dt.$$

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.

(d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.



Graph of f

(a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \leq 0$ for $8 \leq x \leq 12$.

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$.

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

(d) $g(x) \leq 0$ for $-4 \leq x \leq 2$ and $10 \leq x \leq 12$.

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

1 : answer with justification

1 : answer with justification

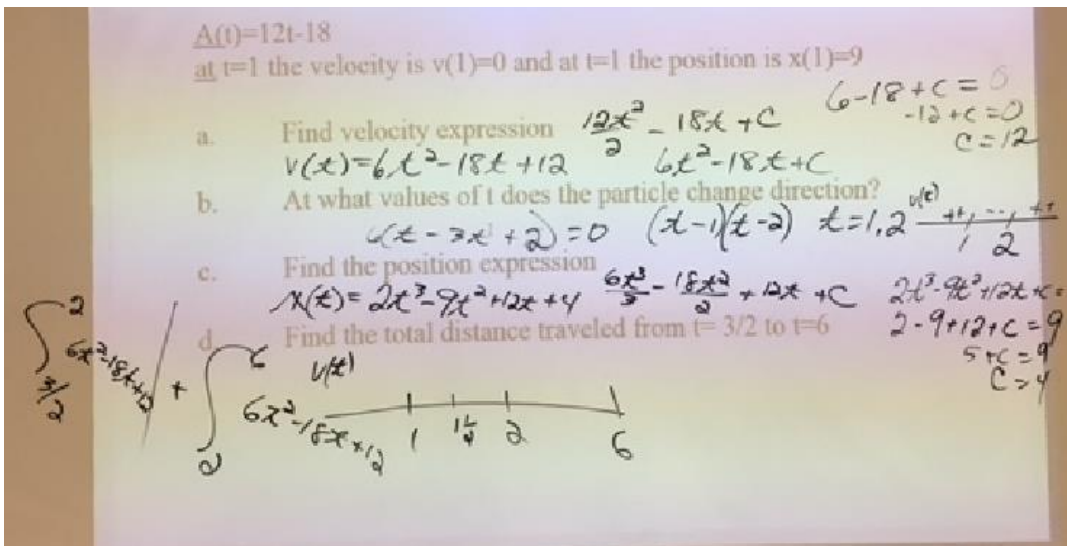
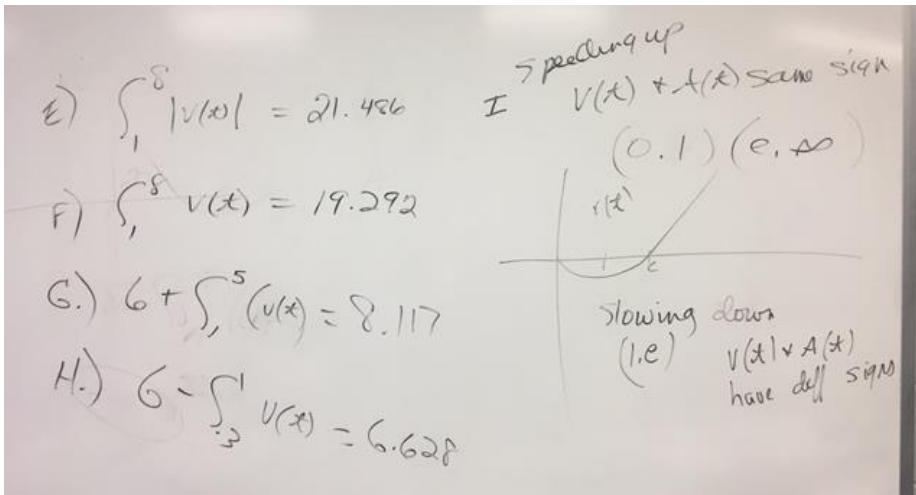
4 : $\left\{ \begin{array}{l} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{array} \right.$

2 : intervals

Lesson #17

Review –leckie’s review #1b, 2c, all parts of 3,4,8,9 a,b,,10,14,

2. Evaluate $\int_0^{\pi} (1 + \sin x) dx$



Chapter TESt 2-7-12 (2-22-13) (2-27-14) 2/1/16, 1-27-17