Contents

Graphs-Optimization-Rectilinear
Approx 2 days on each , + 2 day review and test total (9 days)-
Test –before xmas break

2017 changed order graph, PVA then optimization

Sent home theme 4 worksheets (in folder) night before –

Lesson 1

CW- Group students to do smart notebook question 1-10

go over theme 4 problems p.14,15

HW- 2006 and 1991 problems and 1-2 on WS#93

Calculus: Sketch functions using derivative graphs

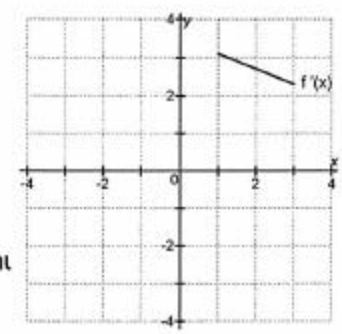
Grade: 9–12

Subject: Mathematics

Date: «date»

title

1 Which statement is true about the graph of ***f*(*x*)** for the piece shown?



A *f*(*x*) is increasing

B *f*(*x*) is decreasing

C *f*(*x*) is concave up

D *f*(*x*) has a local minimu
at ***x* = 2**

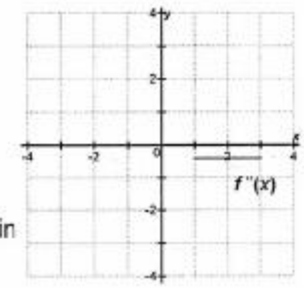
2 Which statement is true about the graph of *f*(*x*) for the piece shown?

A **$f(x)$** is concave up

B **$f(x)$** is concave down

C **$f(x)$** is increasing

D **$f(x)$** has an inflection poin at **$x = 2$**



Q2

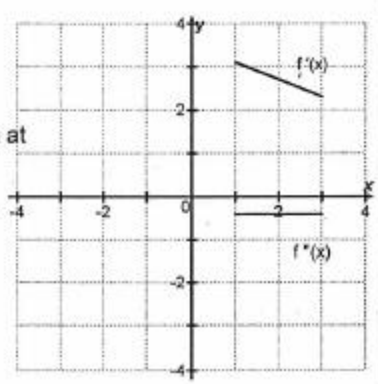
3 Which statement is false about the graph of *f*(*x*) over **[1, 3]** ?

A **$f(x)$** is increasing

B **$f(x)$** is concave down

C **$f(x)$** has a local maximum at **$x = 3$**

D **$f(x)$** has no inflection point in the interval



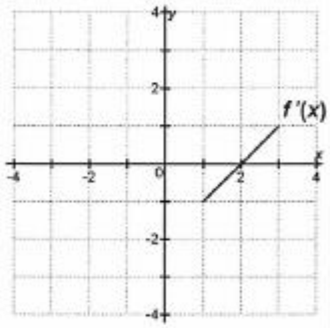
4 Which statement is true about the graph of $f(x)$ for the piece shown?

A $f(x)$ is concave down

B $f(x)$ is decreasing

C $f(x)$ has a local maximum at $x = 2$

D $f(x)$ has a local minimum at $x = 2$



Q4

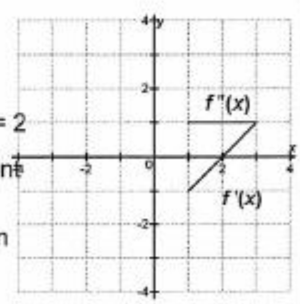
5 Which statement is false about the graph of $f(x)$ over [1, 3] ?

A $f(x)$ is concave up

B $f'(x)$ changes sign at $x = 2$

C $f(x)$ has an inflection point at $x = 2$

D $f(x)$ has a local minimum at $x = 2$



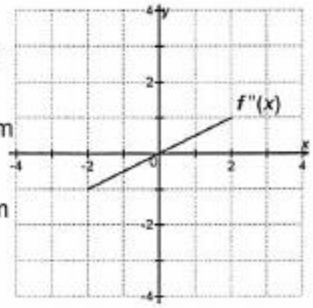
6 Which statement is true about the graph of $f(x)$ for the piece shown?

A $f(x)$ has an inflection poi at $x = 0$

B $f(x)$ has a local maximum at $x = 0$

C $f(x)$ has a local minimum at $x = 0$

D $f(x)$ is concave up



Q6

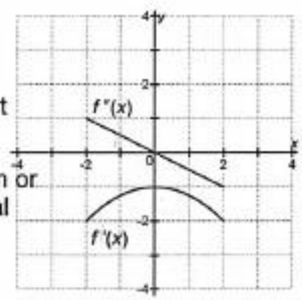
7 Which statement is false about the graph of $f(x)$ over $[-2, 2]$?

A $f(x)$ is decreasing

B $f(x)$ has an inflection point at $x = 0$

C $f(x)$ has no local minimum or maximum over the interval

D $f(x)$ is increasing



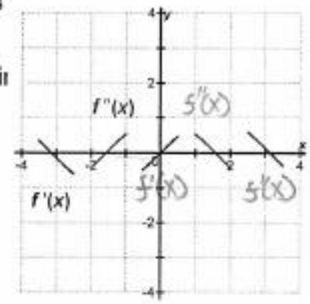
8 Which statement about the graph of ***f*(*x*)** is true?

A $f(x)$ has two inflection points at about $x = -1.5, 1.5$

B $f(x)$ has three inflection poi at about $x = -3.1, 0, 3.1$

C $f(x)$ has local maxima at about $x = -1.5, 1.5$

D $f(x)$ has two local minima at about $x = -1.5, 1.5$



Q8

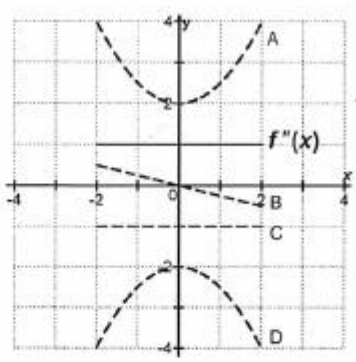
9 If $f''(x)$ is as shown, which line could be $f(x)$?

A line A

B line B

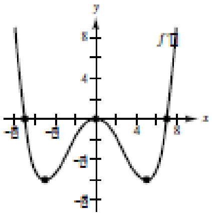
C line C

D line D



Worked Example

Consider the graph of f', the derivative of $y = f(x)$ defined on the domain $-9 < x < 9$.



(a) For what values of x does f have a relative minimum?

(b) For what values of x does f have a relative maximum?

(c) Determine the open intervals where the graph of f is concave downwards. Show the analysis that leads to your conclusion.

(d) Sketch the graph of f on the interval $(-9, 9)$ if $f(0) = 0$. Show the analysis that leads to your graph.

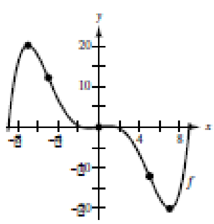
SOLUTION

(a) There is a relative minimum at $x = 7$.

(b) There is a relative maximum at $x = -7$.

(c) The graph of f' is decreasing on the intervals $(-9, -5)$ and $(0, 5)$. By the definition of concavity, this means that the graph of f is concave downwards on these intervals.

(d) The graph of f is increasing to the left of $x = -7$. The graph is decreasing on the intervals $(-7, 0)$ and $(0, 7)$, and increasing on the interval $(7, 9)$. There is an inflection point at $x = 0$ because the concavity changes from concave upwards to concave downwards.



Date ______________________________

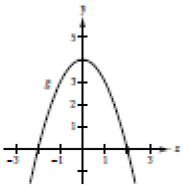
Sample Questions

Show all your work on a separate sheet of paper. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your answers.

Multiple Choice

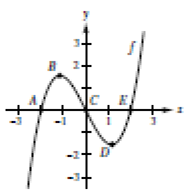
1. Given the graph of $y = g(x)$, estimate the value of $g'(2)$.

(a) -4 (b) -1 (c) 0 (d) 1 (e) 4



2. At which point A, B, C, D, or E on the graph of $y = f(x)$ are both y' and y'' positive?

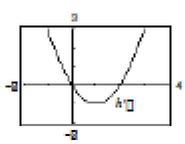
(a) A (b) B (c) C (d) D (e) E



3. Given the graph of $h'(x)$, which of the following statements are true about the graph of h?

I. The graph of h has a point of inflection at $x = 1$.
II. The graph of h has a relative extremum at $x = 0$.
III. The graph of h has a relative extremum at $x = 1$.

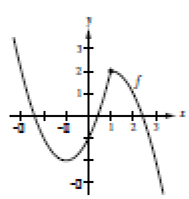
(a) I only (b) II only (c) III only (d) I and II only (e) I and III only



Free Response

The graph of the function f is shown in the figure.

(a) Estimate $f'(0)$.
(b) On what open intervals is f increasing?
(c) On what open intervals is f concave downwards?
(d) What are the critical numbers of f?
(e) Sketch the graph of f'.



SOLUTIONS

Multiple Choice

1. Answer (a). The slope of the tangent line to the graph of $y = g(x)$ is clearly negative. Furthermore, by analyzing the tick marks on the x and y axes you see that the slope is approximately -4.

2. Answer (e). y' is positive at the points where the graph is increasing. y'' is positive at the points where the graph is concave upwards. The only point satisfying these two criteria is point E.

3. Answer (d). At $x = 1$ the derivative of h changes from decreasing to increasing. So, the concavity changes at this point, so (I) is true. The derivative changes from positive to negative at $x = 0$ which implies that there is a relative maximum at $x = 0$. So, (II) is true. However, (III) is false because the derivative does not change sign at $x = 1$. In conclusion, only (I) and (II) are true.

Free Response

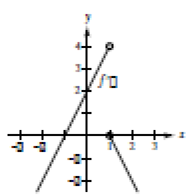
(a) $f'(0)$ is approximately 2.

(b) The graph of f is increasing on the interval $(-1, 1)$.

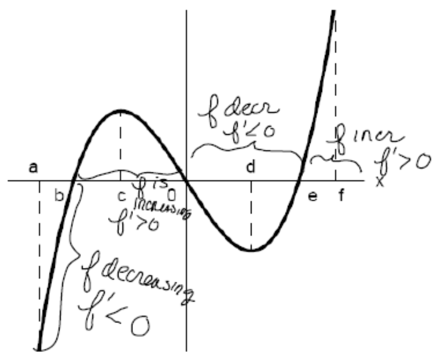
(c) The graph of f is concave downwards on the interval $(1, 3)$.

(d) $x = -1$ is a critical number because $f'(-1) = 0$. Furthermore, $x = 1$ is a critical number because the derivative does not exist there. The derivative from the left is positive, whereas the derivative from the right is 0.

(e) Because f is not differentiable at $x = 1$, you can expect a break in the graph of f' at this point. The graph of f is decreasing to the left of $x = -1$ and increasing on $(-1, 1)$. The graph of f is decreasing to the right of $x = 1$. So, the graph of f' looks like the following.



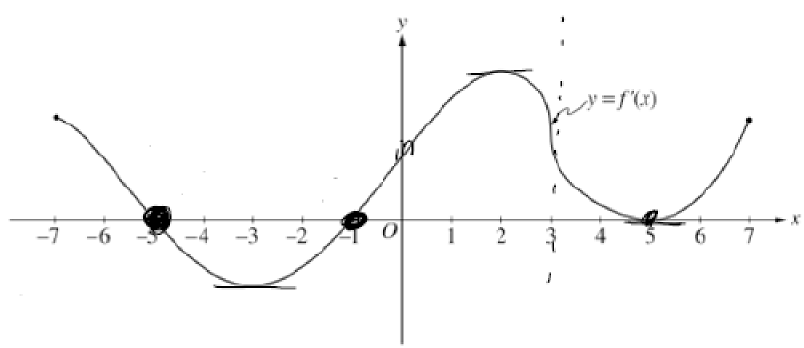
Let $y = f'(x)$ be the graph shown below



Based on the graph of $f'(x)$ shown above find where the graph of $f(x)$ is increasing or decreasing AND find the x-values of any relative extrema.

at $x = b$ and $x = e$ f' changes from negative to positive values Hence f has a rel min at $x = b, x = e$

at $x = 0$ f' changes from positive to negative values Hence f has a rel max at $x = 0$



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangents lines at $x = -3$, $x = 2$, and $x = 5$ and a vertical tangent at $x = 3$. Find all relative extrema on the open interval $-7 < x < 7$. Justify completely.

Lesson 2

Go over HW-
Cw- below problems   WS #93 # 5-13
HW- 1991, 2006 and multiple choice 12-30

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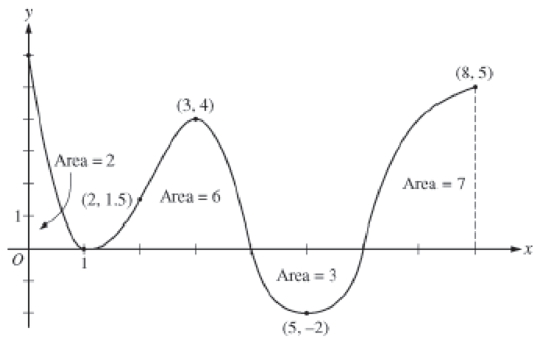


Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

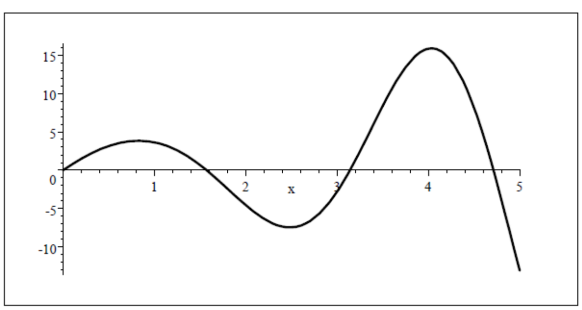
(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

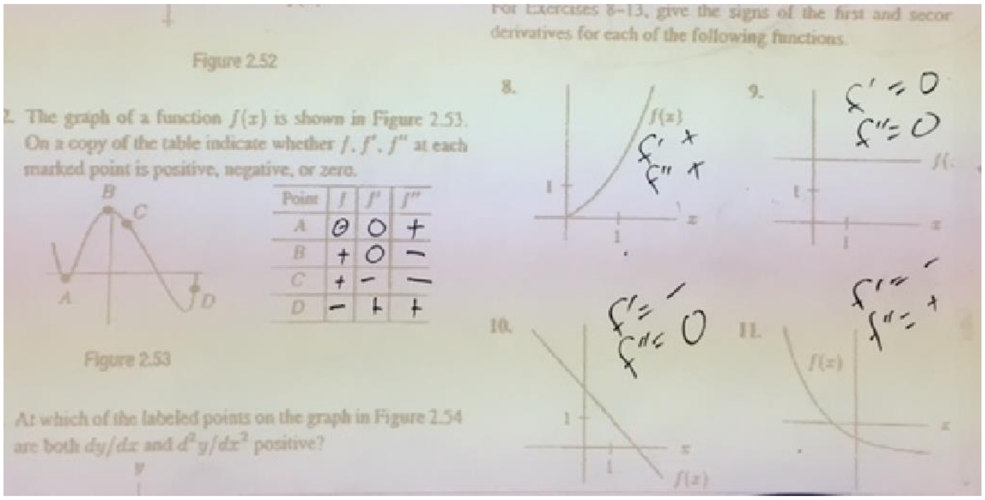
Example *Let the graph of $f'(x)$ be given below.   Find*

a. *the x –coordinate of each inflection point of f;*

b. *where the graph of f is concave up and is concave down.*



$f'(x)$



For Exercises 8–13, give the signs of the first and secor derivatives for each of the following functions.

Figure 2.52

2. The graph of a function $f(x)$ is shown in Figure 2.53. On a copy of the table indicate whether f, f', f'' at each marked point is positive, negative, or zero.

Point	f	f'	f''
A	0	0	+
B	+	0	−
C	+	−	−
D	−	+	+

Figure 2.53

At which of the labeled points on the graph in Figure 2.54 are both dy/dx and d^2y/dx^2 positive?

8. f' + f'' +

9. $f' = 0$ $f'' = 0$

10. $f' = -$ $f'' = 0$

11. $f' = -$ $f'' = +$

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2005 SCORING GUIDELINES

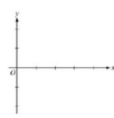
Question 4

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$
$f(x)$	−1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	−3	Negative
$f''(x)$	−2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.
(Note: Use the axes provided in the pink test booklet.)



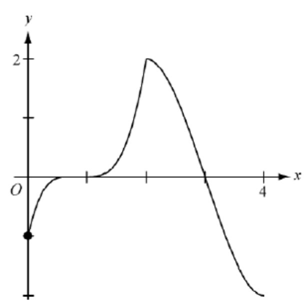
(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

(a) f has a relative maximum at $x = 2$ because f' changes from positive to negative at $x = 2$.

$2: \begin{cases} 1: \text{relative extremum at } x = 2 \\ 1: \text{relative maximum with justification} \end{cases}$

(b)



$2: \begin{cases} 1: \text{points at } x = 0,\ 1,\ 2,\ 3 \text{ and behavior at } (2, 2) \\ 1: \text{appropriate increasing/decreasing and concavity behavior} \end{cases}$

(c) $g'(x) = f(x) = 0$ at $x = 1, 3$.
g' changes from negative to positive at $x = 1$ so g has a relative minimum at $x = 1$. g' changes from positive to negative at $x = 3$ so g has a relative maximum at $x = 3$.

$3: \begin{cases} 1: g'(x) = f(x) \\ 1: \text{critical points} \\ 1: \text{answer with justification} \end{cases}$

(d) The graph of g has a point of inflection at $x = 2$ because $g'' = f'$ changes sign at $x = 2$.

$2: \begin{cases} 1: x = 2 \\ 1: \text{answer with justification} \end{cases}$

1989 AB5
Solution

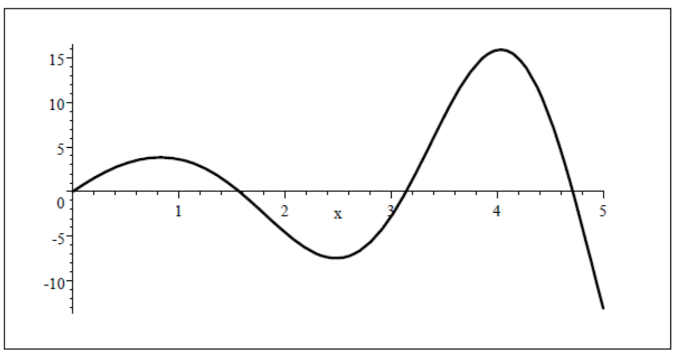
(a) horizontal tangent $\Leftrightarrow f'(x)=0$

$x=-7,-1,4,8$

(b) Relative maxima at $x=-1,\ 8$ because f' changes from positive to negative at these points

(c) f concave downward $\Leftrightarrow f'$ decreasing

$(-3,2),(6,10)$



$f'(x)$

$f''(x) = 0$ when $x = 0.8,\ x = 2.5,\ x = 4.$

$f'' > 0$ (f' is increasing) for $0 < x < 0.8,\ 2.5 < x < 4;$

$f'' < 0$ $\left(f' \text{ is decreasing}\right)$ for $0.8 < x < 2.5,\ 4 < x < 5.$

a. So, $x = 0.8,\ x = 2.5,\ x = 4$ are the x –coordinates of inflection points of f.

b. The graph of f is concave up on $(0,\ 0.8) \cup (2.5,\ 4)$ and is concave down on $(0.8,\ 2.5) \cup (4,\ 5)$.

Lesson 3

Go over HW-scoring guidelines-
CW -2003-2007-

HW-MC #25-26, 42-43 with calc, 56-58,75 82-84

Warm up
Ap problem 2003- part a,b,c
on your own- will be graded AP style

Lesson 4

--Find the absolute maximum on the interval [1,4]. Justify

$$f(x) = x^3 - 6x^2 + 12x - 8$$

NOTES FOR OPTIMIZATION PROBLEMS:

Whenever you are required to Maximize or Minimize a function, you MUST justify whether or not your answer is actually a maximum or a minimum. You may use the FIRST DERIVATIVE TEST (testing points to the left and right of the critical points in the first derivative to see if the sign of the first derivative changes from positive to negative or vice-versa), or the SECOND DERIVATIVE TEST (plugging in the critical points to the second derivative to see if the critical points occur when the original function was concave up or down).

ALWAYS REMEMBER that both of these tests are checking for relative extrema. If you have a CLOSED interval, you must check the endpoints to make sure the absolute maximum or minimum values do not happen to occur there. If you have a closed interval, it is best just to check ALL critical points and endpoints.

Optimization- HW –McCleary worksheet (other side) solutions on website

Primary equation- is what you are trying to maximize or minimize-(we will take the derivative of this equation to find Critical Values

Secondary equation- an equation that helps you get to 1 variable

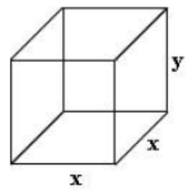
1. Find 2 positive numbers that satisfy the given requirements-
   The second number is the reciprocal of the first and the sum is a minimum

Primary-
Secondary-

2. I have 100 ft of fence to make a rectangular dog pen, what is the maximum area I can construct?

   Primary-
   Secondary-

3 A box with a square base with NO top has a surface area of 108 square feet. Find the dimensions that will maximize the volume. [You must use Calculus!]

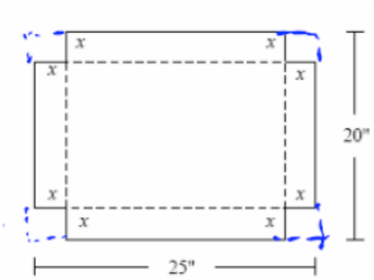


Primary-
Secondary-
Optimization- McCleary - Homework

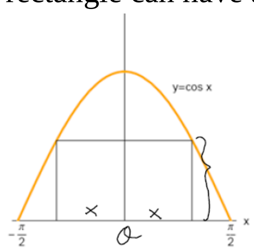
1) Find 2 positive numbers such that the product is 192 and the sum is a minimum.
Primary
Secondary-

2) Find the length and width of a rectangle that has a perimeter of 64 feet and a maximum area.
Primary
Secondary-

3) An open top box is to be made by cutting congruent squares of side length x from the corners of a 20 by 25 inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible
Primary
Secondary-

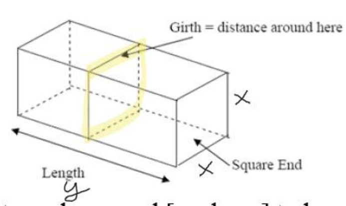


4) A rectangle is to be inscribed under one arch of a cosine curve. What is the largest area the rectangle can have and what dimensions give that area?



Primary
Secondary

5) A rectangular package to be send by UPS can have a maximum combined length and girth of 300cm. Find the dimensions of the package of maximum volume that can be sent (assume that the cross sections is a square)



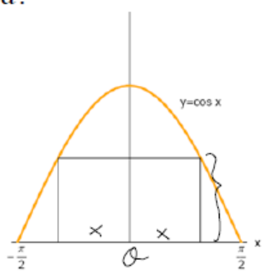
Primary

Secondary

Lesson 5
Go over HW
HW- p.231 #1A,6,9,13,14,16,

Reiterate that max and min can occur at endpoints (if a closed interval) or critical values- you should evaluate all of them –use 1st or 2nd derivative test to justify- make sure you answer the question that is being asked

A rectangle is to be inscribed under one arch of the cosine curve. What is the largest area the rectangle can have and what dimensions give that area?



$a = lw$

$l = 2x$

$w = \cos x$

$a(x) = 2x\cos x$

Domain: $(0, \frac{\pi}{2})$

$a'(x) = 2\cos x + -2x\sin x$

at $x \approx .860$ $a'(x)$ changes from positive to negative values Hence $a(x)$ has a rel max at $x \approx .860$

Our max area ≈ 1.122

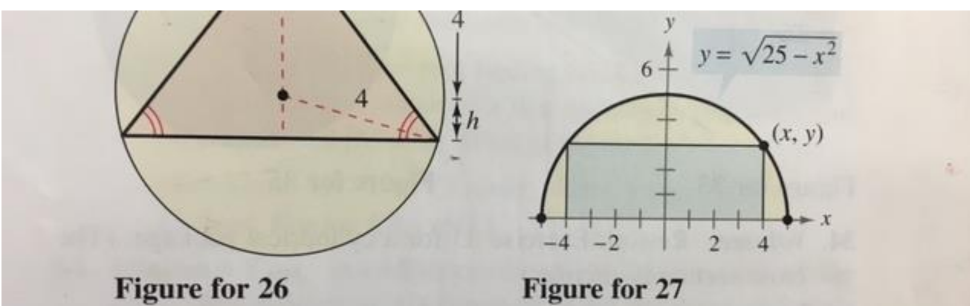
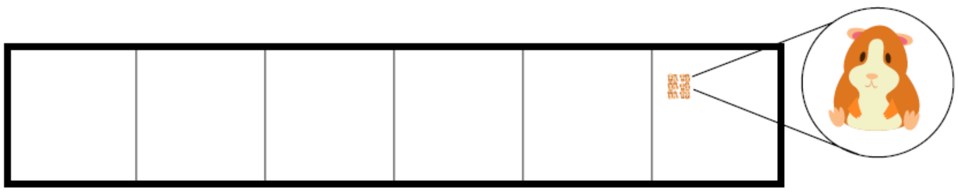


Figure for 26 **Figure for 27**

27. ***Area*** A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$ (see figure). What length and width should the rectangle have so that its area is a maximum?

Lesson 5L- go over homework- optimization
cw-#27 2 prob below distance and area and p. 235 #51-54
HW- review sheet
Use #10 only if time allows

10) Your dream of becoming a hamster breeder has finally come true.
You are constructing a set of rectangular pens in which to breed your furry friends. The overall area you are working with is 60 square feet, and you want to divide the area up into six pens of equal size as shown below.



The cost of the outside fencing is \$10 a foot. The inside fencing costs \$5 a foot. You wish to minimize the cost of the fencing.

a) Labeling variables, write down a constrained optimization problem that describes this problem.

b) Using any method learned in this course, find the exact dimensions of each pen that will minimize the cost of the breeding ground. What is the total cost?

27. ***Area*** A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$ (see figure). What length and width should the rectangle have so that its area is a maximum?

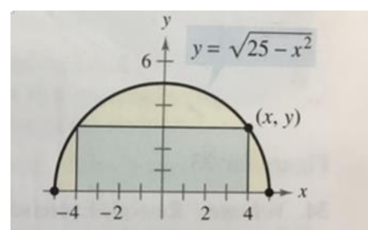


Figure for 27

4. What points on the graph $y=4-x^2$ are closest to the point (0,2)

Primary

Secondary

$x = 0, 2$ (select $x = 2$)

Then $y = 4$ and $A = 4$.

Vertices: $(0, 0)$, $(2, 0)$, $(0, 4)$

27. $A = 2xy = 2x\sqrt{25 - x^2}$ (see figure)

$$\frac{dA}{dx} = 2x\left(\frac{1}{2}\right)\left(\frac{-2x}{\sqrt{25 - x^2}}\right) + 2\sqrt{25 - x^2}$$

$$= 2\left(\frac{25 - 2x^2}{\sqrt{25 - x^2}}\right) = 0 \text{ when } x = y = \frac{5\sqrt{2}}{2} \approx 3.54.$$

By the First Derivative Test, the inscribed rectangle of maximum area has vertices

$$\left(\pm\frac{5\sqrt{2}}{2}, 0\right), \left(\pm\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right).$$

Width: $\frac{5\sqrt{2}}{2}$; Length: $5\sqrt{2}$

29. $xy = 30 \Longrightarrow y = \frac{30}{x}$

10) The cost of the outside fencing is \$10 a foot. The inside fencing costs \$5 a foot. You wish to minimize the cost of the fencing.

a) Let x be the width of each individual pen, and y be the length as shown above. Since the total area is 60 sq. ft., each individual pen will have an area of 10 sq. ft.
The constraint is $xy = 10$. The objective function is the cost. Examining the fencing above, there is $5y$ feet of interior fencing, and $2y + 12x$ feet of exterior fencing.
So the total cost is $C = \$5 \cdot (5y) + \$10 \cdot (2y + 12x)$, or $C = 45y + 120x$.

The constrained optimization problem is: Minimize $C = 45y + 120x$ subject to the constraint $xy = 10$.

b) Solving $xy = 10$ of y gives $y = \frac{10}{x}$. Substituting this into C gives $C = \frac{450}{x} + 120x$, as the function to minimize over $x \in (0, \infty)$.

$C' = -\frac{450}{x^2} + 120$, and so critical points are $x = 0$, and $x = \frac{\sqrt{15}}{2}$, so

$y = \frac{10}{x} = \frac{4\sqrt{15}}{3}$.

The cost is $C = 45y + 120x = 120\sqrt{15} \approx \$464.76$.

Lesson 6

Rectilinear – PVA
CW. Mellina packet page 42,46
HW Mellina packet p 40,41,43

and worksheet with notes particle problem parts 1-4

Rectilinear motion is motion along a line either horizontal or vertical

Position function- where along the x or y axis
Often written as s(t), x(t) or y(t)

Velocity function- v(t) =s'
Acceleration function – a(t) , s", v'

	-	**0**	**+**
Position	left of origin or below origin	at origin	at right of origin or above the origin
Velocity	moving left Or down	no motion	moving to right or up
Acceleration	decreasing vel	constant vel	velocity increasing

Speed is the absolute value of velocity

Velocity has direction + or –

Ex- -30m/s 20 m/s

Which is the greater velocity? 20
Which is the greater speed? -30

Example- look for in smart board----

Motion along the x-axis position function

$S(t) = 2t^3 – 21t^2+60t + 3$

Describe the initial position and motion when t=o

What is s(0) 3 units to the right of origin

Describe the initial velocity? v(0) 60 moving to right at 60 units per sec

Describe the initial acceleration. What is a(0) -42 slowing down because acceleration is negative -
(Note- vel and acc opposite signs)

At what time does the particle stop and turn around?

When the Velocity = 0 it stops, if it changes sign it is turning around.

$V(t) = 6t^2- 42t+60 =0$
T= 5,2

Same sign
Slowing down - opp sign

Particle Motion

Formulas

The formulas for the position, velocity, acceleration and speed of a moving object are given by the following derivatives.

$x(t)$	Position
$v(t) = x'(t)$	Velocity
$a(t) = v'(t) = x''(t)$	Acceleration
$\lvert v(t) \rvert$	Speed

Worked Example

The position function of a particle moving along the x-axis is given by $x(t) = t^3 - 12t^2 + 36t - 20$, $0 \le t \le 8$.

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals when the particle is moving to the left.

(c) Find the velocity of the particle when the acceleration is 0.

(d) Describe the motion of the particle.

SOLUTION

(a) $x(t) = t^3 - 12t^2 + 36t - 20$ Position

$v(t) = 3t^2 - 24t + 36$ Velocity

$a(t) = 6t - 24$ Acceleration

(b) $v(t) = 3t^2 - 24t + 36 = 3(t - 2)(t - 6) < 0$ when $2 < t < 6$.

(c) $a(t) = 6t - 24 = 0$ when $t = 4$. So, $v(4) = 3(4)^2 - 24(4) + 36 = -12$.

(d) You can analyze the motion of the particle by building a table of values for $x(t)$, $v(t)$, and $a(t)$ at $t = 0, 1, \ldots, 8$.

t	0	1	2	3	4	5	6	7	8
$x(t)$	-20	5	12	7	-4	-15	-20	-13	12
$v(t)$	36	15	0	-9	-12	-9	0	15	36
$a(t)$	-24	-18	-12	-6	0	6	12	18	24

From this table you can see that the particle starts at $x = -20$ and moves to the right to $x = 12$ when $t = 2$. The velocity is zero when $t = 2$ and the particle reverses direction and returns to $x = -20$ when $t = 6$. Again, the velocity is zero here, and the particle reverses direction once more and moves off to the right.

Notes

(a) You can obtain the velocity and acceleration functions by differentiating the given position function, $v(t) = x'(t)$ and $a(t) = x''(t) = v'(t)$.

(b) The particle is moving to the left when $v(t) < 0$, so you have to solve a quadratic inequality. Note that the particle is moving to the right when $0 < t < 2$ and $6 < t < 8$.

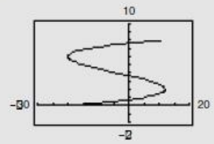
(c) The velocity is negative because the particle is moving to the left.

(d) The *table* feature on the *TI-83* is especially useful for constructing a table of values. You can obtain a schematic graph of the position of the particle by plotting (in parametric mode) the curve

$$x_1(t) = t^3 - 12t^2 + 36t - 20$$

$$y_1(t) = t$$

for $0 \le t \le 8$, using the viewing window $[-30, 20] \times [-2, 10]$.



Particle is speeding up if V(t) and A(t) have the same sign
Particle slowing down if V(t) and A(t) have opposite signs

A particle is moving along the x-axis. Its position at time, *t*, is given by the equation:

$$s(t) = \frac{t^4}{4} - \frac{7t^3}{3} + 5t^2.$$

1. What is the velocity of the particle at $t = 3$? Is the velocity increasing or decreasing at this time? Explain and justify your answer.

2. At what values of *t* does the particle change direction? Explain and justify your answer.

3. For which values of *t* is the position graph concave downwards? For which values is it concave upwards? Explain and justify your answer.

4. For which values of *t* is the particle speeding up? For which values is it slowing down? Explain and justify your answer.

6. A particle is moving along the x-axis. Its position at time t, is given by the equation:

$$s(t) = \frac{t^4}{4} - \frac{7t^3}{3} + 5t^2 = \frac{1}{4}t^4 - \frac{7}{3}t^3 + 5t^2$$

a. What is the velocity of the particle at $t = 3$? Is the velocity increasing or decreasing at this time? Explain and justify your answer.

$v(t) = t^3 - 7t^2 + 10t$

$t(t^2 - 7t + 10)$

$t(t-5)(t-2)$

$t = 0, 5, 2$

$v(3) = 27 - 63 + 30$

$\boxed{-6}$

$A(3) = -$ decreasing because $A(t)$ is neg

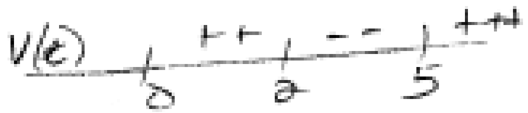
b. At what values of t does the particle change direction? Explain and justify your answer.

$v(t) = t^3 - 7t^2 + 10t$

$t(t^2 - 7t + 10)$

$t(t-5)(t-2)$

$t = 0, 5, 2$



$t = 2, 5$ $v(t) = 0$ and changes sign

c. For which values of t is the position graph concave downwards? For which values is it concave upwards? Explain and justify your answer.

$A(t) = 3t^2 - 14t + 10$

$\dfrac{14 \pm \sqrt{(-14)^2 - 4(3)(10)}}{2(3)}$ .880 3.786

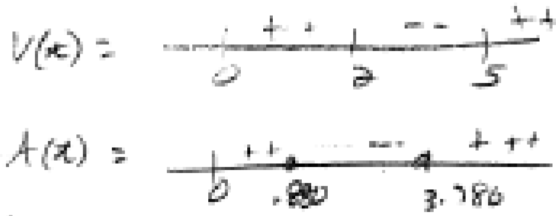
concave up because f'' is pos $(0, .880)$ $(3.786, \infty)$

concave down $(.880, 3.786)$ b.c. $f''(x)$ is neg

d. For which values of t is the particle speeding up? For which values is it slowing down? Explain and justify your answer.

Speeding up - $v(t)$ + $A(t)$ same sign

$(0, .880) \cup (2, 3.786) \cup (5, \infty)$



slowing down $v(t)$ + $A(t)$ have opp signs

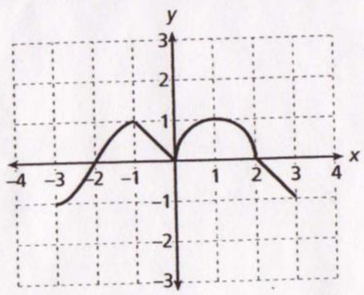
$(3.786, 5) \cup (.880, 2)$

lesson 7

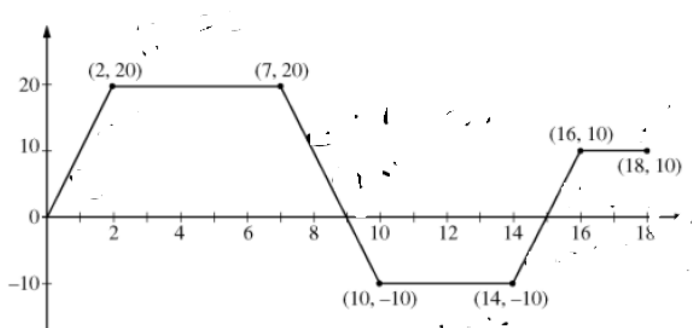
cw/hw mellina packet **page 43-49**

HW- Tomorrow night's HW is MC packet # 11-13, 35-41, 72-73

6. The graph of $f'(x)$ is given below for $x \in [-3,3]$. On which interval(s) is the function $f(x)$ both increasing and concave up?



(A) $(-2,2)$
(B) $(-2,0)\cup(0,2)$
(C) $(-3,-2)$
(D) $(-2,-1)\cup(0,1)$
(E) none of these



Graph of v

This is a graph of v(t), when does the particle stop and change direction?

Car A has positive velocity $v(t)$ as it travels on a straight road, where v is a differentiable function of t. The velocity is recorded for selected values over the time interval $0 \leq t \leq 10$ seconds, as shown in the table below.

t (sec)	0	2	5	7	10
$v(t)$ (ft/sec)	0	9	36	61	115

1

a. Use data from the table to approximate the acceleration of Car A at $t = 8$ seconds. Indicate units of measure.

2

A particle moves along the y-axis for $0 \leq t \leq 40$ seconds. The velocity of the particle at time t is given by $v(t) = \sin\left(\frac{\pi}{8}t\right)$ in meters/second. The particle is at position $y = 10$ meters at time $t = 0$.

INITIAL POSITION

(a) Find the acceleration of the particle at time t

Is the speed of the particle increasing, decreasing, or neither at time $t = 10$ seconds?

14) a. $v(t) = \dfrac{(t^2+4)-(t)(2t)}{(t^2+4)^2} = \dfrac{4-t^2}{(t^2+4)^2}$

$a(t) = \dfrac{(t^2+4)^2(-2t)-(4-t^2)\left[2(t^2+4)(2t)\right]}{(t^2+4)^4}$

$= (t^4+8t^2+16)(-2t)-(4-t^2)\left[4t^3+16t\right]$

$= -2t^5-16t^3-32t-\left[\cancel{16t^3}+64t-4t^5-\cancel{16t^3}\right]$

$= -2t^5-16t^3-32t-64t+4t^5 = 2t^5-16t^3-96t$

$= \dfrac{2t^5-16t^3-96t}{(t^2+4)^4}$

b. $s(1) = \frac{1}{5}$

$v(1) = \frac{3}{25}$

$a(1) = -22/125$

c. $v(t) = \dfrac{4-t^2}{(t^2+4)^2} = 0$

$t = 2; \cancel{-2}$

v(t)

+++++++++ | - - - - - - -
2

d. $a(t) = \dfrac{2t^5-16t^3-96t}{(t^2+4)^4} = 0$

$t = 0; 2\sqrt{3}$

a(t)

| - - - - - - - | + + + + +
0 $2\sqrt{3}$

speeding up: $(2, 2\sqrt{3})$

slowing down: $(0,2)(2\sqrt{3},\infty)$

e. $s(0) = 0$

$s(5) = 5/29$

$D = \sqrt{(x-x_0)^2+(y-y_0)^2}$

$= \sqrt{(5-0)^2+(5/29-0)^2}$

$= \dfrac{5\sqrt{842}}{29} \approx 5.003$

Test topics- ch 5- AP Style
~~Mean value theorem-Rolles Thm~~ not on 2019 test
Tangent line approximation
Critical values (numbers)
Relative min and max
Absolute max and min
Reading f'(x) graphs and v(t) graphs
Optimization
Rectilinear motion PVA
Speeding up or slowing down
Inflection points, concavity
Increasing, decreasing

$P(t) = t^3 - t^2 - t$

Find- initial position and Initial motion
(WHAT SHOULD WE LOOK AT)

When does the particle change direction?
(WHAT SHOULD WE LOOK AT)

Is the velocity increasing or decreasing at t=4
(WHAT SHOULD WE LOOK AT)

When is the particle speeding up, slowing down
(WHAT SHOULD WE LOOK AT)

Lesson 8

REVIEW.
p.51 of packet with calculator, last page #20,21,22

Multiple choice packet # 11-13, # 35-41, 72-73
Multiple choice packet 35-41, 72-73

with a calculator- and justify

Worked Example

The position function of a particle moving along the x-axis is given by $x(t) = t^3 - 12t^2 + 36t - 20$, $0 \leq t \leq 8$.

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals when the particle is moving to the left.

(c) Find the velocity of the particle when the acceleration is 0.

(d) Describe the motion of the particle.

818 (1989BC). Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

a) Find the absolute maximum and minimum values of $f(x)$.

b) Find intervals on which f is increasing.

c) Find the x-coordinate of each point of inflection of the graph of f.

1. $F(x) = 2x^2$. Write an equation of the tangent line at x=3. Use it to approximate f(3.1).
   Does the approximation overestimate or underestimate the actual value?

LESSON 10- test