

## Flipped classroom

Watch Mr Leckie's video on implicit differentiation up to the 11 minute mark- follow along with the worksheet- **complete the worksheet for homework credit**

[http://www.chaoticgolf.com/vodcasts/calc/lesson3\\_7/lesson3\\_7.html](http://www.chaoticgolf.com/vodcasts/calc/lesson3_7/lesson3_7.html)

### 3.7 IMPLICIT DIFFERENTIATION

#### *Implicit and Explicit Functions*

Equations that are solved for  $y$  are called explicit functions, whereas equations that are not solved for  $y$  are called implicit.

For instance, the equation  $x + 2y - 3 = 0$ , implies that  $y$  is a function of  $x$ , even though it is not written in the form

$y = -\frac{1}{2}x + \frac{3}{2}$ . Up to this point in this class we have been using functions of  $x$  expressed in the form  $y = f(x)$  such as

$$y = \frac{x+1}{x+2} \quad \text{or} \quad y = \sin x.$$

If we have an equation that involves both  $x$  and  $y$  in which  $y$  has not been solved for  $x$ , then we say the equation defines  $y$  as an *implicit* function of  $x$ . In this case, we may (or may not) be able to solve for  $y$  in terms of  $x$  to obtain an explicit function (or possibly several functions).

First a few skills that you will need ...

*Example 1:* Find the derivative of the following expressions:

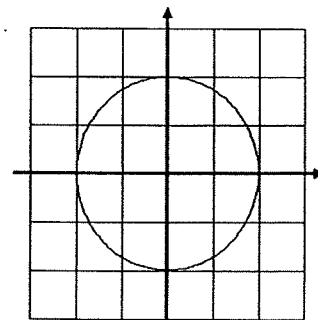
a) Find  $\frac{dy}{dx}$ :  $x^3$

b) Find  $\frac{dy}{dx}$ :  $y^3$

c) Find  $\frac{dy}{dx}$ :  $xy$

d) Find  $\frac{dy}{dx}$ :  $x^3 + y^2 - 3xy$

*Example 2:* Solve the equation  $x^2 + y^2 = 4$  to obtain  $y$  written as an explicit function of  $x$ . The graph of this equation is a circle. What is the graph of each explicit function?



Example 3: Find  $\frac{dy}{dx}$  for the circle  $x^2 + y^2 = 4$ :

a) by differentiating the explicit functions of  $x$

b) by differentiating implicitly

c) Find where the derivative is positive, where it is negative, where it is zero, and where it is undefined.

If we have  $y$  written as an explicit function of  $x$ ,  $y = f(x)$ , then we know how to compute the derivative  $\frac{dy}{dx}$ . For an equation which defines  $y$  as an implicit function of  $x$ , we can compute the derivative  $\frac{dy}{dx}$  without solving for  $y$  in terms of  $x$  with the following procedure. **The key to this entire procedure is to remember that even though you did not (or cannot) write  $y$  as a function of  $x$ ,  $y$  is implicitly defined as a function of  $x$ .**

#### Guidelines for Implicit Differentiation

1. Differentiate both sides of the equation **with respect to  $x$** . Remember,  $y$  is a function of  $x$  (use the Chain Rule)
2. Collect all  $\frac{dy}{dx}$  terms on the left side of the equation and move all other terms to the right side of the equation.
3. Factor  $\frac{dy}{dx}$  out of the left side of the equation.
4. Solve for  $\frac{dy}{dx}$ . (It is okay to have both  $x$ 's and  $y$ 's in your answer)

To find  $\frac{dy}{dx}$  at a given point, plug both the  $x$  and  $y$  value into the equation you obtained in step 4.