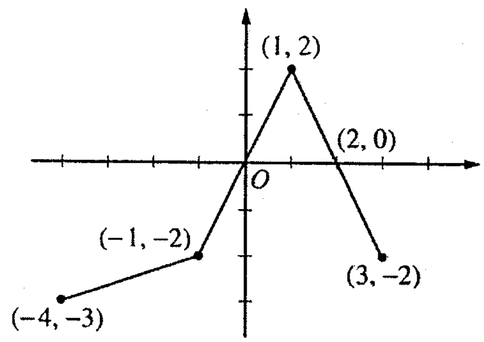
AP® CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 4

The graph of the function f above consists of three line segments.



Graph of f

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

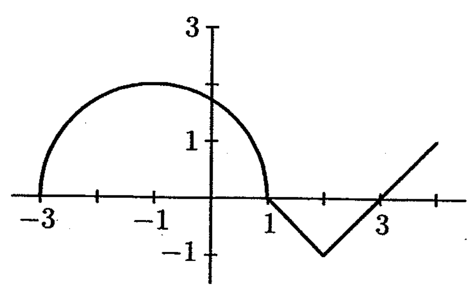
For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

1091. The graph of a function f consists of a semicircle and two line segments as shown below. Let $g(x) = \int_{1}^{x} f(t)\,dt$.



a) Find $g(1)$.

b) Find $g(3)$.

c) Find $g(-1)$.

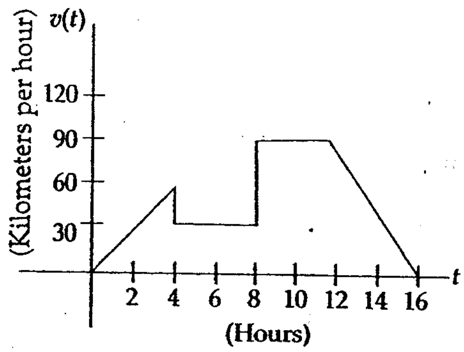
d) Find all the values of x on the open interval $(-3, 4)$ at which g has a relative maximum.

e) Write an equation for the line tangent to the graph of g at $x = -1$.

f) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-3, 4)$.

g) Find the range of g.

P435 #10



A car's velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

(A) 360 (B) 390 (C) 780 (D) 1000 (E) 1360

2. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

(a) Write an expression for the velocity of the particle $v(t)$.

(b) At what values of t does the particle change direction?

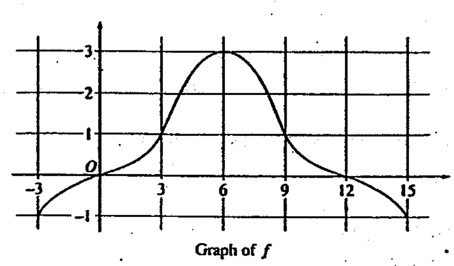
(c) Write an expression for the position $x(t)$ of the particle.

(d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

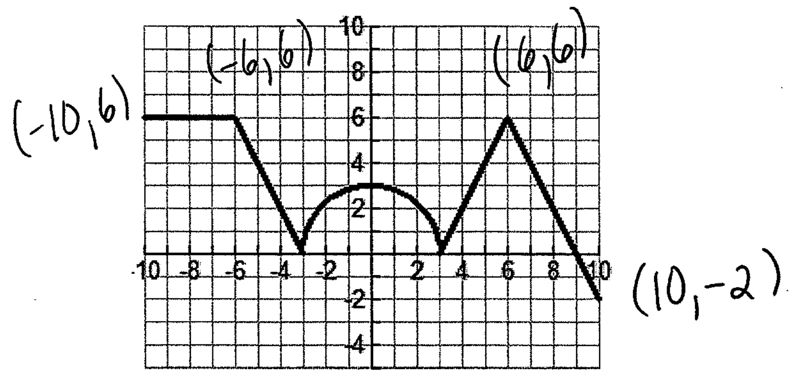
(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

My Big FTC Picture Problem ____________________________

Show all work/steps. Justify fully.



Graph of f

A continuous function f is defined on the closed interval $-10 \le x \le 10$. The graph of f consists of a semi-circle and four line segments as shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $\lim_{x \to 5} f(x)$

(b) Find the average rate of change for f on the interval $-10 \le x \le 10$

(c) Does the Mean Value Theorem guarantee a value c, $-10 < c < 10$ such that $f'(c)$ will equal the average rate of change from part (b)?

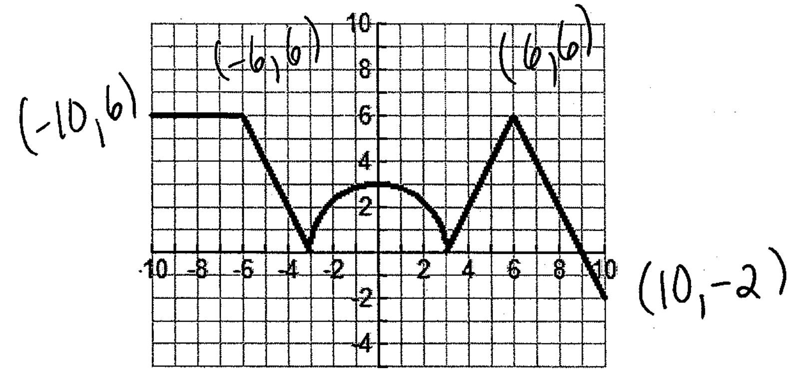
(d) Show [using Calculus] that $f'(6)$ does not exist

(e) Find the value of $g(-3)$

(f) Find the value of $g(3)$

(g) Find the value of $g(-10)$

(h) Find the value of $g(10)$



Graph of f

(i) Find $g'(x)$

(j) Find the x–value(s) of the critical value(s) for the graph of g and classify as relative minimum, relative maximum, or neither

(k) Find all intervals where the graph of g is increasing

(l) Find all intervals where the graph of g is decreasing

(m) Find the absolute extrema for the graph of g

(n) Find $g'(5)$

(o) Write the equation of the line tangent to the graph of g at $x = 5$

(p) Use the tangent line from part (o) to estimate $g(5.1)$

(q) Does the tangent line from part (o) lie above or below the graph of g?

(r) Is the estimate you found in part (p) an over- or an under-estimate?

(s) Find $g'(-4)$

(t) Write the equation of the line tangent to the graph of g at $x = -4$

(u) Use the tangent line from part (t) to find an estimate for $g(-4.1)$

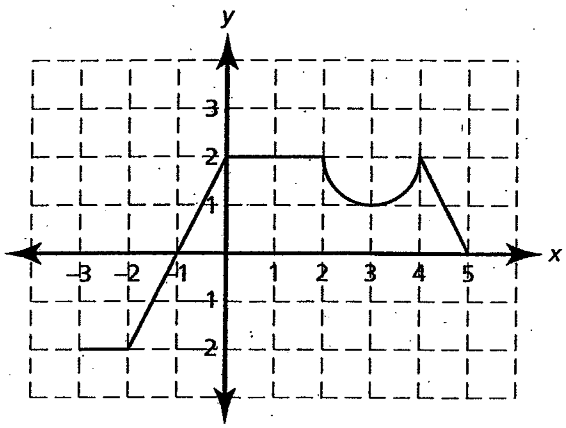
(v) Does the tangent line from part (t) lie above or below the graph of g?

AP. Prep Book

Ch 4 2nd Fundamental thm.

up to # 24

No calc



The graph of $f(x)$ consists of four line segments and a semicircle as shown above in the closed interval $-3 \le x \le 5$. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$. Use this information for problems 5–7.

5. What is $g(-1) + g'(-1) + g''(-1)$?
   (A) -1
   (B) 0
   (C) 1
   (D) 2
   (E) 3

6. What is $\int_{-3}^{5} f(t)\,dt$?
   (A) $7-\pi$
   (B) $7-\frac{\pi}{2}$
   (C) $7-\frac{\pi}{4}$
   (D) $12-\frac{\pi}{2}$
   (E) $12-\frac{\pi}{4}$

7. Which of the following statements is false for $g(x)$?
   (A) The absolute maximum for $g(x)$ occurs at $x = 5$.
   (B) A relative minimum for $g(x)$ occurs at $x = -1$.
   (C) A point of inflection for $g(x)$ occurs at $x = 3$.
   (D) $g(x)$ has roots at $x = 0$ and $x = -2$.
   (E) $g(x)$ is concave down in the open interval $-2 < x < -1$.

13. If $h''(x) = e^{x-1}(2x-1)^2(x-3)^3(4x+5)$, then $h(x)$ has how many points of inflection?
   (A) 4
   (B) 3
   (C) 2
   (D) 1
   (E) 0

22. Let $y = 2x(\sin 2x + x\cos 2x)$ in the interval $0 \le x \le \frac{\pi}{2}$. What is the average rate of change of y with respect to x in this interval?
   (A) $-\pi$
   (B) $-\frac{\pi}{2}$
   (C) 0
   (D) $\frac{\pi}{2}$
   (E) π

24. The concentration of an anti-inflammatory drug in the bloodstream t mins after taking a single dose is $C(t) = \frac{2t}{8100+t^2}$, $t \ge 0$. At what time is the concentration the greatest?
   (A) 90 minutes
   (B) $30\sqrt{6}$ minutes
   (C) $30\sqrt{3}$ minutes
   (D) $15\sqrt{6}$ minutes
   (E) none of these

Calc

29. Suppose $g(0) = 4$, $g'(0) = 8$, and $g''(0) = -12$. If $h(x) = \sqrt{g(x)}$, what is $h''(0)$?
   (A) -5
   (B) $-\frac{13}{4}$
   (C) $-\frac{1}{32}$
   (D) $\frac{3}{8}$
   (E) 1

10 spark note

If $F(x) = \int_1^{\ln x} \sqrt{\cos t}\, dt$, then $F'(x) =$

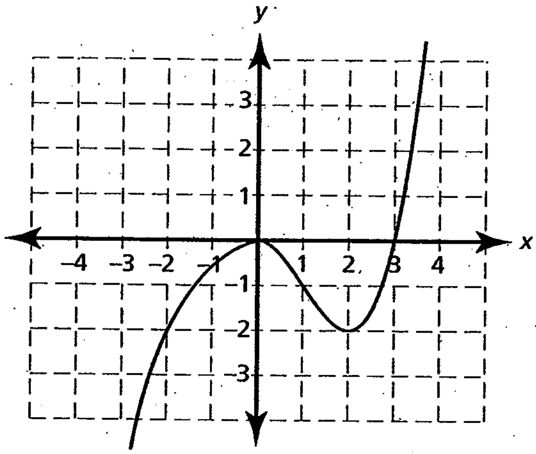
(A) $\sqrt{\cos x}$

(B) $\sqrt{\cos\left(\frac{1}{x}\right)}$

(C) $\sqrt{\cos(\ln x)}$

(D) $\dfrac{\sqrt{\cos(\ln x)}}{x}$

(E) $-\dfrac{\sin(\ln x)}{2\sqrt{\cos(\ln x)}}$



Let $f(x) = x^2 + \int_{-2}^{x} g(t)\, dt$, where $g(x)$ is shown in the graph above. Use this graph to answer problems 39–41.

Calc

39. What is $f(-2)$?
    (A) –6
    (B) –4
    (C) 0
    (D) 2
    (E) 4

Calc

40. What is $f'(-2)$?
    (A) –6
    (B) –4
    (C) 0
    (D) 2
    (E) 4

Calc

41. What is $f''(2)$?
    (A) –6
    (B) –4
    (C) 0
    (D) 2
    (E) 4

5. Consider the function $h(x) = 3x^2 - \sqrt{x+1}$.
   a. Evaluate $\frac{1}{3-(-1)}\int_{-1}^{3}\left(3x^2 - \sqrt{x+1}\right) dx$ and interpret its meaning.
   b. What is the equation of the tangent to $h(x)$ at $x = 0$?
   c. Use the tangent found in part b to approximate $h(x)$ at $x = -0.01$.
   d. Is the approximation, found in part c, greater or less than the actual value of $h(x)$ at $x = -0.01$? Justify your answer using calculus.

N/C