Everything I am supposed to remember about motion problems:

The position function is commonly called $s(t)$ but can also be called $x(t)$ if the particle is moving along the x-axis or $y(t)$ if the particle is moving along the y-axis. [Beware of the dreaded "f-disease" – if the problem is stated in terms of time, then do NOT start calling every function f (x) since no one will assume that you know what you are talking about!] Since we are not "time lords", then time is given with the domain $t \geq 0$

The position at time $t = 0$ is commonly called the initial position. If a problem states that an object is thrown from the ground, then its initial position, $s(0) = 0$. If a problem states, for example, that it is dropped from the top of a 50-foot building, then its initial position is $s(0) = 50$. Please learn how to read the problem thoroughly so that you can find all of the "clues" given in the problem's description.

How to find the position function if you are given the velocity function:

$$\int v(t)\,dt = s(t) + C$$

You will be able to determine the value of "C" based on information given to you about a specific position at some given time.

How to find a position if you are given the velocity function and some value of s(t):

Example: Find $s(7)$ if $s(3) = 4$ and $v(t) = \sin t\left(e^{\cos t}\right)$

$$s(7) = s(3) + \int_3^7 v(t)\,dt$$

Think of this as the initial position plus the accumulated rate of change of the position. Why does this work? Well, since $v(t) = s'(t)$, then

$s(7) = s(3) + \int_3^7 v(t)\,dt$ can be thought of as

$s(7) = s(3) + \int_3^7 s'(t)\,dt$ which equals

$$s(7) = s(3) + \left[s(7) - s(3)\right]$$

This technique can be used to find specified values of other functions when given some value of the function and the function's derivative or rate of change.

Velocity is given as the function $v(t)$. Velocity tells us how fast the object/particle is going and in which direction. The initial velocity of an object is given as $v(0)$. If $v(t)=0$, then the particle is "at rest" [not moving]. If $v(t)>0$, then the particle is moving right if it is traveling on the x-axis or moving upwards if it is traveling along the y-axis. Likewise, if $v(t)<0$, then the particle is moving left if it is on the x-axis or moving downwards if it is traveling on the y-axis. If you are asked to final the "terminal velocity", then find the velocity at the time the object reaches its final destination, i.e. the ground, which means that the s(t) would be equal to zero.

The instantaneous rate of change of the position is the velocity. In other words, s'(t)=v(t). If you are asked for the average velocity on an interval [a, b] AND you are only given a table of position values that include s(a) and s(b), then

Average velocity on [a, b] = $\dfrac{s(b)-s(a)}{b-a}$

If you are asked to find the average value of the velocity on [a,b] AND you are given the velocity function, then

Average value of the velocity on [a, b] = $\dfrac{1}{b-a}\displaystyle\int_a^b v(t)\,dt$

Speed = $|v(t)|$

Speed is either equal to zero or a positive value [think of your speedometer]. Speed tells us how fast the object/particle is going regardless of its direction.

To determine if the speed is increasing or decreasing at some time t = c

If $v(c)>0$ AND $a(c)>0$, then speed is increasing at t = c

If $v(c)<0$ AND $a(c)<0$, then speed is increasing at t = c

If $v(c)<0$ AND $a(c)>0$, then speed is decreasing at t = c

If $v(c)>0$ AND $a(c)<0$, then speed is decreasing at t = c

If $v(t)$ is not negative on [a, b], then the total distance traveled on [a, b] is equal to

TDT on [a, b] = $\int_a^b v(t)\,dt$

If $v(t)$ has some negative values on [a, b], then the total distance traveled on [a, b] is equal to

TDT on [a, b] = $\int_a^b |v(t)|\,dt$

To find displacement on [a, b] which is not the same as total distance traveled,

Displacement on [a, b] = $\int_a^b v(t)\,dt$

Acceleration is the instantaneous rate of change of the velocity. In other words, $a(t) = v'(t) = s''(t)$.

If you are given a table of velocity values which include v(a) and v(b), then the average acceleration on [a, b] is equal to:

Average acceleration on [a, b] = $\frac{v(b) - v(b)}{b - a}$

If you are given the acceleration function, then the average acceleration on [a, b] is equal to:

Average acceleration on [a, b] = $\frac{1}{b-a}\int_a^b a(t)\,dt$

Since $a(t) = v'(t)$, then $\int a(t)\,dt = v(t) + C$

The value of "C" can be determined if given a specified value of v(t).

Free response problems often ask for correct units in their solutions, so you need to be mindful of the units given in the problem.

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