

Set 3: Multiple-Choice Questions on Differentiation

In each of Questions 1–5 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. $y = (4x + 1)(1 - x)^3$

- (A) $-12(1 - x)^2$ (B) $(1 - x)^2(1 + 8x)$ (C) $(1 - x)^2(1 - 16x)$
(D) $3(1 - x)^2(4x + 1)$ (E) $(1 - x)^2(16x + 7)$

2. $y = \frac{2 - x}{3x + 1}$

- (A) $-\frac{7}{(3x + 1)^2}$ (B) $\frac{6x - 5}{(3x + 1)^2}$ (C) $-\frac{9}{(3x + 1)^2}$
(D) $\frac{7}{(3x + 1)^2}$ (E) $\frac{7 - 6x}{(3x + 1)^2}$

3. $y = \sqrt{3 - 2x}$

- (A) $\frac{1}{2\sqrt{3 - 2x}}$ (B) $-\frac{1}{\sqrt{3 - 2x}}$ (C) $-\frac{(3 - 2x)^{3/2}}{3}$
(D) $-\frac{1}{3 - 2x}$ (E) $\frac{2}{3}(3 - 2x)^{3/2}$

4. $y = \frac{2}{(5x + 1)^3}$

- (A) $-\frac{30}{(5x + 1)^2}$ (B) $-30(5x + 1)^{-4}$ (C) $\frac{-6}{(5x + 1)^4}$
(D) $-\frac{10}{3}(5x + 1)^{-4/3}$ (E) $\frac{30}{(5x + 1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

- (A) $2x^{1/3} - 2x^{-1/2}$ (B) $3x^{-1/3} - 2x^{-1/2}$ (C) $\frac{9}{5}x^{5/3} - 8x^{3/2}$
(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ (E) $2x^{-1/3} - 2x^{-1/2}$

In Questions 6–13, differentiable functions f and g have the values shown in the table.

| x | f | f' | g | g' |
|-----|-----|------|-----|------|
| 0 | 2 | 1 | 5 | -4 |
| 1 | 3 | 2 | 3 | -3 |
| 2 | 5 | 3 | 1 | -2 |
| 3 | 10 | 4 | 0 | -1 |

6. If $A = f + 2g$, then $A'(3) =$
(A) -2 (B) 2 (C) 7 (D) 8 (E) 10
7. If $B = f \cdot g$, then $B'(2) =$
(A) -20 (B) -7 (C) -6 (D) -1 (E) 13
8. If $D = \frac{1}{g}$, then $D'(1) =$
(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$
9. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$
(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) 2 (D) $\frac{2}{\sqrt{10}}$ (E) $4\sqrt{10}$
10. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$
(A) $\frac{-13}{25}$ (B) $-\frac{1}{4}$ (C) $\frac{13}{25}$ (D) $\frac{13}{16}$ (E) $\frac{22}{25}$
11. If $M(x) = f(g(x))$, then $M'(1) =$
(A) -12 (B) -6 (C) 4 (D) 6 (E) 12
12. If $P(x) = f(x^3)$, then $P'(1) =$
(A) 2 (B) 6 (C) 8 (D) 12 (E) 54

In Questions 14–21 find y' .

14. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

(A) $x + \frac{1}{x\sqrt{x}}$ (B) $x^{-1/2} + x^{-3/2}$ (C) $\frac{4x-1}{4x\sqrt{x}}$

(D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ (E) $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

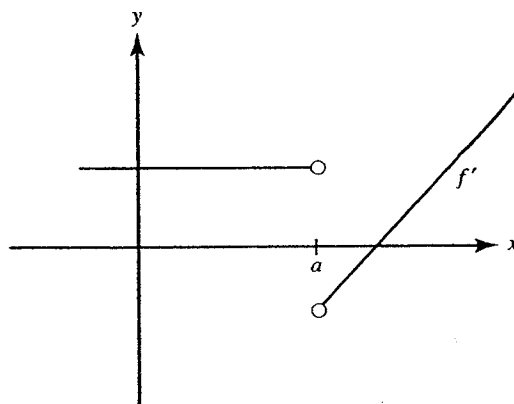
15. $y = \sqrt{x^2 + 2x - 1}$

(A) $\frac{x+1}{y}$ (B) $4y(x+1)$ (C) $\frac{1}{2\sqrt{x^2 + 2x - 1}}$

(D) $-\frac{x+1}{(x^2 + 2x - 1)^{3/2}}$ (E) none of these

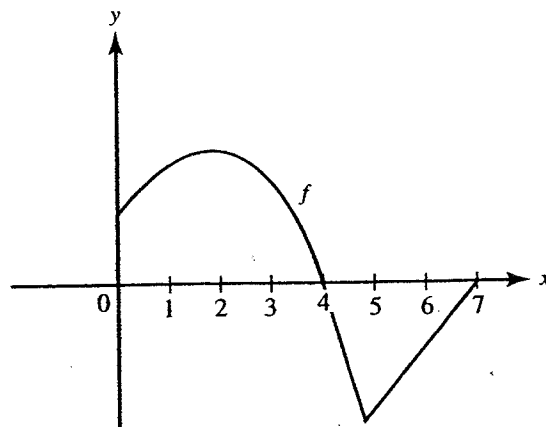
23. A function f has the derivative shown. Which of the following statements must be false?

- (A) f is continuous at $x = a$.
- (B) $f(a) = 0$.
- (C) f has a vertical asymptote at $x = a$.
- (D) f has a jump discontinuity at $x = a$.
- (E) f has a removable discontinuity at $x = a$.



24. The function f whose graph is shown has $f' = 0$ at $x =$

- (A) 2 only
- (B) 2 and 5
- (C) 4 and 7
- (D) 2, 4, and 7
- (E) 2, 4, 5, and 7



25. A differentiable function f has the values shown. Estimate $f'(1.5)$.

| | | | | |
|--------|-----|-----|-----|-----|
| x | 1.0 | 1.2 | 1.4 | 1.6 |
| $f(x)$ | 8 | 10 | 14 | 22 |

- (A) 8 (B) 12 (C) 18 (D) 40 (E) 80

In Questions 27–33, find $\frac{dy}{dx}$.

27. $y = x^2 \sin \frac{1}{x}$ ($x \neq 0$)

(A) $2x \sin \frac{1}{x} - x^2 \cos \frac{1}{x}$ (B) $-\frac{2}{x} \cos \frac{1}{x}$ (C) $2x \cos \frac{1}{x}$

(D) $2x \sin \frac{1}{x} - \cos \frac{1}{x}$ (E) $-\cos \frac{1}{x}$

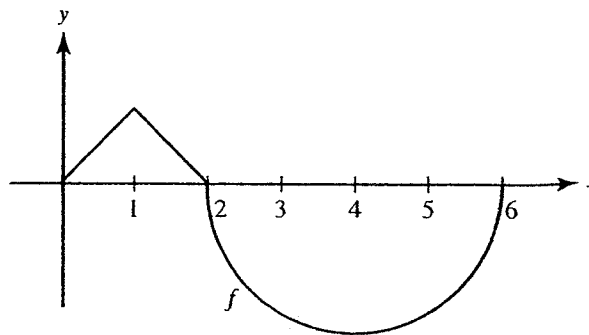
32. $y = \frac{1+x^2}{1-x^2}$

(A) $-\frac{4x}{(1-x^2)^2}$ (B) $\frac{4x}{(1-x^2)^2}$ (C) $\frac{-4x^3}{(1-x^2)^2}$

(D) $\frac{2x}{1-x^2}$ (E) $\frac{4}{1-x^2}$

Use the graph to answer Questions

34–36. It consists of two line segments and a semicircle.



34. $f'(x) = 0$ for $x =$

- (A) 1 only
 (B) 2 only
 (C) 4 only
 (D) 1 and 4
 (E) 2 and 6

35. $f'(x)$ does not exist for $x =$

- (A) 1 only (B) 2 only (C) 1 and 2
 (D) 2 and 6 (E) 1, 2, and 6

36. $f'(5) =$

- (A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) 2 (E) $\sqrt{3}$

In each of Questions 43–46, y is a differentiable function of x . Choose the alternative that is the derivative $\frac{dy}{dx}$.

43. $x^3 - xy + y^3 = 1$

- (A) $\frac{3x^2}{x-3y^2}$ (B) $\frac{3x^2-1}{1-3y^2}$ (C) $\frac{y-3x^2}{3y^2-x}$
(D) $\frac{3x^2+3y^2-y}{x}$ (E) $\frac{3x^2+3y^2}{x}$

44. $x + \cos(x+y) = 0$

- (A) $\csc(x+y) - 1$ (B) $\csc(x+y)$ (C) $\frac{x}{\sin(x+y)}$
(D) $\frac{1}{\sqrt{1-x^2}}$ (E) $\frac{1-\sin x}{\sin y}$

45. $\sin x - \cos y - 2 = 0$

- (A) $-\cot x$ (B) $-\cot y$ (C) $\frac{\cos x}{\sin y}$
(D) $-\csc y \cos x$ (E) $\frac{2-\cos x}{\sin y}$

46. $3x^2 - 2xy + 5y^2 = 1$

- (A) $\frac{3x+y}{x-5y}$ (B) $\frac{y-3x}{5y-x}$ (C) $3x+5y$
(D) $\frac{3x+4y}{x}$ (E) none of these

49. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

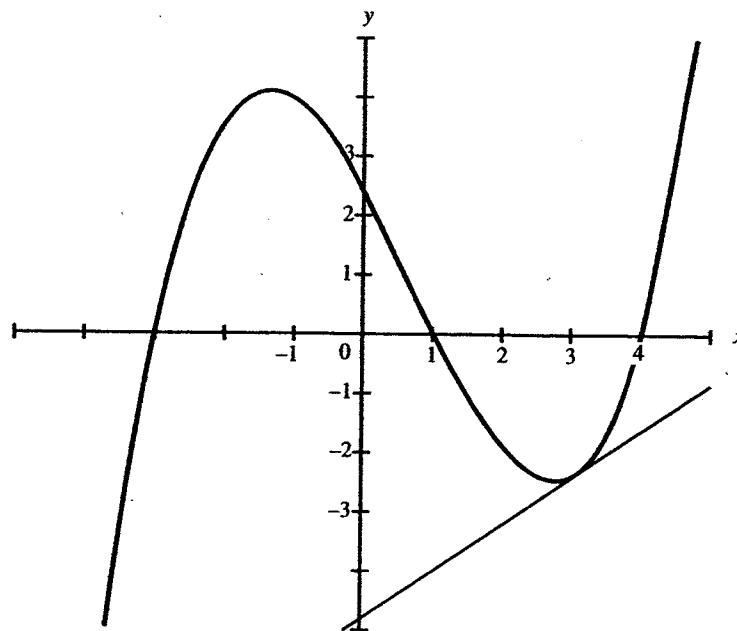
- (A) -32 (B) -16 (C) -4 (D) -2 (E) $-\frac{1}{2}$

51. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

- (A) 0 (B) $\frac{1}{5}$ (C) -5 (D) $-\frac{1}{5}$ (E) nonexistent

Chapter 3: Differentiation

Use this graph of $y = f(x)$ for Questions 76 and 77.



76. $f'(3)$ is most closely approximated by
(A) 0.3 (B) 0.8 (C) 1.5 (D) 1.8 (E) 2
77. The rate of change of $f(x)$ is least at $x \approx$
(A) -3 (B) -1.3 (C) 0 (D) 0.7 (E) 2.7