

AP[®] CALCULUS AB
2010 SCORING GUIDELINES

Question 6

R

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

- (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.
- (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.
- (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

R

5. 1993 AB 33

If $dy/dx = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

- (A) $-2/3$ (B) $-1/3$ (C) 0 (D) $1/3$ (E) $2/3$

F

6. 1993 AB 42

A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

- (A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds
(D) 5.6 pounds (E) 6.5 pounds

R

7. 1993 BC 13

If $dy/dx = x^2y$, then y could be

- (A) $3 \ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

R = Thursday
F = Friday

1

2020

10. 1997 BC 83 (calculator allowed)

If $dy/dx = (1 + \ln x)y$ and if $y = 1$ when $x = 1$, then $y =$

- (A) $e^{\frac{x^2-1}{x^2}}$ (B) $1 + \ln x$ (C) $\ln x$ (D) $e^{2x + x \ln x - 2}$ (E) $e^{x \ln x}$

F 11. 1998 AB 21

If $dy/dt = ky$ and k is a nonzero constant, then y could be

- (A) $2e^{ky}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $ky + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

F 12. 1998 AB 84 (calculator allowed)

Population y grows according to the equation $dy/dt = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

- (A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

V 13. 1988 BC 8

If $dy/dx = \sin x \cos^2 x$ and if $y = 0$ when $x = \pi/2$, what is the value of y when $x = 0$?

- (A) -1 (B) -1/3 (C) 0 (D) 1/3 (E) 1

F 1992 AB 6

At time t , $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius.

At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with radius r is $V = (4/3)\pi r^3$.)

- (a) Find the radius of the sphere as a function of t .
- (b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

1991 BC 6

A certain rumor spreads through a community at the rate $dy/dt = 2y(1 - y)$, where y is the proportion of the population that has heard the rumor at time t .

- (a) What proportion of the population has heard the rumor when it is spreading the fastest?
- (b) If at time $t = 0$ ten percent of the people have heard the rumor, find y as a function of t .
- (c) At what time t is the rumor spreading the fastest?

1989 AB 6

Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $dy/dt = ky$, where y is the amount of oil left in the well at any time t . Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

- (a) Write an equation for y , the amount of oil remaining in the well at any time t .
- (b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
- (c) In order not to lose money, at what time t should oil no longer be pumped from the well?

2000

R

6. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

- (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.
- (b) Find the domain and range of the function f found in part (a).

2001

R

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

2003

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2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.
- (a) Find $f''(3)$.
- (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.