Derivatives of inverse functions

To find the derivative of f^{-1} *at the point* (p,q) *we find the reciprocal of the derivative of* f *at the point* (q,p)

$$\left.\frac{df^{-1}}{dx}\right|_p = \frac{1}{\left.\frac{df}{dx}\right|_q}$$

Find $(f^{-1})'(a)$ *for the function* f *at* a:

1. $f(x) = x^3 + 2x - 1 \qquad a = 2$

2. $f(x) = \frac{1}{27}(x^5 + 2x^3) \qquad a = -11$

3. $f(x) = \cos 2x \qquad 0 \le x \le \frac{\pi}{2} \quad a = 1$

4. $f(x) = \sqrt{x-4} \qquad a = 2$

<u>Answers</u>: 1) 1/5 2) 1/17 3) 1/0 = undefined 4) 4

AP Calculus AB - **Derivatives of Inverse Trigonometric Functions**

Know the following Theorems.

$\frac{d}{dx}\left[\sin^{-1}\boxed{u}\right]=\frac{1}{\sqrt{1-\boxed{u}^2}}\cdot\frac{d\boxed{u}}{dx}$	$\frac{d}{dx}\left[\tan^{-1}\boxed{u}\right]=\frac{1}{1+\boxed{u}^2}\cdot\frac{d\boxed{u}}{dx}$	$\frac{d}{dx}\left[\sec^{-1}\boxed{u}\right]=\frac{1}{\left\lvert\boxed{u}\right\rvert\sqrt{\boxed{u}^2-1}}\cdot\frac{d\boxed{u}}{dx}$
$\frac{d}{dx}\left[\cos^{-1}\boxed{u}\right]=\frac{-1}{\sqrt{1-\boxed{u}^2}}\cdot\frac{d\boxed{u}}{dx}$	$\frac{d}{dx}\left[\cot^{-1}\boxed{u}\right]=\frac{-1}{1+\boxed{u}^2}\cdot\frac{d\boxed{u}}{dx}$	$\frac{d}{dx}\left[\csc^{-1}\boxed{u}\right]=\frac{-1}{\left\lvert\boxed{u}\right\rvert\sqrt{\boxed{u}^2-1}}\cdot\frac{d\boxed{u}}{dx}$

Find the derivative of y with respect to the appropriate variable.

1. $y=\cos^{-1}\left(x^2\right)$
2. $y=\sin^{-1}\sqrt{2t}$
3. $y=\sin^{-1}\frac{3}{t^2}$
4. $y=x\sin^{-1}x+\sqrt{1-x^2}$
5. $y=\sec^{-1}5s$
6. $y=\cot^{-1}\sqrt{t-1}$

7. Which of the following is $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right)$?

A) $-\frac{2}{\sqrt{4-x^2}}$ B) $-\frac{1}{\sqrt{4-x^2}}$ C) $\frac{2}{4+x^2}$ D) $\frac{2}{\sqrt{4-x^2}}$ E) $\frac{1}{\sqrt{4-x^2}}$

Answers

1. $y'=-\frac{2x}{\sqrt{1-x^4}}$
2. $y'=\frac{1}{\sqrt{2t}\sqrt{1-2t}}$
3. $y'=-\frac{6}{t\sqrt{t^4-9}}$
4. $y'=\sin^{-1}x$
5. $y'=\frac{1}{|s|\sqrt{25s^2-1}}$
6. $y'=-\frac{1}{2t\sqrt{t-1}}$
7. E