AP Calculus
2.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the definition of continuity?

2. Sketch a possible graph for each function described.

   a) $f(5)$ exists, but $\lim_{x \to 5} f(x)$ does not exist.

   b) The $\lim_{x \to 5} f(x)$ exists, but $f(5)$ does not exist.

3. If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$, and if f is continuous at $x = 2$, then $k = ?$ Justify your response.

4. Let $f(x) = \begin{cases} x^2 - a^2x & ; x < 2 \\ 4 - 2x^2 & ; x \geq 2 \end{cases}$. Find all values of a that make f continuous at 2. Justify your response.

5. Let f be the function defined by the following:

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous?

6. If the function f is continuous for all real numbers and if $f(x)=\frac{x^2-4}{x+2}$, when $x\neq -2$, then $f(-2)=$

7. Let f be the function given by $f(x)=\frac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers?

A) None B) 1 only C) 2 only D) 4 only E) 1 and 4

8. Let $h(x)=\frac{x^2+5x+6}{x^2+7x+10}$.

a) Find the domain of $g(x)$.

b) Find the $\lim\limits_{x\to c} g(x)$ for all values of c where $g(x)$ is not defined.

c) Find any horizontal asymptotes and justify your response.

d) Find any vertical asymptotes and justify your response.

e) Write an extension to the function so that $g(x)$ is continuous at $x=-2$.

9. Without using a picture, give a written explanation of why the function $f(x)=x^2-4x+3$ has a zero in the interval [2, 4].

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2.3 Worksheet

Solutions

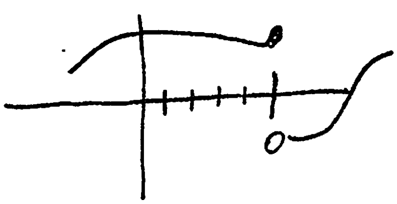
All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the definition of continuity?

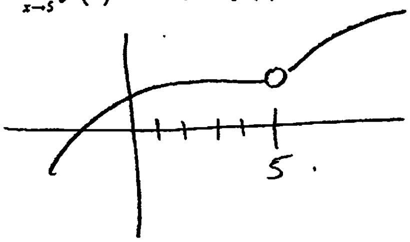
$f(x)$ is continuous at $x=a$ if $\lim\limits_{x\to a} f(x)=f(a)$.

2. Sketch a possible graph for each function described.

a) $f(5)$ exists, but $\lim\limits_{x\to 5} f(x)$ does not exist.



b) The $\lim\limits_{x\to 5} f(x)$ exists, but $f(5)$ does not exist.



3. If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & \text{if } x\neq 2\\ k & \text{if } x=2\end{cases}$, and if f is continuous at $x=2$, then $k=$? Justify your response.

$$\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}=\lim_{x\to 2}\frac{(\sqrt{2x+5}-\sqrt{x+7})(\sqrt{2x+5}+\sqrt{x+7})}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$=\lim_{x\to 2}\frac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}=\lim_{x\to 2}\frac{\cancel{x-2}}{(\cancel{x-2})(\sqrt{2x+5}+\sqrt{x+7})}$$

$$=\frac{1}{\sqrt{9}+\sqrt{9}}=\frac{1}{6}.$$

Make $\boxed{K=1/6}$

4. Let $f(x)=\begin{cases}x^2-a^2x & ;x<2\\ 4-2x^2 & ;x\geq 2\end{cases}$. Find all values of a that make f continuous at 2. Justify your response.

$$2^2-a^2(2)=4-2(2)^2$$

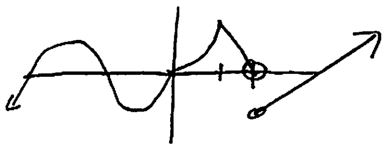
$$4-2a^2=4-8$$

$$-2a^2=-8$$

$$a^2=4$$

$$\boxed{a=\pm 2}$$

5. Let f be the function defined by the following:



$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

$\sin(0)=0$, $(0)^2=0$ > cont @ x

$1^2=1$, $2-1=1$ > cont @ $x=1$

$2-2=0$, $2-3=-1$ > Not cont @ $x=2$

For what values of x is f NOT continuous?

$f(x)$ is NOT continuous at $x=2$ since $\lim_{x\to 2} f(x)$ DNE.

6. If the function f is continuous for all real numbers and if $f(x)=\frac{x^2-4}{x+2}$, when $x\neq -2$, then $f(-2)=$

$f(x)=\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}=x-2$ $f(-2)=-2-2$
$=\boxed{-4}$

7. Let f be the function given by $f(x)=\frac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers?

[✓ A) None] B) 1 only C) 2 only D) 4 only E) 1 and 4

Even if $a=4$ there are holes at 2 & -2.

8. Let $g(x)=\frac{x^2+5x+6}{x^2+7x+10}$. $\frac{(x+3)(x+2)}{(x+5)(x+2)}$

a) Find the domain of $g(x)$. $\mathbb{R}$ $x\neq -5, -2$

b) Find the $\lim_{x\to c} g(x)$ for all values of c where $g(x)$ is not defined.

$\lim_{x\to -5} g(x)=$ DNE $\lim_{x\to -2} g(x)=\frac{-2+3}{-2+5}=\boxed{\frac{1}{3}}$

c) Find any horizontal asymptotes and justify your response.

$\lim_{x\to\pm\infty}\frac{x^2+5x+6}{x^2+7x+10}=\boxed{1}$ $y=1$

d) Find any vertical asymptotes and justify your response.

hole @ $x=-2$ (Removable Disc) VA @ $x=-5$

e) Write an extension to the function so that $g(x)$ is continuous at $x=-2$.

$g(x)=\frac{x+3}{x+5}$ OR $\longrightarrow$ $g(x)=\begin{cases}\frac{x^2+5x+6}{x^2+7x+10}, & x\neq -\\ 1/3, & x=-2\end{cases}$

9. Without using a picture, give a written explanation of why the function $f(x)=x^2-4x+3$ has a zero in the interval [2, 4].

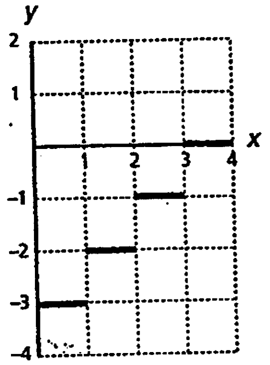
$f(2)=-1$
$f(4)=3$

Since $f(x)$ is a continuous function on the interval $[2,4]$ and $f(2)=-1$ and $f(4)=3$, and $-1<0<3$, by the Intermediate value theorem there must exist a c in $[2,4]$ such that $f(c)=0$.

Calc Ch1

5. Estimate the limit, if it exists: $\lim_{x \to 3} f(x)$, where f(x) is represented by the given graph:

(A) 0
(B) –1
(C) 3
(D) 1
(E) The limit does not exist.



6. Given the function:

$$f(x) = \begin{cases} \sin 2x, & x \le \pi \\ 2x + k, & x > \pi \end{cases}$$

what value of k will make this piecewise function continuous?

(A) -2π
(B) $-\pi$
(C) 0
(D) π
(E) 2π

7. Find the limit: $\lim_{x \to 0} x\left(e^x + \frac{1}{x}\right)$.

(A) 0
(B) 1
(C) 2
(D) The limit does not exist.
(E) None of these

8. Identify the vertical asymptotes for $f(x) = \frac{x^2 + 3x - 4}{x^2 + x - 2}$.

(A) $x = -2, x = 1$
(B) $x = -2$
(C) $x = 1$
(D) $y = -2, y = 1$
(E) $y = -2$

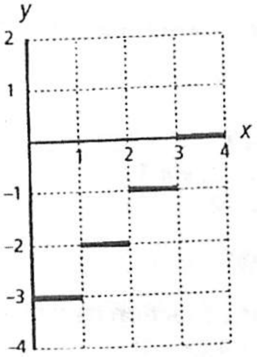
9. If $p(x)$ is a continuous function on the closed interval [1, 3], with $p(1) \le K \le p(3)$ and c is in the closed interval [1, 3], then which of the following statements must be true?

(A) $p(c) = \frac{p(3) + p(1)}{2}$
(B) $p(c) = \frac{p(3) - p(1)}{2}$
(C) There is at least one value c, such that $p(c) = K$.
(D) There is only one value c, such that $p(c) = K$.
(E) $c = 2$

10. How many vertical asymptotes exist for the function $f(x) = \frac{1}{2\sin^2 x - \sin x - 1}$ in the open interval $0 < x < 2\pi$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

5. Estimate the limit, if it exists: $\lim_{x\to 3} f(x)$, where $f(x)$ is represented by the given graph:
(A) 0
(B) –1
(C) 3
(D) 1
(E) The limit does not exist.



ANSWER: (E)

$\lim_{x\to 3^-} f(x) = -1$ and $\lim_{x\to 3^+} f(x) = 0$. Therefore, $\lim_{x\to 3} f(x)$ does not exist.

(*Calculus* 7th ed. pages 68–76 / 8th ed. pages 70–78)

6. Given the function:

$$f(x) = \begin{cases} \sin 2x, & x \le \pi \\ 2x + k, & x > \pi \end{cases}$$

what value of k will make this piecewise function continuous?
(A) -2π
(B) $-\pi$
(C) 0
(D) π
(E) 2π

ANSWER: (A)

For this piecewise function, $\lim_{x\to \pi^+} f(x) = 2\pi + k$ and $\lim_{x\to \pi^-} f(x) = 0$.

Therefore, $2\pi + k = 0$, and $k = -2\pi$.

(*Calculus* 7th ed. pages 68–76 / 8th ed. pages 70–78)

7. Find the limit: $\lim_{x\to 0} x\left(e^x + \frac{1}{x}\right)$.

(A) 0
(B) 1
(C) 2
(D) The limit does not exist.
(E) None of these

ANSWER: (B)

Rewriting $\lim_{x\to 0} x\left(e^x + \frac{1}{x}\right)$, we get $\lim_{x\to 0}\left(xe^x + 1\right) = 0\left(e^0\right) + 1 = 0 + 1 = 1$.

(*Calculus* 7th ed. pages 341–346 / 8th ed. pages 350–355)

8. Identify the vertical asymptotes for $f(x)=\dfrac{x^2+3x-4}{x^2+x-2}$.

(A) $x=-2,\ x=1$
(B) $x=-2$
(C) $x=1$
(D) $y=-2,\ y=1$
(E) $y=-2$

ANSWER: (B)

Simplify the function by canceling the common factor. Thus $f(x)=\dfrac{(x+4)(x-1)}{(x+2)(x-1)}$ becomes $f(x)=\dfrac{(x+4)}{(x+2)},\ x\neq 1$. Since $\lim\limits_{x\to -2^-} f(x)=-\infty$ and $\lim\limits_{x\to -2^+} f(x)=+\infty$, this indicates a vertical asymptote at $x=-2$. The $\lim\limits_{x\to 1} f(x)=\dfrac{5}{3}$ indicates that there is no vertical asymptote at $x=1$. (There is a hole in the graph at $x=1$.) Thus, there is only one vertical asymptote which is located at $x=-2$.

(*Calculus* 7th ed. pages 80–84 / 8th ed. pages 83–87)

9. If $p(x)$ is a continuous function on the closed interval [1, 3], with $p(1)\le K\le p(3)$ and c is in the closed interval [1, 3], then which of the following statements must be true?

(A) $p(c)=\dfrac{p(3)+p(1)}{2}$
(B) $p(c)=\dfrac{p(3)-p(1)}{2}$
(C) There is at least one value c, such that $p(c)=K$.
(D) There is only one value c, such that $p(c)=K$.
(E) $c=2$

ANSWER: (C)

This statement illustrates the Intermediate Value Theorem.

(*Calculus* 7th ed. pages 75–76 / 8th ed. pages 77–78)

10. How many vertical asymptotes exist for the function $f(x)=\dfrac{1}{2\sin^2 x-\sin x-1}$ in the open interval $0<x<2\pi$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

ANSWER: (D)

By factoring

$$f(x)=\frac{1}{2\sin^2 x-\sin x-1}=\frac{1}{(2\sin x+1)(\sin x-1)}.$$