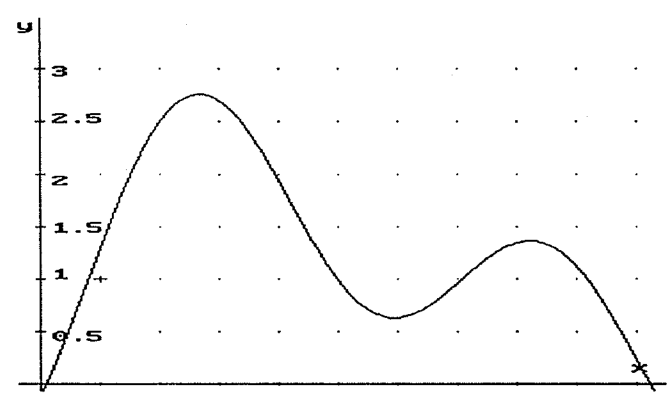
1.1 On the graph below, indicate the following:

1.1.1 A point where the tangent to the curve is horizontal. Draw the tangent line and label it A. What is the slope of the tangent line?

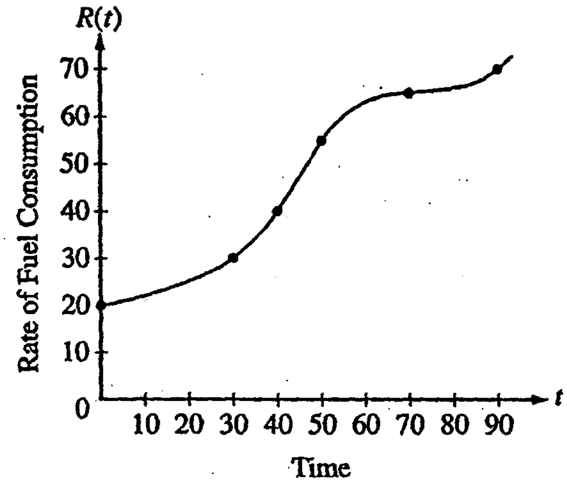
1.1.2 A point on the curve where the tangent line has a positive slope. Draw the tangent line and label it B. Estimate the slope of the tangent line.

1.1.3 A point on the curve where the slope of the tangent line is negative. Draw the tangent line and label it C. Estimate the slope of the tangent line as best you can.



AP CALCULUS AB
AP FREE RESPONSE PROBLEM

APPROX. OF DERIVATIVE



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

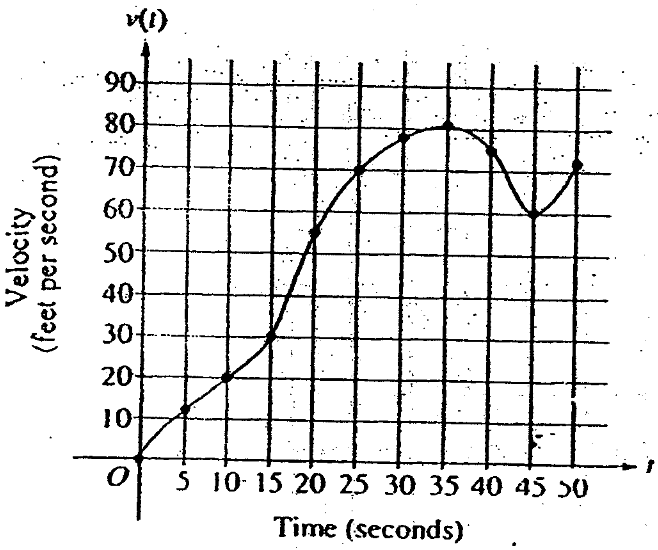
(b) Give three approximations for the slope of R when $t = 70$ minutes.
Indicate units of measure.

(c) When is the rate of fuel consumption greatest?

(d) When is the rate of change of the rate of fuel consumption greatest?

AP CALCULUS AB

QUESTIONS ON DERIVATIVES



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

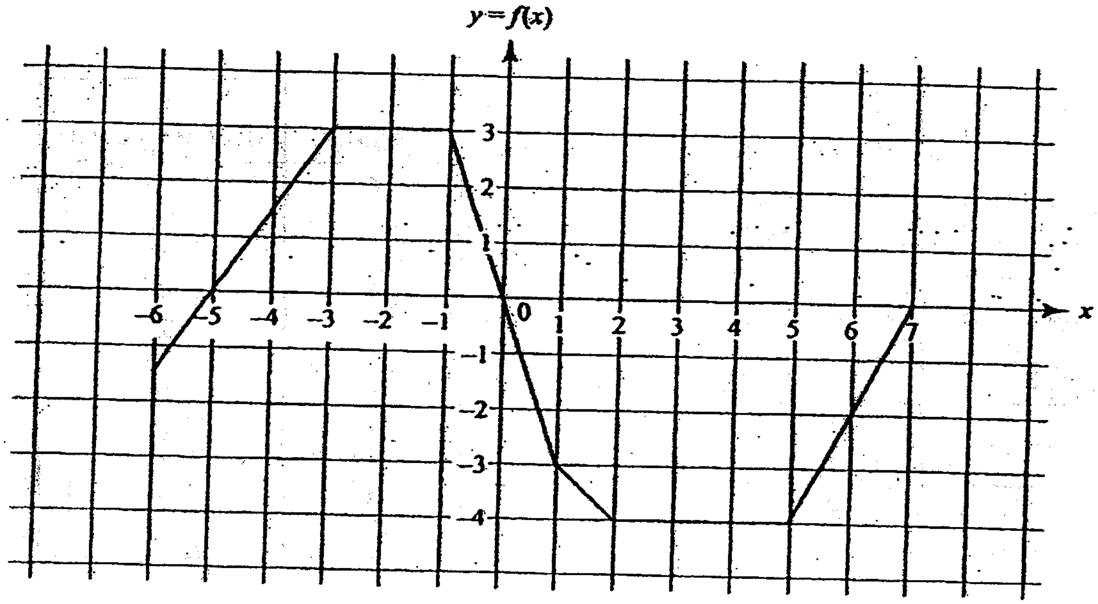
The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

(c) Estimate the acceleration of the car when $t = 30$ seconds.

Questions are based on the following graph of $f(x)$, sketched on $-6 \le x \le 7$. Assume the horizontal and vertical grid lines are equally spaced at unit intervals.



On the interval $1 < x < 2, f(x)$ equals

(A) $-x-2$ (B) $-x-3$ (C) $-x-4$ (D) $-x+2$ (E) $x-2$

Over which of the following intervals does $f'(x)$ equal zero?

I. $(-6,-3)$ II. $(-3,-1)$ III. $(2,5)$

(A) I only (B) II only (C) I and II only
(D) I and III only (E) II and III only

How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

(A) none (B) 2 (C) 3 (D) 4 (E) 5

For $-6 < x -3, f'(x)$ equals

(A) $-\frac{3}{2}$ (B) -1 (C) 1 (D) $\frac{3}{2}$ (E) 2

Which of the following statements about the graph of $f'(x)$ is false?

(A) It consists of six horizontal segments.
(B) It has four jump discontinuities.
(C) $f'(x)$ is discontinuous at each x in the set $\{-3,-1,1,2,5\}$.
(D) $f'(x)$ ranges from -3 to 2.
(E) On the interval $-1 < x < 1, f'(x) = -3$.

The previous two examples show how to compute the derivatives of power functions of the form $f(x) = x^n$, when n is 2 or 3. We can use the Binomial Theorem to show the *power rule* for a positive integer n:

$$\text{If } f(x) = x^n \text{ then } f'(x) = nx^{n-1}.$$

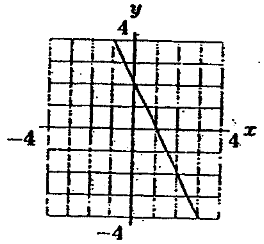
This result is in fact valid for any real value of n.

Exercises and Problems for Section 2.4

Exercises

For Exercises 1–9, sketch a graph of the derivative function of each of the given functions.

1.



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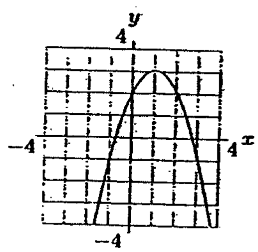

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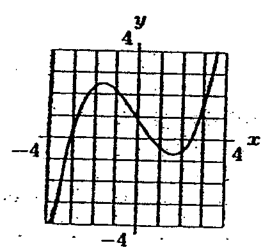

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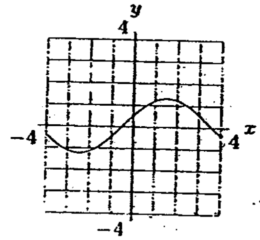

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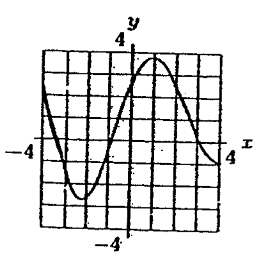

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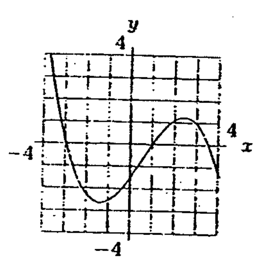

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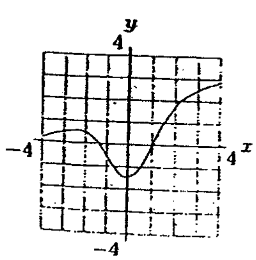

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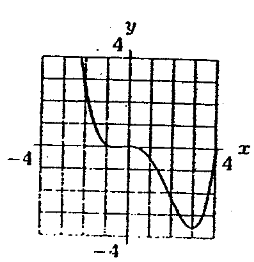
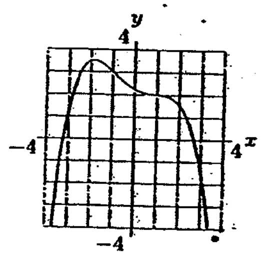


9.



10. For $f(x) = \ln x$, construct tables, rounded to four decimals, near $x = 1$, $x = 2$, $x = 5$, and $x = 10$. Use the tables to estimate $f'(1)$, $f'(2)$, $f'(5)$, and $f'(10)$. Then guess a general formula for $f'(x)$.

11. (a) Estimate $f'(2)$ using the values of f in the table.
(b) For what values of x does $f'(x)$ appear to be positive? Negative?

x	0	2	4	6	8	10	12
$f(x)$	10	18	24	21	20	18	15

12. Find approximate values for $f'(x)$ at each of the x-values given in the following table.

x	0	5	10	15	20
$f(x)$	100	70	55	46	40

In Exercises 13–14, find a formula for the derivative using the power rule. Confirm it using difference quotients.

13. $k(x) = 1/x$

14. $l(x) = 1/x^2$

Find a formula for the derivatives of the functions in Exercises 15–16 using difference quotients.

15. $g(x) = 2x^2 - 3$

16. $m(x) = 1/(x + 1)$

For Exercises 17–20, sketch the graph of $f(x)$, and use this graph to sketch the graph of $f'(x)$.

17. $f(x) = x^2$

18. $f(x) = x(x - 1)$

19. $f(x) = \cos x$

20. $f(x) = \log x$

(a) Sketch the graph of a function whose derivative is positive and increasing for all x.

(b) Sketch the graph of a function whose derivative is negative for $x < 2$, equals zero when $x = 2$, and is positive for $x > 2$. Also, the function has roots at $x = -1$ and $x = 3$.

c) Sketch the graph of a function which has a constant positive derivative for $x < 0$, a constant negative derivative for $0 < x < 2$, and a derivative that is positive and decreasing for $x > 2$. The function is not differentiable at $x = 0$ and $x = 2$.

(d) Sketch the graph of a function with a derivative that is negative and increasing for all x. The function has one root at $x = -1$.