Name: ________________ **Period:** _____

Date: ________________ **AP Calc AB**

Mr. Mellina

Chapter 5: Applications of Derivatives Part 2

Sections:

- ❖ *5.3 Cont.: Connecting f' and f'' with the Graph of f*
- ❖ *5.4 Modeling and Optimization*
- ❖ *5.3 Cont.: PVA*

HW Sets

Set A (Section 5.3) *Worksheet #93: #'s 1 & 2. 2005 & 1989 AP problems.*

Set B (Section 5.3) *Worksheet #93: #'s 5-7 & Drawing Graphs of Functions Worksheet.*

Set C (Section 5.3) *AP Practice (1991/2006), MC #'s 12-30.*

Set D (Section 5.4) *Optimization Worksheet #'s 1-8.*

Set E (Section 5.4) *Page 231, #1, 6, 9, 13, 14, 16.*

Set F (Section 5.3) *5.3 PVA Practice #'s 1-6.*

Set G (Section 5.3) *5.3 PVA Practice #'s 12-14.*

5.3 Cont. Connecting f' and f'' with the Graph of f

Topics

- *First Derivative Test for Local Extrema*
- *Concavity*
- *Points of Inflection*
- *Second Derivative Test for Local Extrema*
- *Learning about Functions from Derivatives*

Warm Up!

Find all points of inflection of the graph of $f(x) = e^{-x^2}$.

Formulas to Remember

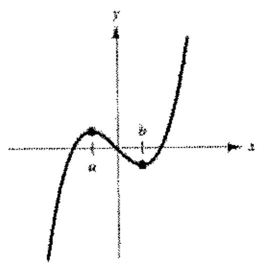
Let $y = f(x)$ be defined on an open interval I containing c.

- f is increasing on I if $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
- f is decreasing on I if $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
- If $f'(x) > 0$ then f is increasing on I.
- If $f'(x) < 0$ then f is decreasing on I.
- If f' changes from negative to positive at c, there is a relative minimum at c.
- If f' changes from positive to negative at c, there is a relative maximum at c.
- f is concave upwards on I if f' is increasing on I.
- f is concave downwards on I if f' is decreasing on I.
- If $f'' > 0$ on I then f is concave upwards on I.
- If $f'' < 0$ on I then f is concave downwards on I.
- The point $(c, f(c))$ is a point of inflection if the concavity changes there.

Connecting Graphs

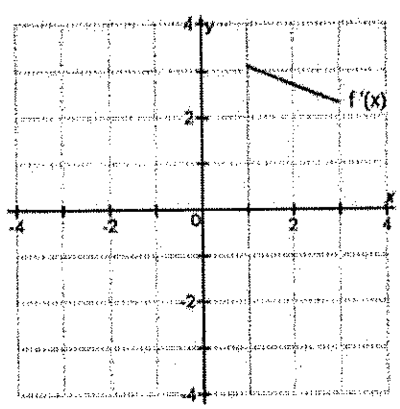
The first and second derivatives of a function provide an enormous amount of useful information about the shape of the graph of the function, as indicated by the properties above. An important skill to develop is that of producing the graph of the derivative of a function, given the graph of the function. Conversely, it is important to be able to produce the graph of a function given the graph of its derivative.

For instance, consider the graph of f below. Because $f'(x) > 0$ on the interval $(-\infty, a)$, f is increasing on that interval. Furthermore $f'(x)$ is decreasing near $x = a$, which implies that the graph of f is concave downwards near $x = a$.



Example 1: Multiple Choice

1 Which statement is true about the graph of $f(x)$ for the piece shown?



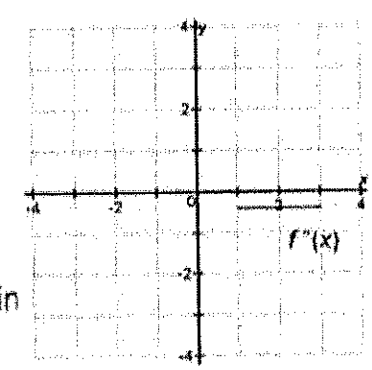
A $f(x)$ is increasing

B $f(x)$ is decreasing

C $f(x)$ is concave up

D $f(x)$ has a local minimu at $x = 2$

2 Which statement is true about the graph of $f(x)$ for the piece shown?



A $f(x)$ is concave up

B $f(x)$ is concave down

C $f(x)$ is increasing

D $f(x)$ has an inflection poin at $x = 2$

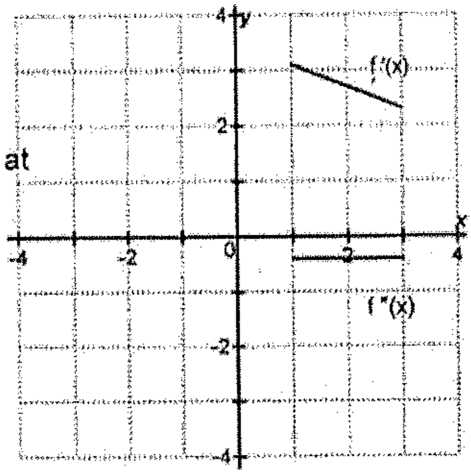
3 Which statement is false about the graph of $f(x)$ over [1, 3] ?

A $f(x)$ is increasing

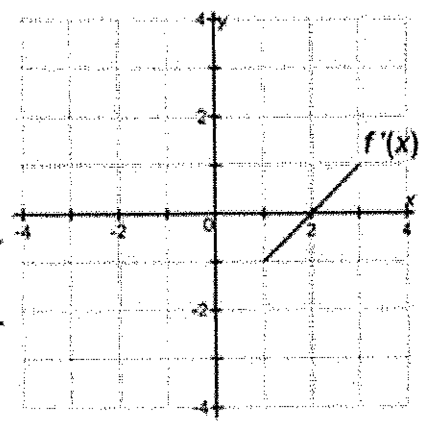
B $f(x)$ is concave down

C $f(x)$ has a local maximum at $x = 3$

D $f(x)$ has no inflection point in the interval



4 Which statement is true about the graph of $f(x)$ for the piece shown?



A $f(x)$ is concave down

B $f(x)$ is decreasing

C $f(x)$ has a local maximum at $x = 2$

D $f(x)$ has a local minimum at $x = 2$

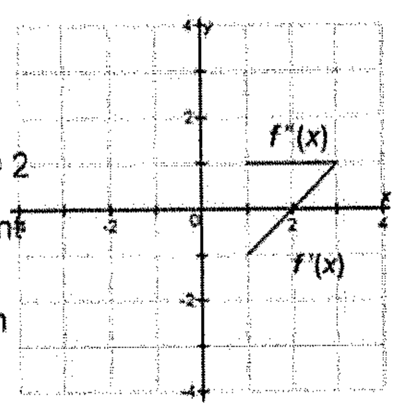
5 Which statement is false about the graph of $f(x)$ over [1, 3] ?

A $f(x)$ is concave up

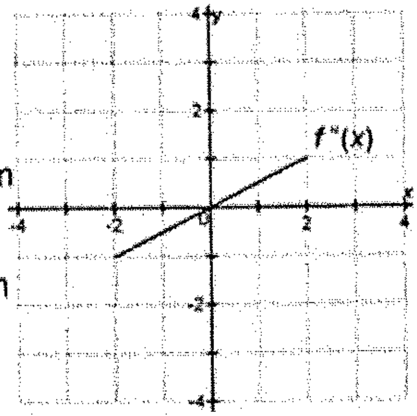
B $f'(x)$ changes sign at $x = 2$

C $f(x)$ has an inflection point at $x = 2$

D $f(x)$ has a local minimum at $x = 2$



6 Which statement is true about the graph of $f(x)$ for the piece shown?



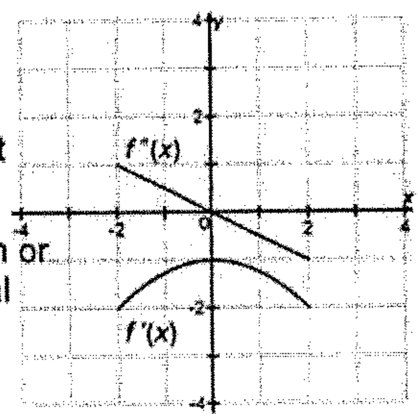
A $f(x)$ has an inflection poi at $x = 0$

B $f(x)$ has a local maximum at $x = 0$

C $f(x)$ has a local minimum at $x = 0$

D $f(x)$ is concave up

7 Which statement is false about the graph of $f(x)$ over [–2, 2] ?



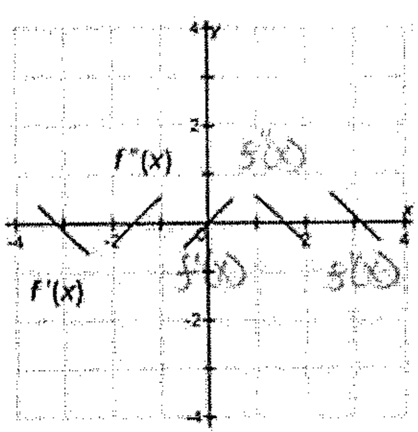
A $f(x)$ is decreasing

B $f(x)$ has an inflection point at $x = 0$

C $f(x)$ has no local minimum or maximum over the interval

D $f(x)$ is increasing

8 Which statement about the graph of $f(x)$ is true?



A $f(x)$ has two inflection points at about $x = -1.5, 1.5$

B $f(x)$ has three inflection poi at about $x = -3.1, 0, 3.1$

C $f(x)$ has local maxima at about $x = -1.5, 1.5$

D $f(x)$ has two local minima at about $x = -1.5, 1.5$

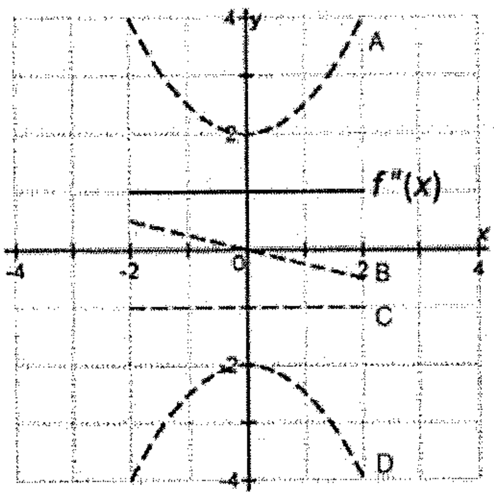
9 If $f''(x)$ is as shown, which line could be $f(x)$?

A line A

B line B

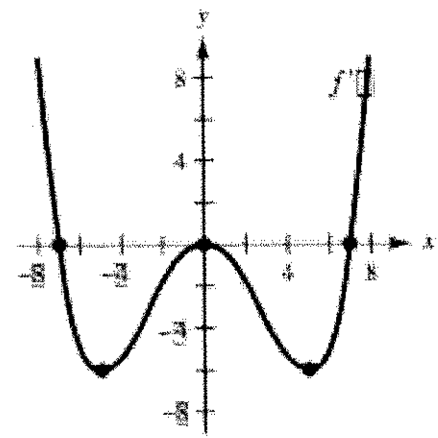
C line C

D line D



Example 2: Connecting Graphs

Consider the graph of f', the derivative of $y = f(x)$ defined on the domain $-9 < x < 9$.



a. For what values of *x* does *f* have a relative minimum?

b. For what values of *x* does *f* have a relative maximum?

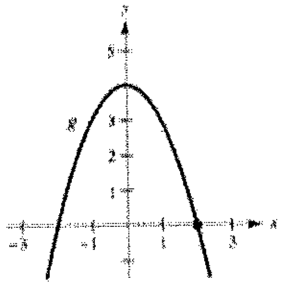
c. Determine the open intervals where the graph of *f* is concave downwards. Show the analysis that leads to your conclusion.

d. Sketch the graph of *f* on the interval $(-9, 9)$ if $f(0) = 0$. Show the analysis that leads to your conclusion.

Example 3: Multiple Choice Practice

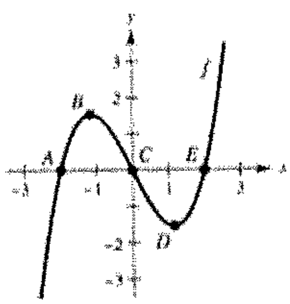
1. Given the graph of $y = g(x)$, estimate the value of $g'(2)$.

(a) -4 (b) -1 (c) 0 (d) 1 (e) 4



2. At which point *A, B, C, D,* or *E* on the graph of $y = f(x)$ are both y' and y'' positive?

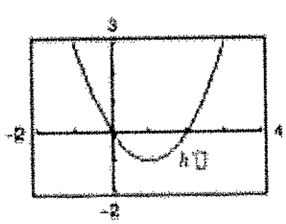
(a) A (b) B (c) C (d) D (e) E



3. Given the graph of $h'(x)$, which of the following statements are true about the graph of h?

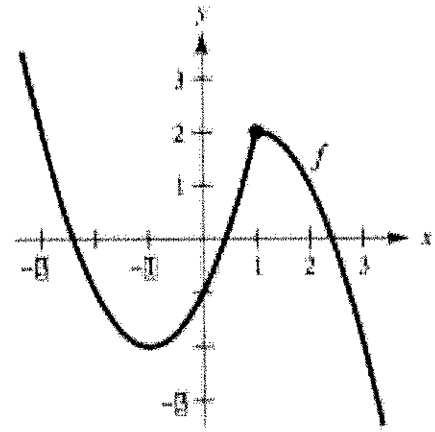
I. The graph of h has a point of inflection at $x = 1$.
II. The graph of h has a relative extremum at $x = 0$.
III. The graph of h has a relative extremum at $x = 1$.

(a) I only (b) II only (c) III only (d) I and II only (e) I and III only



Example 4: Free Response Practice

The graph of the function *f* is shown in the figure.



a. Estimate $f'(0)$.

b. On what open intervals is *f* increasing?

c. On what open intervals is *f* concave downwards?

d. What are the critical numbers of *f*?

e. Sketch the graph of f'.

AP Practice 2005 AP FRQ #4

6. Let f be a function that is continuous on the interval [0, 4]. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

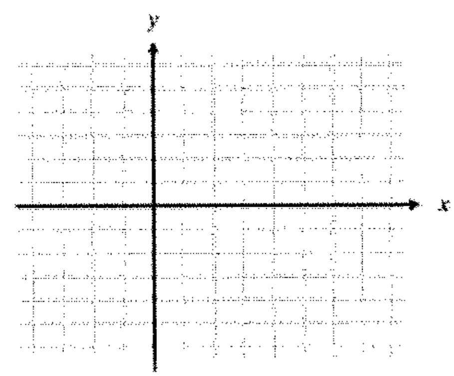
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

a) Describe the behavior of $f(x)$ in each interval using the information above.

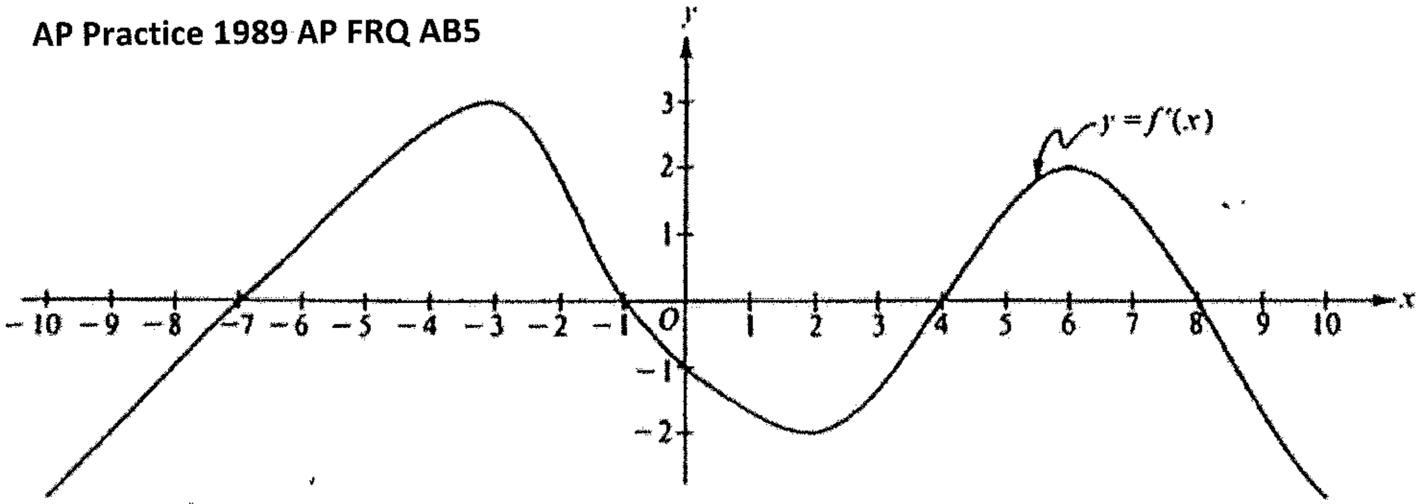
x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$	$3 < x < 4$
$f(x)$				

b) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

c) On the axes provided, sketch the graph of a function that has all the characteristics of f.



AP Practice 1989 AP FRQ AB5



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \leq x \leq 10$.

a. For what values of x does the graph of f have a horizontal tangent?

b. For what values of x in the interval (-10, 10) does f have a relative maximum? Justify your answer.

c. For what values of x is the graph of f concave downward?

d. Draw the graph of f and f'.

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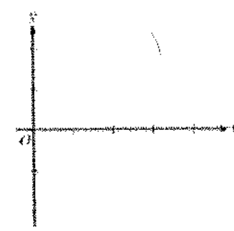
Question 4

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.
(Note: Use the axes provided in the pink test booklet.)



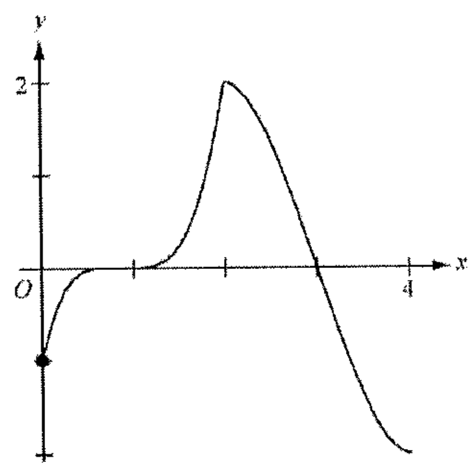
(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

(a) f has a relative maximum at $x = 2$ because f' changes from positive to negative at $x = 2$.

$2 : \begin{cases} 1 : \text{relative extremum at } x = 2 \\ 1 : \text{relative maximum with justification} \end{cases}$

(b)



$2 : \begin{cases} 1 : \text{points at } x = 0, 1, 2, 3 \\ \quad \text{and behavior at } (2, 2) \\ 1 : \text{appropriate increasing/decreasing} \\ \quad \text{and concavity behavior} \end{cases}$

(c) $g'(x) = f(x) = 0$ at $x = 1, 3$.
g' changes from negative to positive at $x = 1$ so g has a relative minimum at $x = 1$. g' changes from positive to negative at $x = 3$ so g has a relative maximum at $x = 3$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{critical points} \\ 1 : \text{answer with justification} \end{cases}$

(d) The graph of g has a point of inflection at $x = 2$ because $g'' = f'$ changes sign at $x = 2$.

$2 : \begin{cases} 1 : x = 2 \\ 1 : \text{answer with justification} \end{cases}$

1989 AB5
Solution

(a) horizontal tangent $\Leftrightarrow f'(x)=0$

$x=-7,-1,4,8$

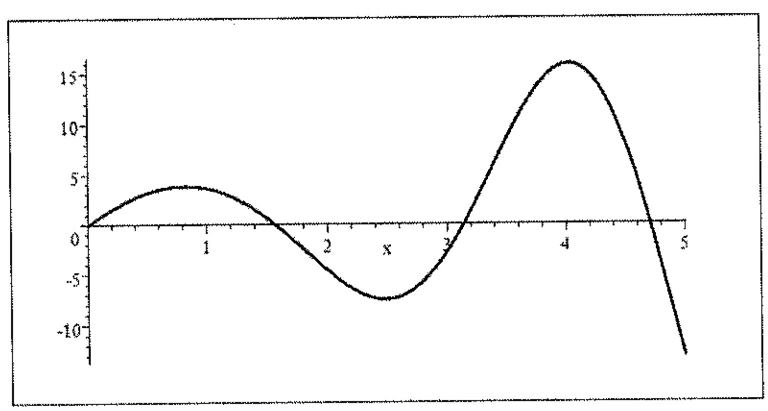
(b) Relative maxima at $x=-1,\ 8$ because f' changes from positive to negative at these points

(c) f concave downward $\Leftrightarrow f'$ decreasing

$(-3,2),(6,10)$

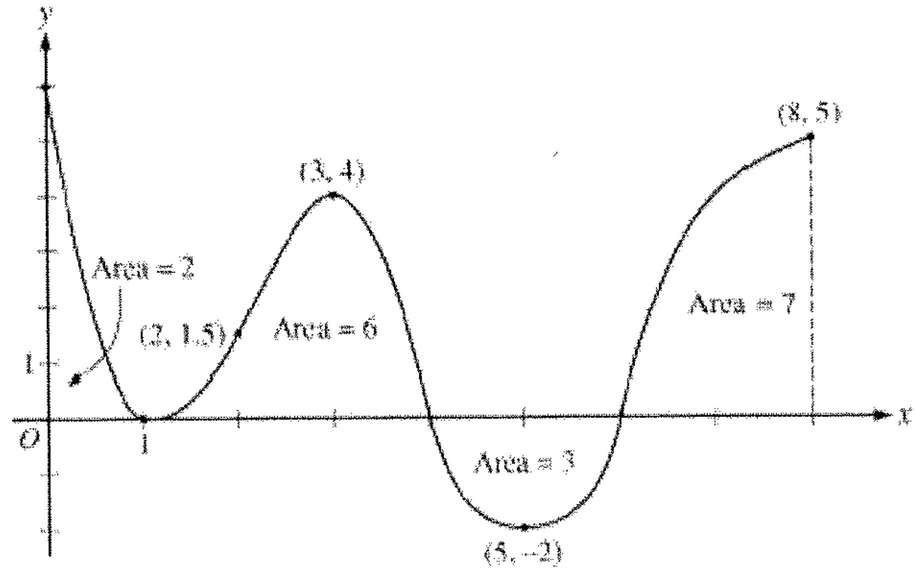
Example 5
Let the graph of $f'(x)$ be given below. Find

a. The x-coordinate of each inflection point of *f*.



$f'(x)$

b. Where the graph of *f* is concave up and is concave down.



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

2013 AP AB FRQ #4 Scoring Rubric

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.	1 : answer with justification
(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint. $f(0) = f(8) + \int_8^0 f'(x)\,dx$ $= f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$ $f(6) = f(8) + \int_8^6 f'(x)\,dx$ $= f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$ $f(8) = 4$ The absolute minimum value of f on the closed interval $[0, 8]$ is -8.	3 : { 1 : considers $x = 0$ and $x = 6$ 1 : answer 1 : justification
(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.	2 : { 1 : answer 1 : explanation
(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$ $g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$	3 : { 2 : $g'(x)$ 1 : answer

Exercises and Problems for Section 2.6

Exercises

1. For the function graphed in Figure 2.52, are the following quantities positive or negative?

(a) $f(2)$ (b) $f'(2)$ (c) $f''(2)$

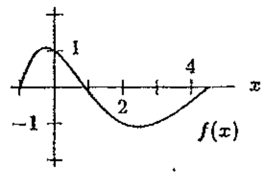
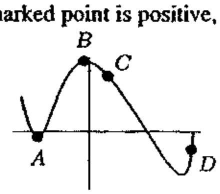


Figure 2.52

2. The graph of a function $f(x)$ is shown in Figure 2.53. On a copy of the table indicate whether f, f', f'' at each marked point is positive, negative, or zero.



Point	f	f'	f''
A			
B			
C			
D			

Figure 2.53

3. At which of the labeled points on the graph in Figure 2.54 are both dy/dx and d^2y/dx^2 positive?

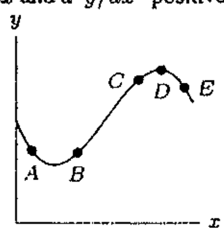


Figure 2.54

4. The distance of a car from its initial position t minutes after setting out is given by $s(t) = 5t^2 + 3$ kilometers. What are the car's velocity and acceleration at time t? Give units.

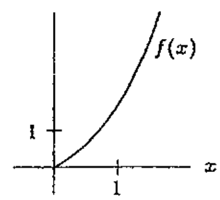
5. Sketch the graph of a function whose first and second derivatives are everywhere positive.

6. Sketch the graph of a function whose first derivative is everywhere negative and whose second derivative is positive for some x-values and negative for other x-values.

7. Sketch the graph of the height of a particle against time if velocity is positive and acceleration is negative.

For Exercises 8–13, give the signs of the first and second derivatives for each of the following functions.

8.



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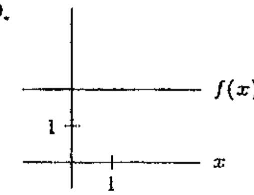

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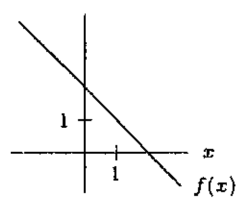

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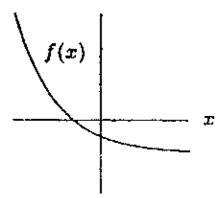

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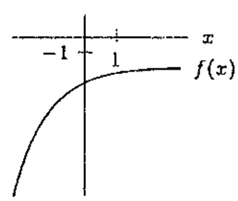
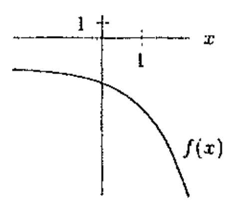


13.



Problems

14. The table gives the number of passenger cars, $C = f(t)$, in millions, in the US in the year t.

(a) Do $f'(t)$ and $f''(t)$ appear to be positive or negative during the period 1940–1980?

(b) Estimate $f'(1975)$. Using units, interpret your answer in terms of passenger cars.

t (year)	1940	1950	1960	1970	1980
C (cars, in millions)	27.5	40.3	61.7	89.3	121.6

15. An accelerating sports car goes from 0 mph to 60 mph in five seconds. Its velocity is given in the following table, converted from miles per hour to feet per second, so that all time measurements are in seconds. (Note: 1 mph is 22/15 ft/sec.) Find the average acceleration of the car over each of the first two seconds.

Time, t (sec)	0	1	2	3	4	5
Velocity, $v(t)$ (ft/sec)	0	30	52	68	80	88

14a

Drawing Graphs of Functions Worksheet

For each question, sketch a possible graph for $f(x)$ based on the given information. Label all zeros, critical points and inflection points. [hint: write out the sign analysis for f' and f'' before drawing the graph.

1. $f(x)$ is continuous and differentiable at all points
   $f(-3) = f(5) = 0$
   $f' > 0$ for $x < -1$ and $x > 4$
   $f' < 0$ for $-1 < x < 4$
   $f'' < 0$ for $x < 0$
   $f'' > 0$ for $x > 0$

2. $f(x)$ is continuous at all points
   $f(1) = 0$
   $f' > 0$ for all x except at $x = 1$
   f' is undefined at $x = 1$
   $f'' > 0$ for $x < 1$
   $f'' < 0$ for $x > 1$

3. $f(x)$ is a piece-wise function that is continuous at all points
   $f(-1) = f(5) = 0$
   $f' > 0$ for all $x > 3$ except at $x = -1$
   $f' < 0$ for $x > 3$
   f' is undefined at $x = -1$
   $f' = 0$ at $x = 3$
   $f'' = 0$ for $x < -1$
   $f'' < 0$ for $x > -1$

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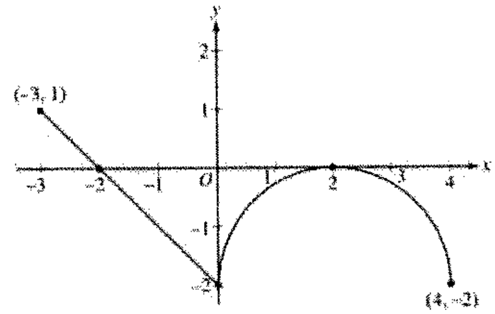
2003 SCORING GUIDELINES

Question 4



Graph of f'

Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0,3)$.

(a) The function f is increasing on $[-3,-2]$ since $f' > 0$ for $-3 \le x < -2$.

$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

(b) $x = 0$ and $x = 2$

f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

$2 : \begin{cases} 1 : x = 0 \text{ and } x = 2 \text{ only} \\ 1 : \text{justification} \end{cases}$

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

1 : equation

Chapter 3 Multiple Choice Practice Worksheet

25. The function $f(x) = x^4 - 4x^2$ has

(A) one relative minimum and two relative maxima
(B) one relative minimum and one relative maximum
(C) two relative maxima and no relative minimum
(D) two relative minima and no relative maximum
(E) two relative minima and one relative maximum

26. The number of inflection points of the curve in Question 25 is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

For Questions 42 and 43, $f'(x) = x \sin x - \cos x$ for $0 < x < 4$. (Calculator)

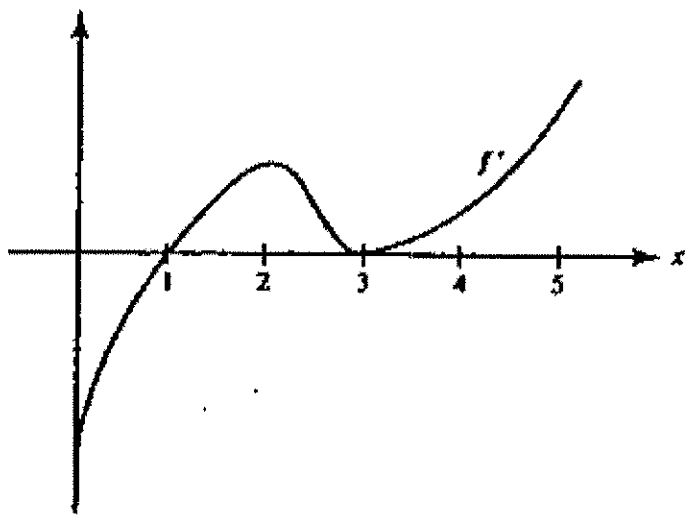
42. f has a local maximum when x is approximately

(A) 0.9 (B) 1.2 (C) 2.3 (D) 3.4 (E) 3.7

43. f has a point of inflection when x is approximately

(A) 0.9 (B) 1.2 (C) 2.3 (D) 3.4 (E) 3.7

Use the graph of f' on [0,5], shown below, for Questions 56 and 57.



56. f has a local minimum at $x =$

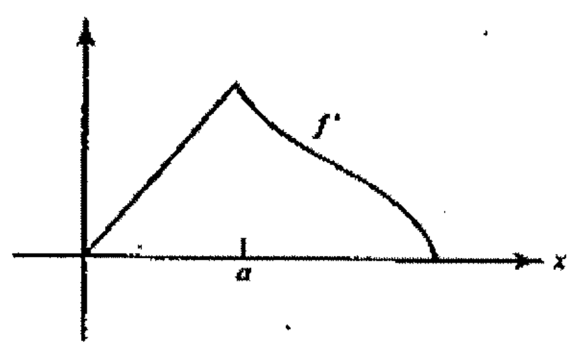
(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

57. f has a point of inflection at $x =$

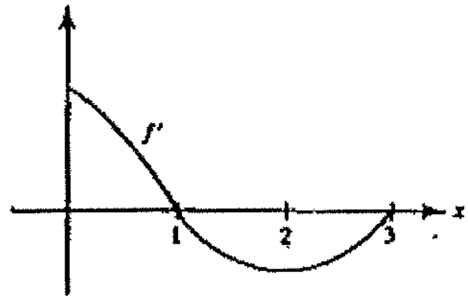
(A) 1 only (B) 2 only (C) 3 only
(D) 2 and 3 only (E) none of these

58. It follows from the graph of f', shown at the right, that

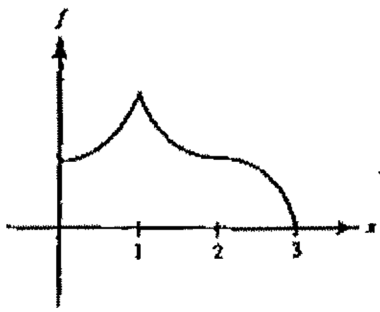
(A) f is not continuous at $x = a$
(B) f is continuous but not differentiable at $x = a$
(C) f has a relative maximum at $x = a$
(D) f has a point of inflection at $x = a$
(E) none of these



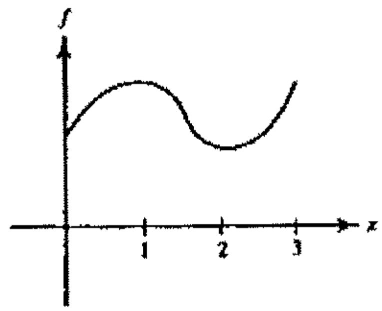
75. Given f' as graphed, which could be the graph of f?



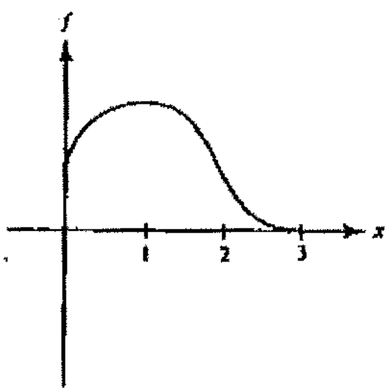
(A)



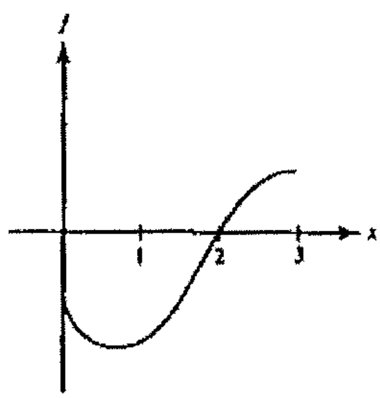
(B)



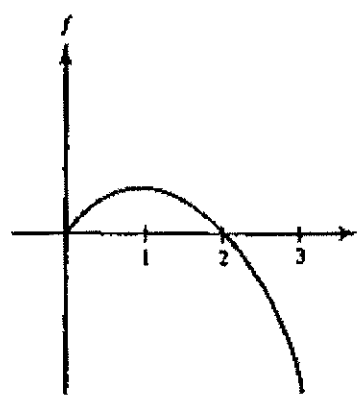
(C)



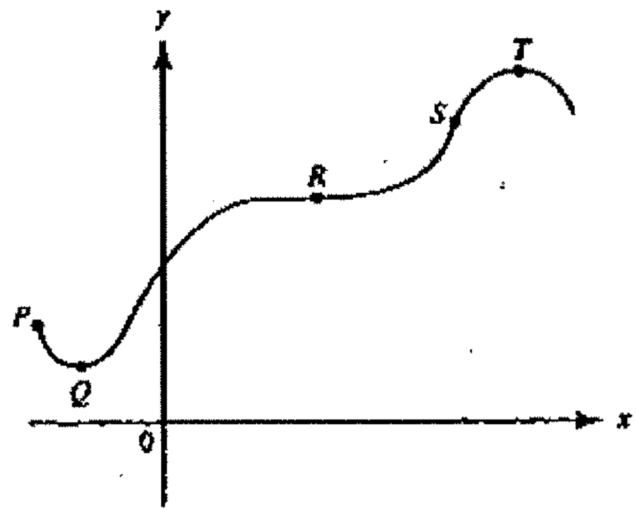
(D)



(E)



Use the following graph for Questions 82–84.



82. At which labeled point do both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ equal zero?

(A) *P* (B) *Q* (C) *R* (D) *S* (E) *T*

83. At which labeled point is $\frac{dy}{dx}$ positive and $\frac{d^2y}{dx^2}$ equal to zero?

(A) *P* (B) *Q* (C) *R* (D) *S* (E) *T*

84. At which labeled point is $\frac{dy}{dx}$ equal to zero and $\frac{d^2y}{dx^2}$ negative?

(A) *P* (B) *Q* (C) *R* (D) *S* (E) *T*

Answers

25. E
26. C

35. B
36. D
37. A
38. E
39. B
40. D
41. A
42. D
43. C

56. B
57. D
~~58. D~~
67. C
68. E
69. C

72. B
73. B
74. D

75. C
92. E

82. C
83. D
84. E

AP Practice (HW)

1991 AB5

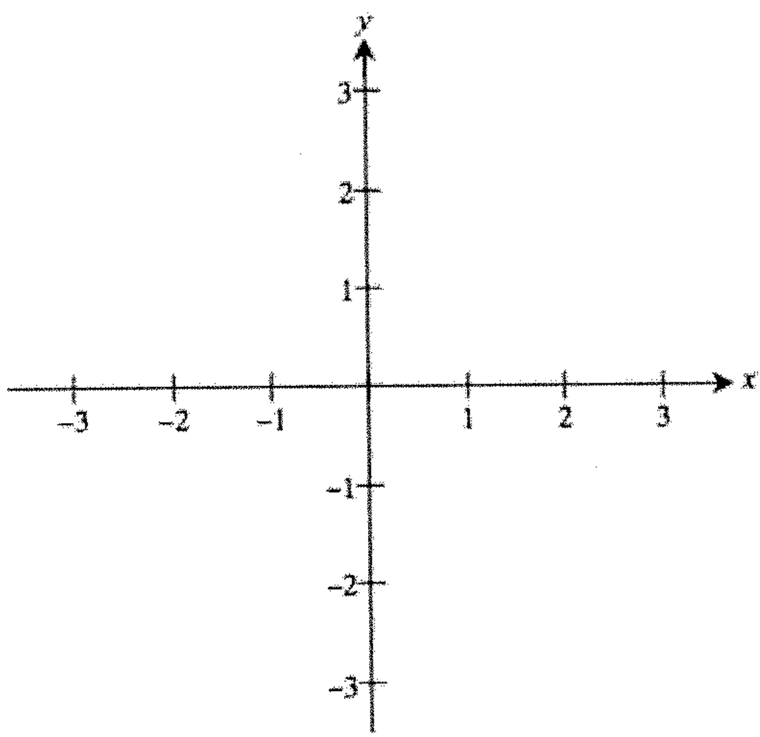
Let f be a function that is even and continuous on the closed interval $[-3,3]$. The function f and its derivatives have the properties indicated in the table below.

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

(a) Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each x-coordinate you give, state whether f attains an absolute maximum or an absolute minimum.

(b) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.

(c) In the xy-plane provided below, sketch the graph of a function with all the given characteristics of f.

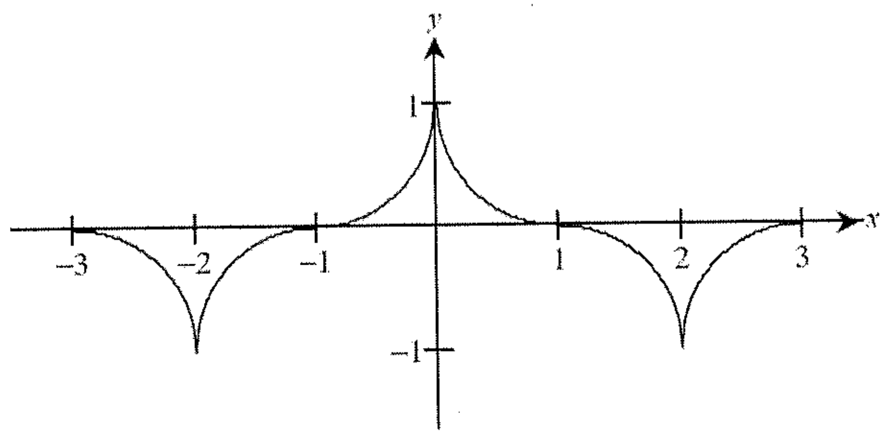


1991 AB5
Solution

(a) Absolute maximum at $x = 0$
Absolute minimum at $x = \pm 2$

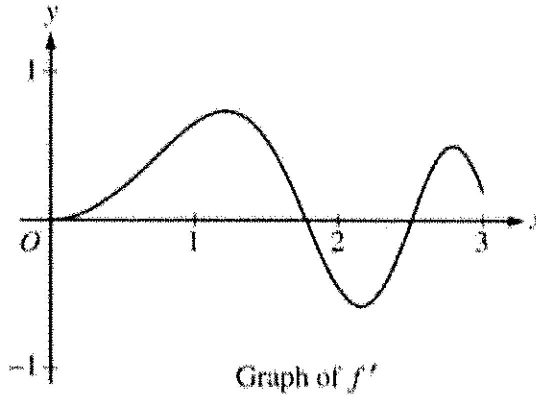
(b) Points of inflection at $x = \pm 1$ because the sign of $f''(x)$ changes at $x = 1$ and f is even

(c)



Question 2

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)}\sin(x^2)$. The graph of $y = f'(x)$ is shown above.



Graph of f'

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

$2: \begin{cases} 1: \text{answer} \\ 1: \text{reason} \end{cases}$

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \ldots$
On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$f(0) = 5$

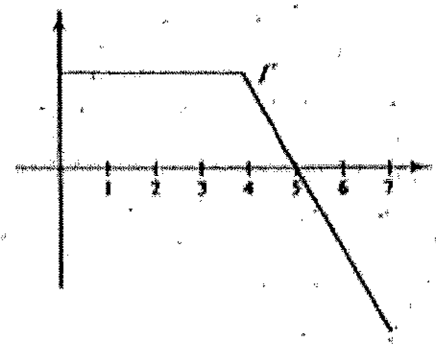
$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x)\,dx = 5.67911$

$f(3) = f(0) + \int_0^3 f'(x)\,dx = 5.57893$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

3 :
- 1 : identifies $\sqrt{\pi}$ and 3 as candidates
  - or -
  indicates that the graph of f increases, decreases, then increases
- 1 : justifies $f(\sqrt{\pi}) > f(3)$
- 1 : answer

Applications of Derivatives Multiple Choice Worksheet



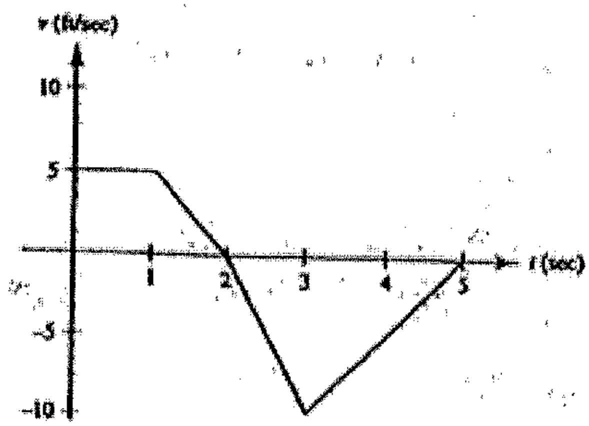
12. Which statement best describes f at $x = 5$?

(A) f has a root. (B) f is a maximum. (C) f is a minimum.
(D) f has a point of inflection. (E) none of these

13. The function is concave downward for which interval?

(A) (0,4) (B) (4,5) (C) (5,7)
(D) (4,7) (E) none of these

Use the graph shown for Questions 18–24. It shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 5$.



18. The object attains its maximum speed when $t =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

19. The speed of the object is increasing during the time interval

(A) (0,1) (B) (1,2) (C) (0,2) (D) (2,3) (E) (3,5)

20. The acceleration of the object is positive during the time interval

(A) (0,1) (B) (1,2) (C) (0,2) (D) (2,3) (E) (3,5)

21. How many times on $0 < t < 5$ is the object's acceleration undefined?

(A) none (B) 1 (C) 2 (D) 3 (E) more than 3

22. During $2 < t < 3$ the object's acceleration (in ft/sec^2) is

(A) −10 (B) −5 (C) 0 (D) 5 (E) 10

23. The object is furthest to the right when $t =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

24. The object's average acceleration (in ft/sec^2) for the interval $0 \leq t \leq 3$ is

(A) −15 (B) −5 (C) −3 (D) −1 (E) none of these

25. The function $f(x) = x^4 - 4x^2$ has

(A) one relative minimum and two relative maxima
(B) one relative minimum and one relative maximum
(C) two relative maxima and no relative minimum
(D) two relative minima and no relative maximum
(E) two relative minima and one relative maximum

26. The number of inflection points of the curve in Question 25 is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

28. The total number of local maximum and minimum points of the function whose derivative, for all x, is given by $f'(x) = x(x-3)^2(x+1)^4$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) none of these

30. On the closed interval $[0, 2\pi]$, the maximum value of the function $f(x) = 4\sin x - 3\cos x$ is

(A) 3 (B) 4 (C) $\frac{24}{5}$ (D) 5 (E) none of these

MC Answers.

12. B
13. D
18. D
19. D
20. E
21. D
22. A
23. C
24. B
25. E
26. C
28. B
30. D

5.4 Modeling and Optimization

Topics

- *Examples from Mathematics*
- *Examples from Business & Industry*
- *Examples from Economics*

Warm Up!

For a, use the First Derivative Test to identify the local extrema of $f(x)$. For b, use the Second Derivative Test to identify the local extrema of $h(x)$.

a. $f(x) = x^3 - 6x^2 + 12x - 8$

b. $h(x) = 2x^3 + 3x^2 - 12x - 3$

Optimization

One of the oldest applications of Differential Calculus was to find maximum and minimum values of functions by finding where horizontal tangent lines might occur. We will use both algebraic and graphical methods in this section to solve "max-min" problems in a variety of contexts, but the emphasis will be on the ***modeling*** *process that both methods have in common.*

Strategy for Solving Max-Min Problems

1. ________________ *the problem. Read the problem carefully. Identify the information you need to solve the problem.*
2. *Develop a Mathematical* __________ *of the problem. Draw pictures and label the parts that are important to the problem. Introduce a variable to represent the quantity to be maximized or minimized. Using that variable, write a function whose extreme value gives the information sought.*
3. __________ *the Function. Find the domain of the function. Determine what values of the variable make sense in the problem.*
4. *Identify the* ____________ *Points and* ______________*. Find where the derivative is zero or fails to exist.*
5. ___________ *the Mathematical Model. If unsure of the result, support or confirm your solution with another method.*
6. _____________ *the Solution. Translate your mathematical result into the problem setting and decide whether the result makes sense.*

______________ *Equation: is what you are trying to maximize or minimize.*
______________ *Equation: an equation that helps you get to 1 variable.*

Example 1: Using the Strategy

Use the information to solve the problem

a. Find two numbers whose sum is 20 and whose product is as large as possible.

b. A rectangle is to be inscribed under one arch of the sine curve. What is the largest area the rectangle can have, and what dimensions give that area?

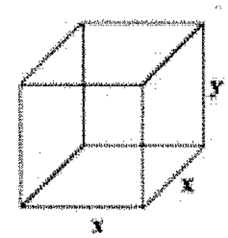
c. Find 2 positive numbers that satisfy the given requirements: The second number is the reciprocal of the first and the sum is a minimum.

d. I have 100 ft of fence to make a rectangular dog pen, what is the maximum area I can construct?

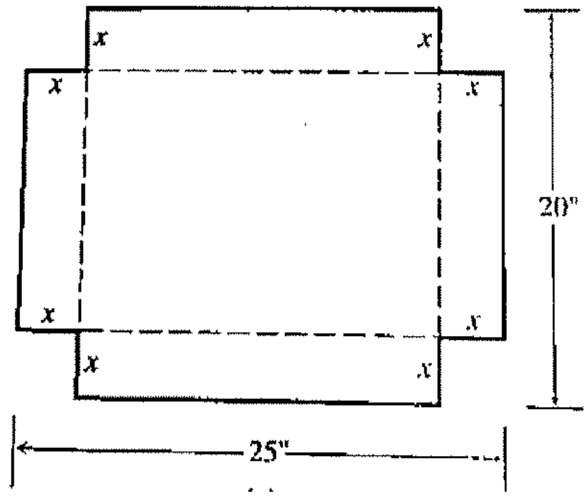
Example 2: Fabricating a box

Use the information to solve

a. A box with a square base with no top has a surface area of 108 square feet. Find the dimensions that will maximize the volume. [Be sure to use calculus]



b. An open-top box is to be made by cutting congruent squares of side length *x* from the corners of a 20- by 25-inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting maximum volume?



Example 3: Graphical Example
Use the information to solve

a. What points on the graph $y = 4 - x^2$ are the closest to the point (0, 2)?

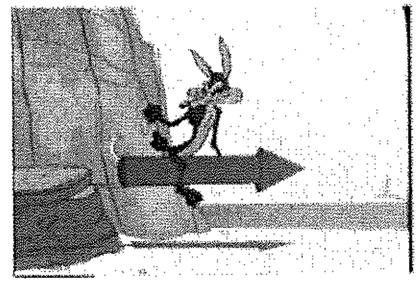
Optimization Worksheet

1.

[Calculator Allowed] Wile E. is after Road Runner again! This time he's got it figured out. Sitting on his ACME rocket he hides behind a hill anxiously awaiting the arrival that "beeping" bird. In his excitement to light the rocket he tips the rocket up. Instead of thrusting himself parallel to the ground where he can catch the Road Runner, he sends himself widely into the air following a path given by function

$$h(t) = .1t^3 - 1.3t^2 + 4.2t + 2,$$

where h is the height of the rocket after t seconds. The rocket fuel lasts for 10 seconds. At that point, Wile E. Coyote stops suddenly and falls straight down to the ground.



a) What is the domain of this function?

b) What is the highest point reached by Wile E. Coyote? [Use Calculus!]

2. Find two positive numbers such that the product is 192 and the sum is a minimum.

3.

[No Calculator] A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$.

a) Draw a sketch the rectangle inscribed under the parabola.

b) Write a formula for the area of the inscribed rectangle as a function of x. What is the domain of this function?

c) What is the largest area the rectangle can have, and what are its dimensions?

4. [Calculator] The Profit P in dollars made by a fast food restaurant selling x hamburgers is given by

$$P = 2.44x - \frac{x^2}{20000} - 5000, \quad 0 \le x \le 35000.$$

a) Find the intervals on which P is increasing or decreasing. [Use Calculus!]

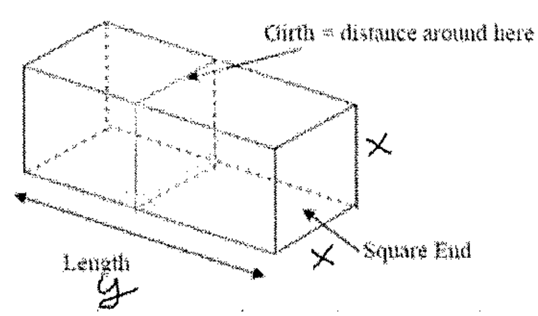
b) Find the maximum profit.

5. A $216 - m^2$ pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

6. Find the length and width of a rectangle that has a perimeter of 64 feet and a maximum area.

7. Suppose you want to build a rectangular pen for your dog using a garage wall on one side and a fence on the other three sides. If you have 40 feet of fencing available, what should be the dimensions of the pen to yield the largest possible area?

8. A rectangular package to be sent by UPS can have a maximum combined length and girth of 300cm. Find the dimensions of the package of maximum volume that can be sent [Assume that the cross section is a square.



9. A rectangle is to be inscribed under one arch of the cosine curve from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. What is the largest area the rectangle can have and what dimensions give that area?

10. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?

5.3 Cont. PVA & Rectilinear Motion

Topics

❖ *Position, Velocity, Acceleration Applications*

❖ *Rectilinear Motion*

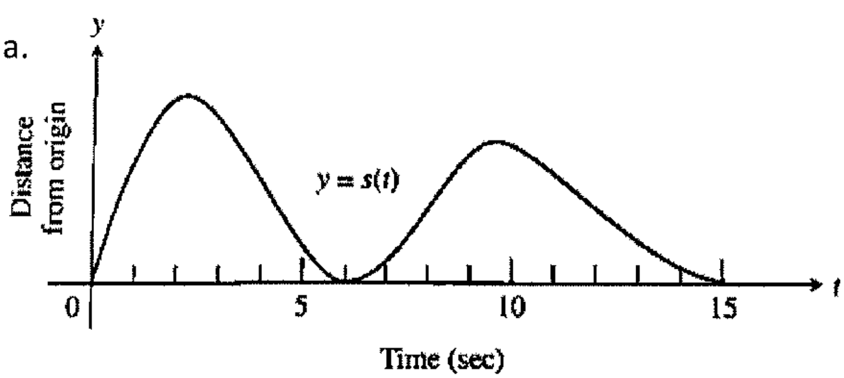
Warm Up!

Explain how the position function $s(t)$, the velocity function $v(t)$, and the acceleration function $a(t)$ are all related.

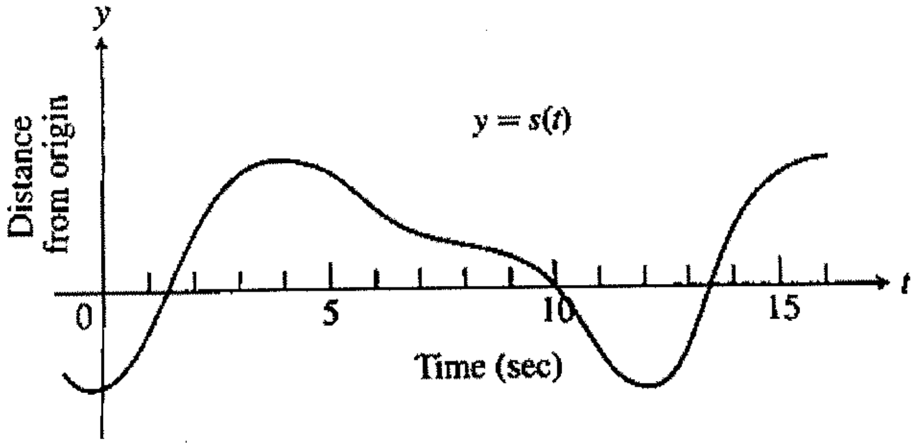
Example 1: Using Graphs

The graph of the position function $y = s(t)$ of a particle moving along a line is given. At approximately what times is the particles velocity equal to zero, velocity equal to zero, and the acceleration equal to zero?

a.



b.



Example 1: Studying Motion Along a Line

A particle is moving along the *x*-axis with the position function provided below. Find the velocity and acceleration. Find the open *t*-intervals when the particle is moving to the left or right. Find the velocity of the particle when the acceleration is 0. Then describe the motion of the particle.

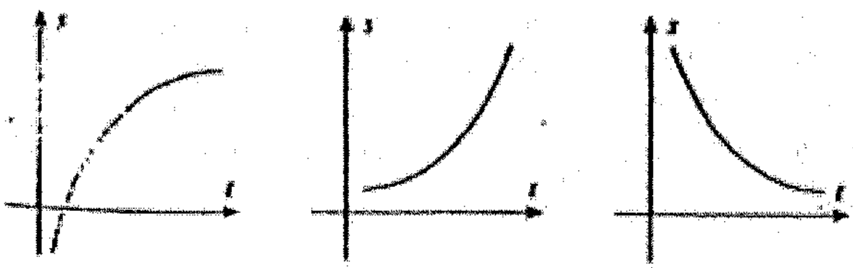
a. $x(t) = 2t^3 - 14t^2 + 22t - 5$

b. $x(t) = t^3 - 3t + 3$

c. $x(t) = 6 - 2t - t^2$

5.3 PVA Practice

1. The graphs of three position functions are shown in the accompanying figure. In each case determine the signs of the velocity and acceleration, then determine whether the particle is speeding up or slowing down.



(*a*) (*b*) (*c*)

2. The position function of a particle moving on a horizontal x-axis is shown in Figure Ex-3.
   (a) Is the particle moving left or right at time t_0?
   (b) Is the acceleration positive or negative at time t_0?
   (c) Is the particle speeding up or slowing down at time t_0?
   (d) Is the particle speeding up or slowing down at time t_1?

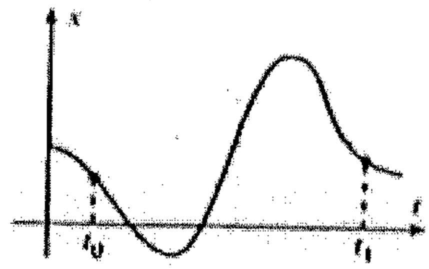
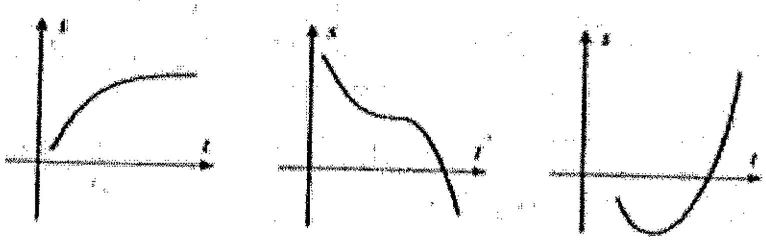
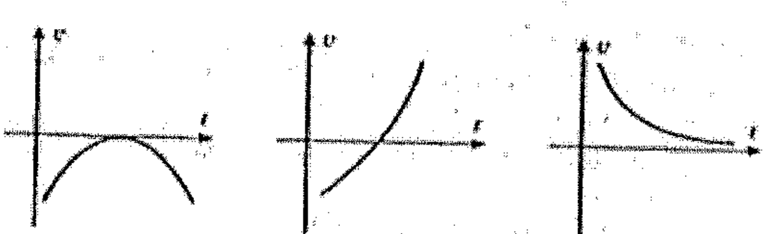


Figure Ex-3

3. For the graphs in the accompanying figure, match the position functions with their corresponding velocity functions.



(*a*) (*b*) (*c*)



(I) (II) (III)

4. The accompanying figure shows the graph of s versus t for an ant that moves along a narrow vertical pipe (an s-axis with the positive direction up).
(a) When, if ever, is the ant above the origin?
(b) When, if ever, does the ant have velocity zero?
(c) When, if ever, is the ant moving down the pipe?

5. The accompanying figure shows the graph of velocity versus time for a particle moving along a coordinate line. Make a rough sketch of the graphs of speed versus time and acceleration versus time.

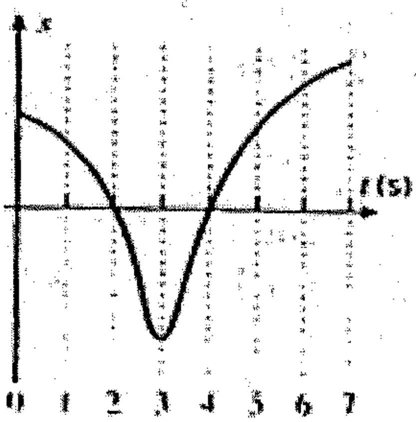


Figure Ex-6

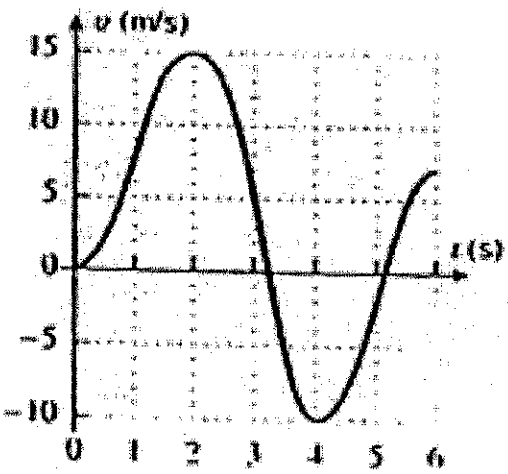


Figure Ex-7

6. A particle is moving along the x-axis. Its position at time t, is given by the equation:

$$s(t) = \frac{t^4}{4} - \frac{7t^3}{3} + 5t^2$$

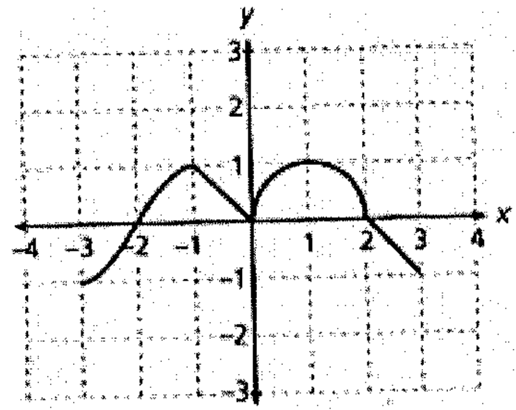
a. What is the velocity of the particle at $t = 3$? Is the velocity increasing or decreasing at this time? Explain and justify your answer.

b. At what values of t does the particle change direction? Explain and justify your answer.

c. For which values of t is the position graph concave downwards? For which values is it concave upwards? Explain and justify your answer.

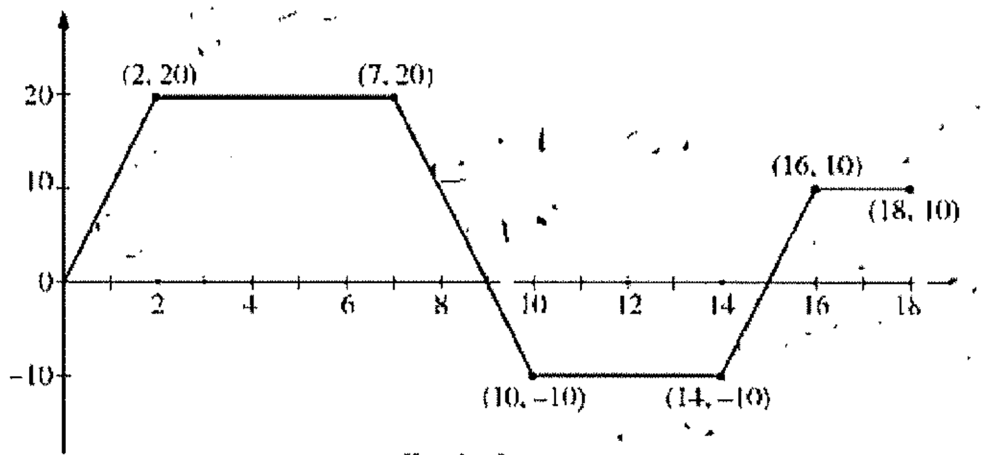
d. For which values of t is the particle speeding up? For which values is it slowing down? Explain and justify your answer.

7. The graph of $f'(x)$ is given below for $x \in [-3,3]$. On which interval(s) is the function $f(x)$ both increasing and concave up?



(A) $(-2,2)$
(B) $(-2,0)\cup(0,2)$
(C) $(-3,-2)$
(D) $(-2,-1)\cup(0,1)$
(E) none of these

8. A graph of $v(t)$ is provided. When does the particle stop and change direction?



Graph of v

9. Car *A* has positive velocity $v(t)$ as it travels on a straight road. Where *v* is a differentiable function of *t*. The velocity is recorded for selected values over the time interval $0 \le t \le 10$ seconds, as shown in the table below. Use the data from the table to approximate the acceleration of Car *A* at *t* = 8 seconds. Indicate units of measure.

t (sec)	0	2	5	7	10
$v(t)$ (ft/sec)	0	9	36	61	115

10. A particle moves along the y-axis for $0 \leq t \leq 40$ seconds. The velocity of the particle at time *t* is given by $v(t) = \sin\left(\frac{\pi}{8}t\right)$ in meters/second. The particle is at position y = 10 meters at time *t* = 0. Find the acceleration of the particle at time *t*. Is the speed of the particle increasing, decreasing, or neither at time *t* = 10 seconds?

11. Use $P(t) = t^3 - t^2 - t$

a. Find initial position and Initial motion.

b. When does the particle change direction?

c. Is the velocity increasing or decreasing at *t* = 4?

d. When is the particle speeding up, slowing down?

For Numbers 12-14: the position function of a particle moving along a coordinate line is given, where s is in feet and t is in seconds. (a) Find the velocity and acceleration functions. (b) Find the position, velocity, speed, and acceleration at time $t = 1$. (c) At what times is the particle stopped? (d) When is the particle speeding up? Slowing down?

12. $s(t) = t^4 - 4t + 2, t \geq 0$

13. $s(t) = 3\cos\left(\frac{\pi}{2}t\right), 0 \leq t \leq 5$

14. $s(t) = \frac{t}{t^2+4}, t \geq 0$

Chapter 5 Part 2 Review Questions

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2003 SCORING GUIDELINES

Question 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

2003 Scoring Rubric Question 2

(a) $a(2) = v'(2) = 1.587$ or 1.588

$v(2) = -3\sin(2) < 0$

Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

$2: \begin{cases} 1: a(2) \\ 1: \text{speed decreasing with reason} \end{cases}$

(b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$

$t = \sqrt{2\pi}$ or 2.506 or 2.507

Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

$2: \begin{cases} 1: t = \sqrt{2\pi} \text{ only} \\ 1: \text{justification} \end{cases}$

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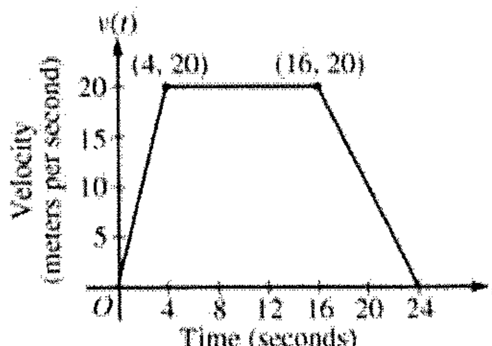
2005 SCORING GUIDELINES

Question 5



A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

2005 Scoring Rubric Question 5

(b) $v'(4)$ does not exist because

$$\lim_{t \to 4^-} \left(\frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \to 4^+} \left(\frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

$3 : \begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

(c)

$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2 : \begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

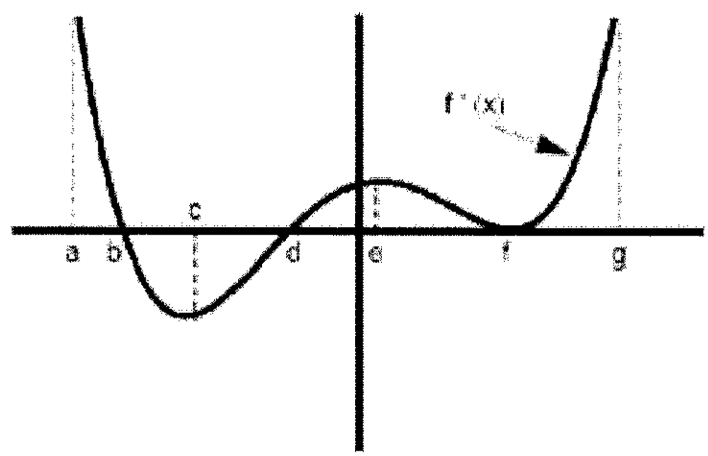
(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2 : \begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

1. Use the graph of $f'(x)$ provided to answer the questions about $f(x)$. Based on the graph of $f'(x)$, find where the graph of $f(x)$ is increasing or decreasing and find the x-values of any relative extrema.



2. Fill in the blanks

   a. When the velocity and acceleration of the particle have the same sign, the particle's speed is ____________________.

   b. When the velocity and acceleration of the particle have opposite signs, the particle's speed is ____________________.

3. Sketch the graph of a function whose first derivative is negative or zero everywhere and whose second derivative starts out negative but becomes positive.

4. Sketch the graph of a position of a particle vs. time if the particle's velocity is negative and its acceleration is positive.

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2006 SCORING GUIDELINES (Form B)

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

2006 Scoring Rubric Question 6 Form B

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.	$2 : \begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$
(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.	$2 : \begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$
Units of ft in (a) and ft/sec in (b)	1 : units in (a) and (b)

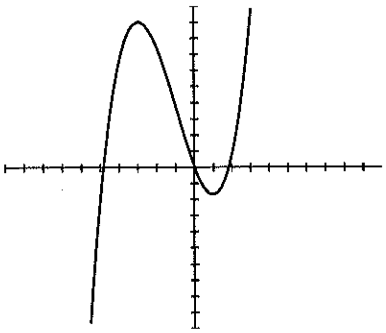
818 (1989BC). Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

a) Find the absolute maximum and minimum values of $f(x)$.

b) Find intervals on which f is increasing.

c) Find the x-coordinate of each point of inflection of the graph of f.

5. The graph of *f* is given below.



(a) For what values of x is $f(x)$ zero? Positive? Negative?

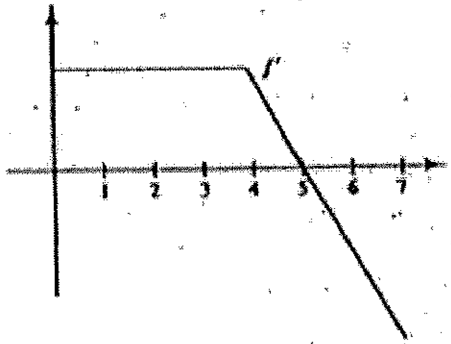
(b) For what values of x is $f'(x)$ zero? Positive? Negative?

(c) For what values of x is $f''(x)$ zero? Positive? Negative?

6. Find all critical values for the function on the closed interval $[0, 2\pi]$. Use the Second Derivative to determine all relative min and relative max, Explain and justify each answer.

Express all x-values in radians.
$f(x) = \sin x + \cos x$

11.



From the graph it follows that.

(A) f is discontinuous at $x = 4$
(B) f is decreasing for $4 < x < 7$
(C) f is constant for $0 < x < 4$
(D) f has a local maximum at $x = 0$
(E) f has a local minimum at $x = 7$

In Questions 35–38, the position of a particle moving along a straight line is given by $s = t^3 - 6t^2 + 12t - 8$.

35. The distance s is increasing for

(A) $t < 2$ **(B)** all t except $t = 2$ **(C)** $1 < t < 3$
(D) $t < 1$ or $t > 3$ **(E)** $t > 2$

36. The minimum value of the speed is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0 **(E)** none of these

37. The acceleration is positive

(A) when $t > 2$ **(B)** for all t, $t \neq 2$ **(C)** when $t < 2$
(D) for $1 < t < 3$ **(E)** for $1 < t < 2$

38. The speed of the particle is decreasing for

(A) $t > 2$ **(B)** $t < 3$ **(C)** all t
(D) $t < 1$ or $t > 2$ **(E)** none of these

In Questions 39–41, a particle moves along a horizontal line and its position at time t is $s = t^4 - 6t^3 + 12t^2 + 3$.

39. The particle is at rest when t is equal to

(A) 1 or 2 (B) 0 (C) $\frac{9}{4}$ (D) 0, 2, or 3 (E) none of these

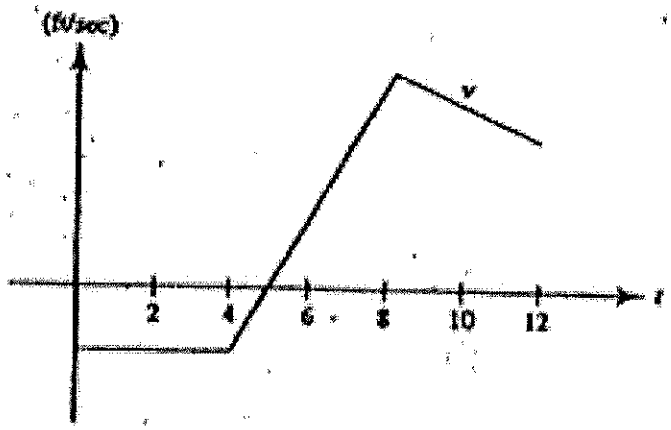
40. The velocity, v, is increasing when

(A) $t > 1$ (B) $1 < t < 2$ (C) $t < 2$
(D) $t < 1$ or $t > 2$ (E) $t > 0$

41. The speed of the particle is increasing for

(A) $0 < t < 1$ or $t > 2$ (B) $1 < t < 2$ (C) $t < 2$
(D) $t < 0$ or $t > 2$ (E) $t < 0$

The graph for Questions 72 and 73 shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 12$.



72. For what t does this object attain its maximum acceleration?

(A) $0 < t < 4$ (B) $4 < t < 8$ (C) $t = 5$ (D) $t = 8$ (E) $t = 12$

73. The object reverses direction at $t =$

(A) 4 only (B) 5 only (C) 8 only
(D) 5 and 8 (E) none of these

MC Answers: 11-E, 35-B, 36-D, 37-A, 38-E, 39-B, 40-D, 41-A, 72-B, 73-B

Calculus AB
Chapter 3.6 – 4.4 Review

CONCEPTS:

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?

2. How do you justify relative extrema?

3. How do you justify that a function is increasing or decreasing?

4. How do you justify that a function is concave up or concave down?

5. How do you justify that a function has a point of inflection?

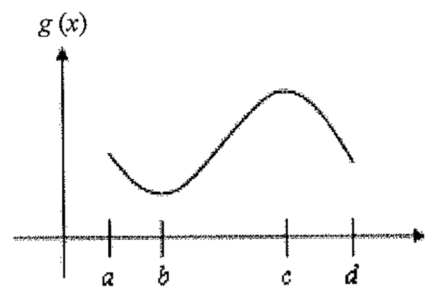
6. Using the graph of $g(x)$ below, determine the signs of $g'(x)$ and $g''(x)$ at each point. Explain your reasoning.

At $x = a$ ...

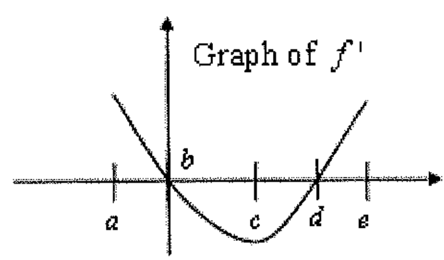
At $x = b$ ...

At $x = c$ ...

At $x = d$ ...



7. Given the graph of f' below answer each of the following questions, and justify your response with a statement that contains the phrase "since f' ____________________ ..."



a) When is f increasing?

b) When is f decreasing?

c) When is f concave up?

d) When is f concave down?

e) When does f have a relative maximum?

f) When does f have a relative minimum?

g) When does f have a point of inflection?

SKILLS:

8. [Calculator Allowed] If $f(x)$ has an inverse, then $f\left(f^{-1}(x)\right)=x$. Find $\left(f^{-1}\right)'(2)$ if $f(x)=x^3+2x-1$.

9. Find the value of c guaranteed by the MVT for $f(x)=4x^2+5x$ on the interval [–2, 1].

10. [Calculator Allowed] Find the value of c guaranteed by the MVT for $f(x)=\sin x$ on the interval [4, 5].
[♬: For those of you doing this problem algebraically, the answer is NOT $c\approx 1.774$ ...Why?]

11. Find the following derivatives:

a) $y=\sin^{-1}\left(3x^2\right)$

b) $y=\tan^{-1}(\sin x)$

c) $y=\sec^{-1}\left(\frac{1}{x}\right)$

d) $y=5^{x^{-3}-8}$

e) $y=e^{8x}$

f) $y=\log_4\left(\sqrt{9x^3-2}\right)$

g) $y=\ln\left(7x^2+3\right)$

h) $y=3^{\sec(x)}$

i) $y=e^{\ln x}$

While none of the previous derivative questions included product and quotient rules, you should be able to combine these rules with any rules we have learned before. See your quiz from 3.8 and 3.9 for examples.

12. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Find the first derivative of the following combinations at the given value of x.

a) $f(g(x))$ at $x = -1$

b) $f^2(x)g^3(x)$ at $x = 0$

c) $g(f(x))$ at $x = -1$

d) $g(x + f(x))$ at $x = 0$

13. Find $\frac{dy}{dx}$ if $x^2y + 3y^2 = x - 2$

14. Find $y'''(x)$ if $y = (4x+1)^{10}$

15. Suppose $y = x^3 - 3x$. [No Calculator]

a) Find the zeros of the function.

b) Determine where y is increasing or decreasing and justify your response.

c) Determine all local extrema and justify your response.

d) Determine the points where y is concave up or concave down, and find any points of inflection. Justify your responses.

e) Use all your information to sketch a graph of this function.

16. If $f'(x)=x^2-9x+1$, what does $f(x)$ equal?

17. Suppose the acceleration of an object in terms of time is given by $a(t)=5$.

a) What is the velocity function if $v(2)=10$?

b) Using your velocity function from part *a*, what is the position function if $s(0)=5$?

18. Suppose $\frac{d^2y}{dx^2}=x^3-4x^2$. Justify each response below.

a) Where is y concave up?

b) Where is y concave down?

c) Are there any inflection points on y? If so, where?

SKILLS AND CONCEPTS APPLIED

19. [Calculator Allowed] The derivative of $h(x)$ is given by $h'(x)=2\cos\left(x-\frac{\pi}{6}\right)+1$ on the interval $[-2\pi,2\pi]$. Justify EVERY response.

a) Where is $h(x)$ increasing?

b) Where is $h(x)$ concave down?

c) Find all extrema of $h(x)$ on the interval $[-2\pi,2\pi]$.

d) Does $h(x)$ have a point(s) of inflection? If so, where?

20. Find the maximum area of a rectangle inscribed under the curve $f(x)=\sqrt{16-x^2}$.

21. [Calculator Allowed] A rectangle is inscribed under one arch of $y=8\cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?

22. The function f is continuous on [0, 3] and satisfies the following:

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

b) Find any points of inflection.

c) Sketch a possible graph of f.

Go back and Review the questions from your assignments in this chapter … especially those in section 4.3.

Understand your notecards!