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## Lesson #1

Chain rule-

HW 2-4 old book p.133-134 Worksheet,7-34 odd, 76,

Find  $f'(x)$

$$f(x) = (x^2 - 3)^2$$

Use what you know to find the derivative

Maybe this is a little bit more helpful:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

This can also be thought of as: Let  $g(x) = u$

$$\text{Then } \frac{d}{dx} f(u) = f'(u) \frac{du}{dx}$$

Many people think of this as the derivative of the “outside” times the derivative of the “inside”

Let's see it at work!

$$y = \sqrt{3x^2 + 17x}$$

$$f(x) = (2x^2 + 5)^7$$

$$f(x) = \sqrt[3]{(x^2 - 1)^2}$$

$$f(x) = (\sqrt{x^3 - 2x^2 + 5})(x^2 + 4)$$

$$f(x) = \frac{2x}{\sqrt[3]{x^2 + 4}}$$

## Lesson #2

Chain rule trig functions- p.158 13-31 odd  
p. 183 91-20 odd)

<http://www.calculus-help.com/funstuff/phobe.html>

$$y = \cos 4x$$

$$y = \cos^2 4x$$

$$y = \cos 4x^2$$

**Beware of poor reading skills!** [When would we need the Chain Rule to find  $y'$  ?]

$$y = \cos 3x^2 \text{ is read as } y = \cos(3x^2)$$

$$y = (\cos 3)x^2 \text{ is read as } y = (\cos 3) \cdot x^2 \text{ where } \cos 3 \text{ is a constant}$$

$$y = \cos(3x)^2 \text{ is read as } y = \cos(9x^2)$$

$$y = \cos^2 x \text{ is read as } y = (\cos x)^2$$

$$y = \cos^2(3x^2) \text{ is read as } y = [\cos(3x^2)]^2$$

$$y = \sqrt{\cos x} \text{ can be read as } y = (\cos x)^{\frac{1}{2}}$$

### Lesson 3

#### 4.4 Derivatives of exponential and logarithmic functions

p. 183 1-20 , 37-40 , p. 158 13-30 even

[Please state your  $u$  and  $\frac{du}{dx}$  and clearly show your steps using standard mathematical notation, blah, blah, blah, ...]

If all you have is the function and the derivative, then NO points will be awarded.

Note: Some problems requires the Chain Rule AND the Product Rule, or they require the Chain Rule AND the Quotient Rule

for which

$f'(x)=0$  and where  $f'(x)$  does not exist

1. Find the derivative

$$\sqrt{x^2 + 1} \sin(2x)$$

2. Find all points on the

$$\text{graph } f(x) = \sqrt[3]{(x^2 - 1)^2}$$

3. Find an equation of the tangent line

$$y = x \cos 4x \quad \text{at } x = 2\pi$$

Chain Rule- find the derivative

SET A-(if it's anything but an x use u-substitution) derivative of  $e^u = u'e^u$

$$F(x) = e^x$$

$$F(x) = e^{3x}$$

$$f(x) = e^{f^x}$$

$$F'(x) =$$

$$F'(x) =$$

$$F'(x) =$$

$$f(x) = 3^x$$

$$f(x) = 3^{4x}$$

$$F(x) = (e^{-x} + e^x)^3$$

$$F'(x) =$$

$$F'(x) =$$

$$F'(x) =$$

SET B (if it's anything but an x use u-substitution) derivative of  $\ln u = \frac{1}{u} u'$

$$f(x) = \ln x \quad f'(x) =$$

$$f(x) = \ln(3x+2)$$

$$f(x) = \ln(x^3+x)$$

$$f(x) = (\ln x^2)^3$$

$$F'(x) =$$

$$F'(x) =$$

$$F'(x) =$$

$$4 \quad F(x) = e^{-x} \ln x$$

**Beware of poor reading skills!** [When would we need the Chain Rule to find  $y'$  ?]

SET C (if it's anything but an  $x$  use u-substitution)

$$y = \cos 4x$$

$$y = \cos 3x^2 \text{ is read as } y = \cos(3x^2)$$

$$y = (\cos 3)x^2 \text{ is read as } y = (\cos 3) \cdot x^2 \text{ where } \cos 3 \text{ is a constant}$$

$$y = \sin 4x^2$$

$$y = \cos(3x)^2 \text{ is read as } y = \cos(9x^2)$$

$$y = \cos^2 x \text{ is read as } y = (\cos x)^2$$

$$y = \cos^2 4x$$

$$y = \cos^2(3x^2) \text{ is read as } y = [\cos(3x^2)]^2$$

$$y = \sqrt{\cos x} \text{ can be read as } y = (\cos x)^{\frac{1}{2}}$$

answers

SET A-(if it's anything but an  $x$  use u-substitution) derivative of  $e^u = u'e^u$

$$F(x) = e^x$$

$$F'(x) = e^x (\ln e)$$

$$F(x) = e^{3x}$$

$$F'(x) = 3e^{3x}(\ln e)$$

$$f(x) = 3^x$$

$$f'(x) = 3^x \ln 3$$

$$f(x) = 3^{4x}$$

$$f'(x) = 3^{4x} \ln 3(4)$$

$$f(x) = e^{\sqrt{x}}$$

$$F(x) = (e^{-x} + e^x)^3$$

SET B (if it's anything but an  $x$  use u-substitution) derivative of  $\ln u = u' \cdot \frac{1}{u}$

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

1.  $f(x)=\ln(3x+2)$   
 $F'(x)=\frac{3}{3x+2}$

2.  $f(x)=\ln(x^3+x)$   
 $F'(x)=\frac{3x^2+1}{x^3+x}$

3.  $f(x)=(\ln x)^3$   
 $F'(x)=$

4  $F(x) = e^{-x} \ln x$



## Lesson 4

Review –go over all homework from last couple of days

CW- extra derivative practice- other side #1,3,6,7

for more practice p. 186 #1,2,3,5,7,8,9,11,13,14,15

HW- Set 3-Multiple choice # 2-55

Quiz on chain rule tomorrow

Find  $f'(x)$

$$f(x)=u^3$$

$$f(x) = \ln u$$

$$f(x) = e^u$$

$$f(x) = \sin u \quad f(x) = 3^u$$

For each of the following, use the fact that

$$\begin{array}{lll} g(5) = -3 & g'(5) = 6 & g'(3) = 1 \\ h(5) = 3 & h'(5) = -2 & \text{to find } f'(5) \end{array}$$

a)  $f(x) = g(x)h(x)$

b)  $f(x) = g(h(x))$

c)  $f(x) = \frac{g(x)}{h(x)}$

d)  $f(x) = [g(x)]^3$  ... ⚠: Your book refers to  $[g(x)]^3$  as  $g^3(x)$

## Lesson 5

1/2 day-Halloween 2019

Chain rule test

## Lesson 6

Implicit differentiation-**reverse classroom**,-

Cw/hw 4.2 p. 167 (1-12) odd ,**27,29** p. 169 59,60,61,63,64

Guidelines for implicit differentiation

1. Differentiate both sides of the equation with respect to  $x$
2. Collect all terms involving  $y'$  on the left side of the equation and move all others to the right.
3. Factor  $y'$  out of the left side of the equation
4. Solve for  $y'$  by dividing both sides of the equation by the left hand

EX 1- find  $dy/dx$   $x^2 - 2y^3 + x^2y = 2$

EX 2 - find  $dy/dx$   $2y = x^2 + \sin y$

EX 3 Write the equation of the tangent line to the graph of  $y^3 + yx^2 + x^2 - 3y^2 = -1$  at the point (1,2)

EX 4. Find  $\frac{d^2y}{dx^2}$   $2x^3 - 3y^2 = 7$

## Higher Order Derivatives

Find  $\frac{d^2y}{dx^2}$  if  $2x^3 - 3y^2 = 7$  .

$$2x^3 - 3y^2 = 7 \quad y'' = \frac{y \cdot 2x - x^2 y'}{y^2}$$

$$6x^2 - 6y y' = 0 \quad y'' = \frac{2x}{y} - \frac{x^2}{y^2} y'$$

$$-6y y' = -6x^2 \quad y'' = \frac{2x}{y} - \frac{x^2}{y^2} \cdot \frac{x^2}{y}$$

Substitute  $y'$  back into the equation.

$$y' = \frac{-6x^2}{-6y}$$

$$y' = \frac{x^2}{y}$$

$$y'' = \frac{2x}{y} - \frac{x^4}{y^3}$$

## LESSON 7

4.2 Implicit differentiation cw- 2005 frq5-  
HW 2004 frq4 and p.168 #,47, 52,

1. Find the  $\frac{d^2y}{dx^2}$  for the curve defined by  
 $x^2+y^2=25$

## Lesson 8

Go over HW from last 2 nights- complete MC packet #35-77  
Tomorrow's lesson leckie related rates

Differentiating with respect to time  $dy/dt$  or  $dx/dt$

1  $y=x^2+3$  find  **$dy/dx$**

2 find  $dy/dt$   $y=x^2+3$  when  $x=1$ , given  $dx/dt =2$

3  $y=\sqrt{x}$  find  **$dy/dt$**  when  $x=4$ , given  $dx/dt =3$

4  $y=2(x^2-3x)$  find  **$dy/dt$**  when  $x=3$  given  $dx/dt=2$

( watch related rates with Mr Leckie

[http://www.chaoticgolf.com/vodcasts/calc/lesson4\\_6b/lesson4\\_6b.html](http://www.chaoticgolf.com/vodcasts/calc/lesson4_6b/lesson4_6b.html)

Determine the slope of the graph when  $x=1$

$$2x^2 + 4y^2 = 9$$

Find the second derivative

$$2x^3 - 3y^2 = 7$$

### **Lesson #9**

Related rates –Mr Leckie-(if it's a Wednesday,  
Otherwise examples 1-3 on each side of paper

chapter Test-- - after vacation

### GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

## LESSON #10

### CW worksheet –both sides

Show how to get down to one variable- 2002 #5 and inverted circular cone, relationship between height and radius

Hw- 2 problems below

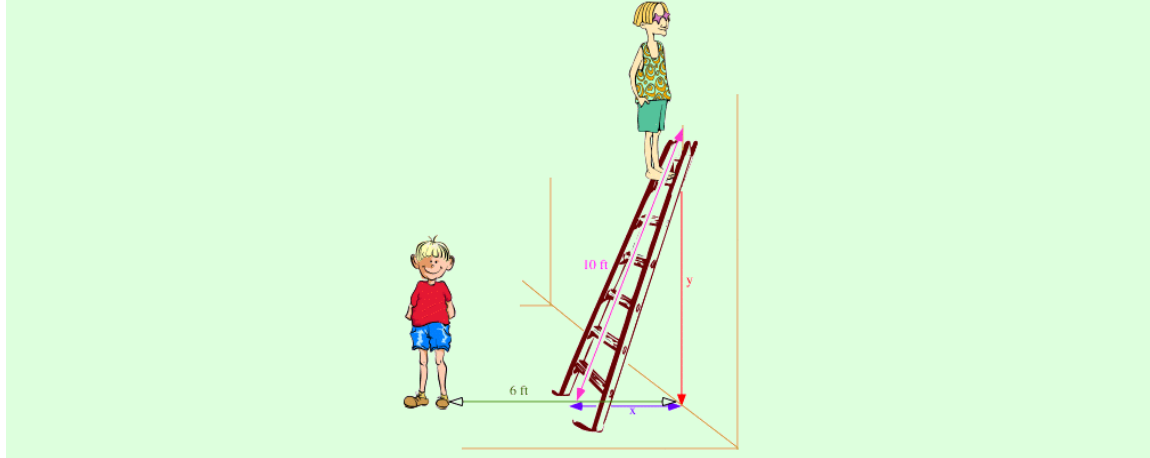
Solutions and in book

<http://slideplayer.com/slide/273790/>

A hot-air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is  $\frac{\pi}{4}$ , the angle is increasing at the rate of 0.14 radians per minute. How fast is the balloon rising at that moment?

A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mpg. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

Joey is perched precariously the top of a 10-foot ladder leaning against the back wall of an apartment building (spying on an enemy of his) when it starts to slide down the wall at a rate of 4 ft per minute. Joey's accomplice, Lou, is standing on the ground 6 ft. away from the wall. How fast is the base of the ladder moving when it hits Lou?



### Lesson #11

CW-AP 2002 #5, 2002 B#6

HW. p. 255 # 9,11,13,17,19

CW-go over HW and BELOW problems

3 A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius  $r$  of the outer circle is increasing at a rate of 1ft per second. When the radius is 4 ft, at what rate is the total area  $A$  of the disturbed water changing?

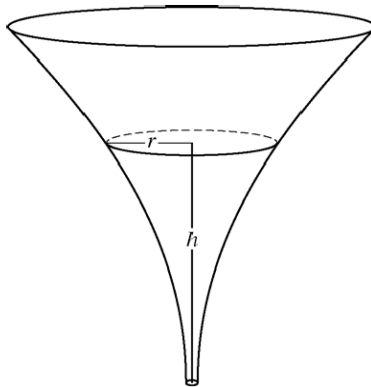
4. A cone-shaped icicle is dripping from the roof. The radius of the icicle is decreasing at a rate of 0.2 cm/hour, while the length is increasing at a rate of 0.8 cm/hour. If the icicle is currently 4 cm in radius and 20 cm long, is the volume of the icicle increasing or decreasing, and at what rate?  $V = \frac{1}{3}\pi r^2 h$



## LESSON 12

Go over HW- tonight's homework- 1-6 of packet

### Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height  $h$ , the radius of the funnel is given by  $r = \frac{1}{20}(3 + h^2)$ , where  $0 \leq h \leq 10$ . The units of  $r$  and  $h$  are inches.

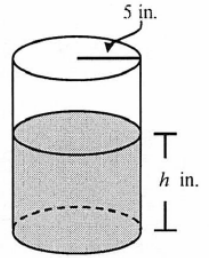
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is  $h = 3$  inches, the radius of the surface of the liquid is decreasing at a rate of  $\frac{1}{5}$  inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

## Lesson 13

### Review sheet- all the u-sub 1-13-hw multiple choice

8. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure to the right. Let  $h$  be the depth of the coffee in the pot, measured in inches, where  $h$  is a function of time  $t$ , measured in seconds. The volume  $V$  of coffee in the pot is changing at the rate of  $-5\pi\sqrt{h}$  cubic inches per second. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)

Show that  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ .



1. Find  $dy/dt$   $3x^2+2y^3=8$

2. Find  $dy/dx$   $y=\sin^2 3x$

#### Topics for chapter 4 test

Derivatives-power, product, quotient, chain,

U-substitution -trig function,  $e^x$ ,  $\ln x$ )

2<sup>nd</sup> derivative

Implicit differentiation

Related rates

## **Lesson 14**

Cw/HW review sheet- chapter 4, and sec 5.6- complete packet

## Lesson 15

pp. 186      1-13

## **LESSON 16**

Test