AP Calculus
Chapter 7 Review WS

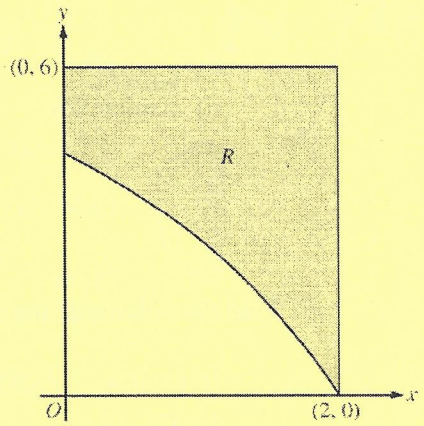
Cofield

1. **Complete the following questions from your textbook on a separate piece of paper. (Your book is that orange one…be sure to find it because you will be returning it soon!!)**

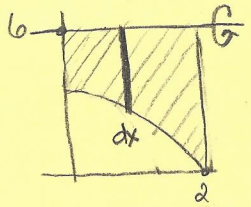
Page 386 #8, 10
Pages 430 – 433 #1 – 5, and 54 } See back!
Don't forget to review your 7.1 worksheets and AP review problem on worksheets from 7.3 days 1-3!

Also, there is only one Disk Method problem on this worksheet…if you need more practice with the Disk Method, try problems 7-14 on page 407.

2. [Calculator] In the figure at the right, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.



a) Find the volume of the solid generated when R is revolved about the line $y = 6$.



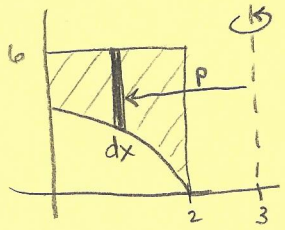
$r = 6 - 4\ln(3-x)$

disk: $V = \pi \int r^2 dx$

$$V = \pi \int_0^2 [6 - 4\ln(3-x)]^2 dx$$

$V \approx 82.519$

b) Find the volume of the solid generated when R is revolved about the line $x = 3$.



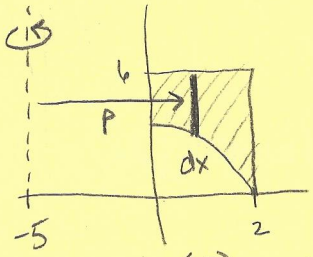
$p = 3 - x$
$h = \text{top} - \text{bottom} = 6 - 4\ln(3-x)$

Shell: $V = 2\pi \int ph\,dx$

$$V = 2\pi \int_0^2 (3-x)[6 - 4\ln(3-x)]dx$$

$V \approx 76.812$

c) Find the volume of the solid generated when R is revolved about the line $x = -5$.



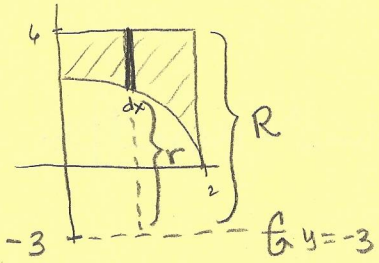
$p = x - (-5) = x + 5$
$h = 6 - 4\ln(3-x)$

Shell: $V = 2\pi \int ph\,dx$

$$V = 2\pi \int_0^2 (x+5)[6 - 4\ln(3-x)]dx$$

$V \approx 265.831$

d) Find the volume of the solid generated when R is revolved about the line $y = -3$.



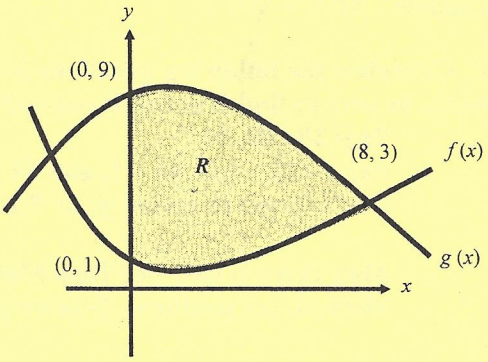
$R = 6 - (-3) = 9$
$r = 4\ln(3-x) - (-3) = 4\ln(3-x) + 3$

washers: $V = \pi \int (R^2 - r^2)dx$

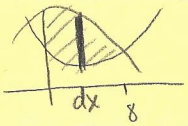
$$V = \pi \int_0^2 \left[9^2 - (4\ln(3-x)+3)^2\right] dx$$

$V \approx 302.954$

3. Each of the questions below refer to the region R as shown in the figure below. Simply $\boxed{\text{set up}}$ the integral expression that would be used to answer each question.



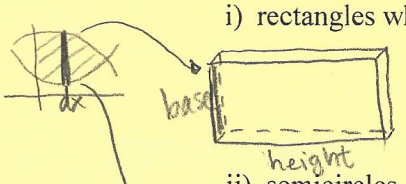
a) Find the area of R.



$$A = \int_0^8 [g(x) - f(x)]\,dx$$

b) Find the volume of the solid whose base is R and where the cross sections perpendicular to the x-axis make the following shape:

i) rectangles whose height equal 3 times its base.



base: $g(x) - f(x)$

ht: $3 \cdot$ base

$$V = \int_0^8 [g(x) - f(x)] \cdot 3[g(x) - f(x)]\,dx \quad \text{or} \quad V = 3\int_0^8 [g(x) - f(x)]^2\,dx$$

ii) semicircles

diameter: $g(x) - f(x)$

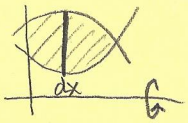
radius: $\frac{1}{2}(g(x) - f(x))$

$V_{slice} = \frac{\pi}{2} r^2 \cdot dx$

$$V = \frac{\pi}{2}\int_0^8 \left[\tfrac{1}{2}(g(x) - f(x))\right]^2 dx \quad \text{or} \quad V = \frac{\pi}{8}\int_0^8 [g(x) - f(x)]^2\,dx$$

c) Find the volume of the solid formed by revolving the region R around each given axis.

i) x–axis



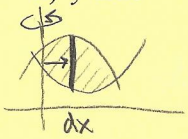
washer: $\pi \int (R^2 - r^2)\,dx$

$R = g(x)$

$r = f(x)$

$$V = \pi \int_0^8 \left[(g(x))^2 - (f(x))^2\right] dx$$

ii) y-axis



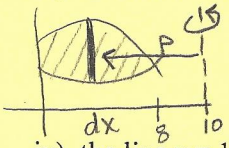
shell: $2\pi \int p h\,dx$

$p = x$

$h = g(x) - f(x)$

$$V = 2\pi \int_0^8 x(g(x) - f(x))\,dx$$

iii) the line $x = 10$



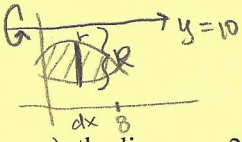
shell: $2\pi \int p h\,dx$

$p = 10 - x$

$h = g(x) - f(x)$

$$V = 2\pi \int_0^8 (10 - x)(g(x) - f(x))\,dx$$

iv) the line $y = 10$



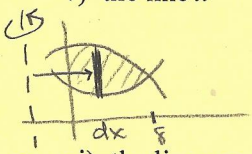
washer: $\pi \int (R^2 - r^2)\,dx$

$R = 10 - f(x)$

$r = 10 - g(x)$

$$V = \pi \int_0^8 \left[(10 - f(x))^2 - (10 - g(x))^2\right] dx$$

v) the line $x = -2$



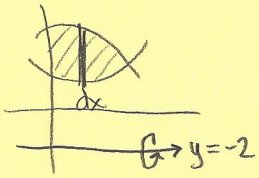
shell: $2\pi \int p h\,dx$

$p = x - (-2) = x + 2$

$h = g(x) - f(x)$

$$V = 2\pi \int_0^8 (x + 2)(g(x) - f(x))\,dx$$

vi) the line $y = -2$



washer $\pi \int (R^2 - r^2)\,dx$

$R = g(x) - (-2) = g(x) + 2$

$r = f(x) - (-2) = f(x) + 2$

$$V = \pi \int_0^8 \left[(g(x) + 2)^2 - (f(x) + 2)^2\right] dx$$

$x = e^y$

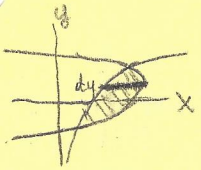
$x = 3 - y^2$

$y^2 = 3 - x$

$y = \pm\sqrt{3-x}$

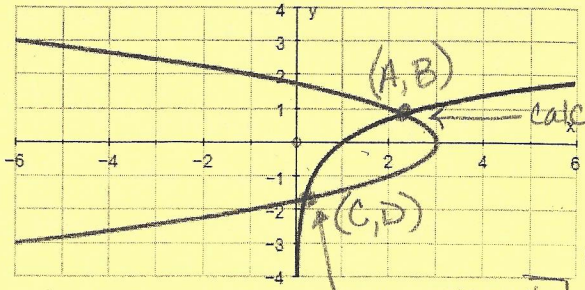
4. Each of the questions below refer to the region R, the region enclosed by the graphs of $y = \ln(x)$ and $x = 3 - y^2$. Set up an integral expression to answer each question, then use your calculator to evaluate.

a) Find the area of R.



$$A_R = \int_D^B \left[(3-y^2) - e^y\right] dy$$

≈ 3.652



calc intersect

$x = 2.3036\ldots$ → Store A

$y = .8344\ldots$ store B

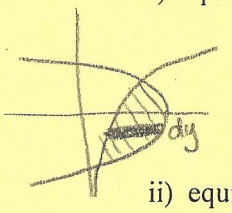
calc intersect

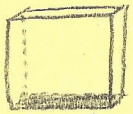
$x = .1868\ldots$ store C

$y = -1.677\ldots$ store D

b) Find the volume of the solid that uses R as a base and has cross sections perpendicular to the y-axis that are …

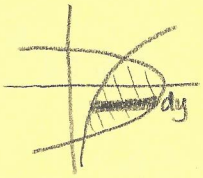
i) squares

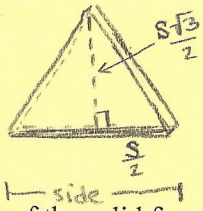




$$V = \int_D^B \left[(3-y^2) - e^y\right]^2 dy \approx 6.358$$

ii) equilateral triangles





$A = \frac{1}{2}(s)\left(\frac{s\sqrt{3}}{2}\right) = \frac{s^2\sqrt{3}}{4}$

side: $(3-y^2) - e^y$

$V_{slice} = \frac{\sqrt{3}}{4}\left[(3-y^2) - e^y\right]^2 \cdot dy$

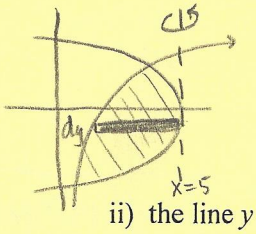
$$V = \frac{\sqrt{3}}{4}\int_D^B \left[(3-y^2) - e^y\right]^2 dy$$

≈ 2.753

c) Find the volume of the solid formed by revolving the region R around each given axis.

i) the line $x = 5$

washer $\pi\int(R^2 - r^2)\,dy$



$R = 5 - e^y$

$r = 5 - (3-y^2)$

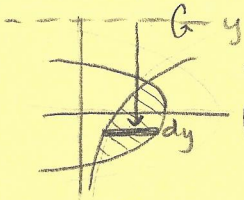
$= 2 + y^2$

$$V = \pi\int_D^B \left[(5-e^y)^2 - (2+y^2)^2\right] dy$$

≈ 76.697

ii) the line $y = 5$

shell $2\pi\int p\,h\,dy$



$p = 5 - y$

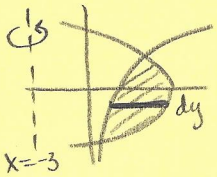
$h = (3-y^2) - e^y$

$$V = 2\pi\int_D^B (5-y)\left[(3-y^2) - e^y\right] dy$$

≈ 123.759

iii) the line $x = -3$

washer $\pi\int(R^2 - r^2)\,dy$



$R = 3 - y^2 - (-3) = 6 - y^2$

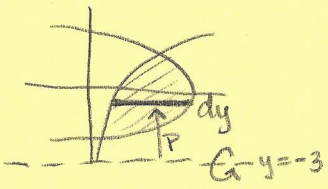
$r = e^y - (-3) = e^y + 3$

$$V = \pi\int_D^B \left[(6-y^2)^2 - (e^y+3)^2\right] dy$$

≈ 106.871

iv) the line $y = -3$

shell $2\pi\int p\,h\,dy$



$p = y - (-3) = y + 3$

$h = (3-y^2) - e^y$

$$V = 2\pi\int_D^B (y+3)\left[(3-y^2) - e^y\right] dy$$

≈ 59.809