

1. [No Calculator] Evaluate using the FTC (the evaluation part)

a) $\int_2^7 \left(\frac{8}{x} + 7\sqrt[3]{x} + x^{-4} \right) dx$

b) $\int_4^9 \left(\frac{5}{x^3} + 7\sqrt{x} + \frac{1}{x} \right) dx$

2. [No Calculator] Evaluate using geometry

a) $\int_{-2}^3 \sqrt{25 - (x+2)^2} dx$

c) $\int_{-6}^1 |8 + 2x| dx$

3. [No Calculator] Evaluate each derivative.

a) $\frac{d}{dx} \left[\int_{10}^x \tan(3t^2 + 9) dt \right]$

b) Find $h'(x)$ if $h(x) = \int_{5x^4}^{\sec x} \sqrt{4t-9} dt$.

c) $\frac{d}{dx} \left[\int_8^x \ln(3t^2 + 9) dt \right]$

d) Find $h'(x)$ if $h(x) = \int_{3x^6}^{\tan x} \frac{9}{x^2 - 1} dt$.

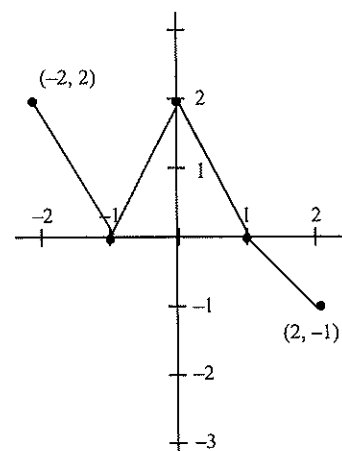
4. [No Calculator] Given the graph of $f(x)$ as shown and the definition of $g(x) = \int_0^x f(t) dt$

a) Find $g(-1)$, $g'(-1)$, $g''(-1)$

b) Over what interval is $g(x)$ increasing.
Show your work and explain your reasoning.

c) Over what interval is $g(x)$ concave up? Show your work and explain your reasoning.

d) Graph $g(x)$

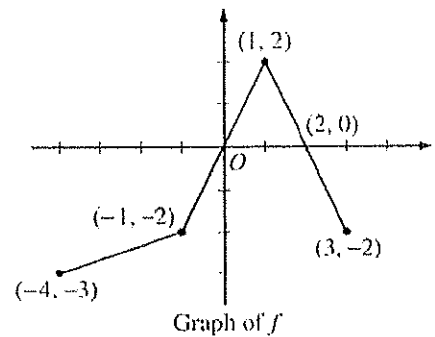


Graph of f

5. [No Calculator] The graph of the function f shown below consists of three line segments.

a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.



b) For the function g defined in part a, find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$.

Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

d) For the function h defined in part c, find all intervals on which h is decreasing. Explain your reasoning.

6. [Calculator] The temperature, in degrees Celsius ($^{\circ}\text{C}$), of the water in a pond is a differentiable function W of time t . The table below shows the water temperature as recorded every 3 days over a 15-day period.

a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

t (days)	$W(t)$ ($^{\circ}\text{C}$)
0	20
3	31
6	28
9	24
12	22
15	21

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

c) A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

d) Use the function P defined in part c to find the average value, in $^{\circ}\text{C}$, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

7. [Calculator] For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

8. [No Calculator] Suppose $\int_1^2 f(x) dx = 3$, $\int_1^5 f(x) dx = -13$, and $\int_1^5 g(x) dx = 7$. Find each of the following:

a) $\int_3^3 g(x) dx$

b) $\int_5^1 f(x) dx$

c) $\int_1^5 [g(x) - f(x)] dx$

d) $\int_2^5 f(x) dx$

e) $\int_1^5 [3f(x) - g(x)] dx$

f) $\int_1^5 \frac{g(x)}{4} dx$

9. [No Calculator] Suppose $H(x) = \int_2^x \ln(t+5) dt$ for the interval $[2, 10]$.

a) Use MRAM to approximate $H(10)$ using 4 equal subdivisions.

b) When is $H(x)$ decreasing? Justify your response.

c) If the average rate of change of $H(x)$ on $[2, 10]$ is k , what is the value of $\int_2^{10} \ln(t+5) dt$ in terms of k .

10. [No Calculator] Let $H(x) = \int_0^x f(t) dt$, where f is the continuous function with domain $[0, 12]$ shown below.

a) Find $H(0)$

b) Is $H(12)$ positive or negative? Explain.

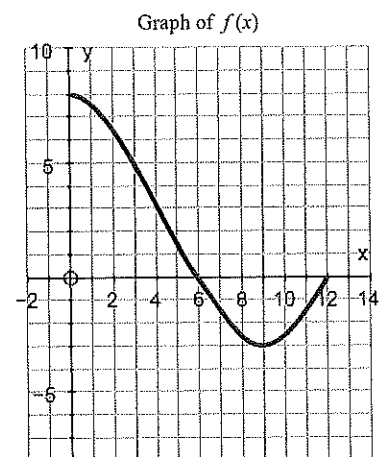
c) Find $H'(x)$ and use it to evaluate $H'(0)$.

d) When is $H(x)$ increasing? Justify your answer.

e) Find $H''(x)$.

f) When is $H(x)$ concave up? Justify your answer.

g) At what x -value does $H(x)$ achieve its maximum value? Justify your answer.



11. [No Calculator] If $\int_a^b f(x) dx = a + 2b$, then $\int_a^b [f(x) + 3] dx =$

A $a + 2b + 3$

B $3b - 3a$

C $4a - b$

D $5b - 2a$

E $5b - 3a$

12. [No Calculator] Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$. At what value of x is $f(x)$ a minimum?

A none

B 0.5

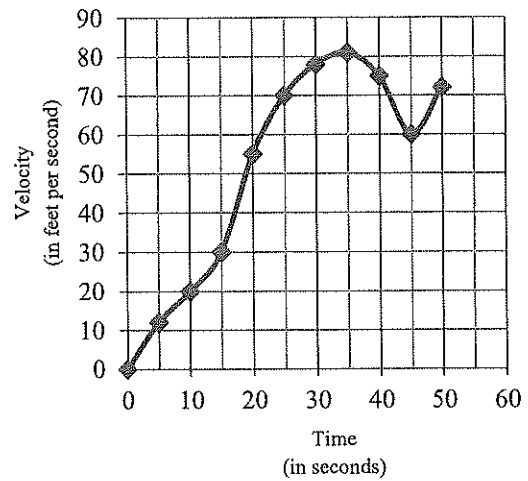
C 1.5

D 2

E 3

13. [Calculator] If $f(x) = \int_a^x \ln(2 + \sin t) dt$, and $f(3) = 4$, what does $f(5) = ?$

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

b) Find the average acceleration of the car, in ft/sec^2 , over the interval $0 \leq t \leq 50$.

c) Approximate $\int_0^{50} v(t) dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

1. [No Calculator] Evaluate using the FTC (the evaluation part)

$$a) \int_2^7 \left(\frac{8}{x} + 7\sqrt{x} + x^{-4} \right) dx = \int_2^7 \left(8 \cdot \frac{1}{x} + 7x^{1/2} + x^{-4} \right) dx$$

$$= 8 \ln|x| + 7 \cdot \frac{2}{3} x^{3/2} - \frac{1}{3} x^{-3} \Big|_2^7$$

$$= \left[8 \ln|7| + \frac{14}{3} (7)^{3/2} - \frac{1}{3} (7)^{-3} \right] - \left[8 \ln|2| + \frac{14}{3} (2)^{3/2} - \frac{1}{3} (2)^{-3} \right]$$

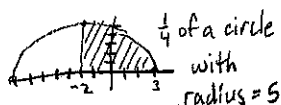
$$b) \int_4^9 \left(\frac{5}{x^3} + 7\sqrt{x} + \frac{1}{x} \right) dx = \int_4^9 \left(5x^{-3} + 7x^{1/2} + \frac{1}{x} \right) dx$$

$$= 5 \cdot \left(-\frac{1}{2}\right) x^{-2} + 7 \cdot \frac{2}{3} x^{3/2} + \ln|x| \Big|_4^9$$

$$= \left[-\frac{5}{2} (9)^{-2} + \frac{14}{3} (9)^{3/2} + \ln|9| \right] - \left[-\frac{5}{2} (4)^{-2} + \frac{14}{3} (4)^{3/2} + \ln|4| \right]$$

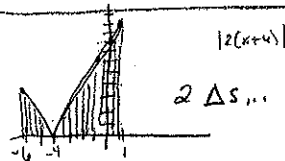
2. [No Calculator] Evaluate using geometry

$$a) \int_{-2}^3 \sqrt{25 - (x+2)^2} dx$$



$$\frac{1}{4} \pi (5)^2 = \frac{25\pi}{4}$$

$$c) \int_{-6}^1 |8+2x| dx$$



$$= \frac{1}{2} (2)(4) + \frac{1}{2} (5)(10)$$

$$= 4 + 25$$

$$= \boxed{29}$$

3. [No Calculator] Evaluate each derivative.

$$a) \frac{d}{dx} \int_{10}^x \tan(3t^2 + 9) dt = \tan(3x^2 + 9)$$

$$b) \text{ Find } h'(x) \text{ if } h(x) = \int_{5x^4}^{\sec x} \sqrt{4t-9} dt.$$

$$h'(x) = \sqrt{4 \sec x - 9} \cdot (\sec x \tan x) - \sqrt{4(5x^4) - 9} \cdot (20x^3)$$

$$c) \frac{d}{dx} \int_8^x \ln(3t^2 + 9) dt = \ln(3x^2 + 9)$$

$$d) \text{ Find } h'(x) \text{ if } h(x) = \int_{3x^6}^{\tan x} \frac{9}{x^2 - 1} dt.$$

$$h'(x) = \frac{9}{\tan^2 x - 1} \cdot (\sec^2 x) - \frac{9}{(3x^6)^2 - 1} \cdot (18x^5)$$

4. [No Calculator] Given the graph of $f(x)$ as shown and the definition of $g(x) = \int_0^x f(t) dt$

a) Find $g(-1)$, $g'(-1)$, $g''(-1)$

$$g(-1) = \int_0^{-1} f(t) dt = - \int_{-1}^0 f(t) dt = -1$$

$$\frac{1}{2} (1)(2) = 1$$

$\begin{cases} g'(x) = f(x) \\ \therefore g'(-1) = f(-1) = 0 \\ g''(x) = f'(x) \end{cases}$

b) Over what interval is $g(x)$ increasing. Show your work and explain your reasoning.

$g(x)$ is increasing if $g'(x) > 0$

Since $g'(x) = f(x)$, $g(x)$ is increasing on $[-2, 1]$.

c) Over what interval is $g(x)$ concave up? Show your work and explain your reasoning.

$g(x)$ is concave up if $g''(x) > 0$

Since $g''(x) = f'(x)$, $g(x)$ is concave up on $(-1, 0)$.

d) Graph $g(x)$

First find some points...

$$g(-2) = \int_0^{-2} f(t) dt = - \int_{-2}^0 f(t) dt = -2$$

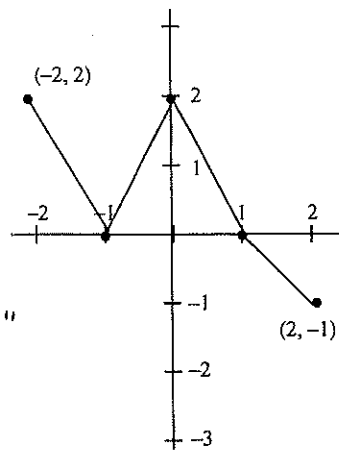
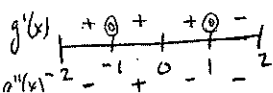
$$g(1) = \int_0^1 f(t) dt = 1$$

$$g(2) = \int_0^2 f(t) dt = \frac{1}{2}$$

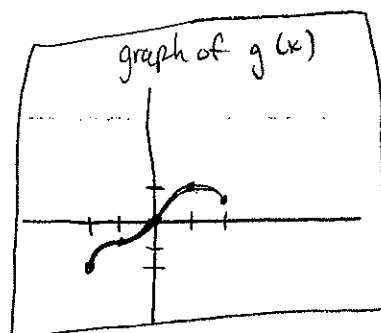
Already know $g(-1) = -1$
point (a)

$$a) \int_0^0 f(t) dt = 0$$

$$\frac{1}{2} (1)(2) + \frac{1}{2} (1)(2)$$



Graph of f



5. [No Calculator] The graph of the function f shown below consists of three line segments.

a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

$$g(-1) = \int_{-4}^{-1} f(t) dt = \frac{-15}{2}$$

Trapezoid
 $\frac{1}{2}(-2 + -3)$

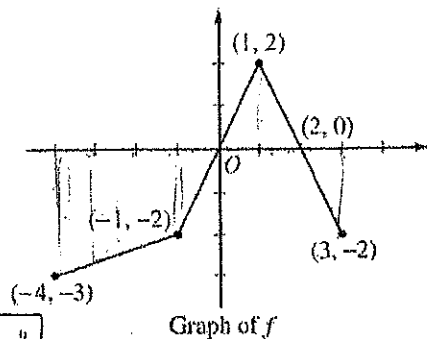
$$g'(x) = f(x)$$

$$\therefore g'(-1) = f(-1) = -2$$

$$g''(x) = f'(x)$$

$$\therefore g''(-1) = f'(-1)$$

DNE ...
"pointy place"



b) For the function g defined in part a, find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

$g(x)$ has a point of inflection when $g''(x)$ changes signs. Since $g''(x) = f'(x)$ we need to look for x -coordinates where $f'(x)$ changes signs... This occurs at $x=1$ only

c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$.

Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

$$h(x) = 0 \text{ when } x=3 \text{ since } \int_3^3 f(t) dt = 0$$

$$h(x) = 0 \text{ when } x=1 \text{ since } h(1) = \int_1^3 f(t) dt = 1 - 1 = 0$$

$$h(x) = 0 \text{ when } x = -1 \text{ since } h(-1) = \int_{-1}^3 f(t) dt = -1 + 1 + 1 - 1 = 0$$

d) For the function h defined in part c, find all intervals on which h is decreasing. Explain your reasoning.

h is decreasing if $h'(x) < 0$

Since $h'(x) = -f(x)$, $h'(x) < 0$ when $f(x) > 0$, ... this occurs on $[0, 2]$

6. [Calculator] The temperature, in degrees Celsius ($^{\circ}\text{C}$), of the water in a pond is a differentiable function W of time t . The table below shows the water temperature as recorded every 3 days over a 15-day period.

a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

$$W'(12) \text{ is the slope of } w \text{ at } t=12 \quad W'(12) \approx \frac{21-24}{15-9} = \frac{-3}{6} = -\frac{1}{2} \text{ } ^{\circ}\text{C/day}$$

The temp of the water is decreasing approximately $\frac{1}{2}$ $^{\circ}\text{C/day}$ at day 12.

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

$$\text{Average temp} = \text{Average value of } w(t) = \frac{\int_0^{15} w(t) dt}{15} = \frac{3\left(\frac{31+20}{2}\right) + 3\left(\frac{20+31}{2}\right) + 3\left(\frac{24+28}{2}\right) + 3\left(\frac{22+24}{2}\right) + 3\left(\frac{21+22}{2}\right)}{15} = \frac{316.5}{15} \approx 21.1^{\circ}\text{C}$$

t (days)	$W(t)$ ($^{\circ}\text{C}$)
0	20
3	31
6	28
9	24
12	22
15	21

c) A student proposes the function P , given by $P(t) = 20 + 10te^{-(t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$$P'(12) \approx -0.549$$

The temperature of the water is decreasing $0.549^{\circ}\text{C/day}$ at the time $t=12$

d) Use the function P defined in part c to find the average value, in $^{\circ}\text{C}$, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

$$\frac{\int_0^{15} P(t) dt}{15} \approx 25.757^{\circ}\text{C}$$

7. [Calculator] For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

$R(t)$ is already a rate of change!
 So $R(t)$ is treated like a "derivative function"

since $R(6) > 0$,
 the # of mosquitoes is increasing
 at $t = 6$

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

We just showed $R(6) > 0$ to show the # of mosquitoes is increasing,

Now, since $R'(6) \approx -1.913 < 0$ the # of mosquitoes is increasing at a decreasing rate
 $R(6) > 0$ $R'(6) < 0$

c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

TOTAL # of mosquitoes = starting # + change in mosquitoes

$$= 1000 + \int_0^{31} R(t) dt \approx 964.35192$$

964

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

$T(x)$ = total # of mosquitoes at time x days

$$T(x) = 1000 + \int_0^x R(t) dt \leftarrow \text{MAXIMIZE THIS! on } [0, 31]$$

$$T'(x) = R(x) = 0 \text{ when } x \approx 0, x \approx 7.8539816, x \approx 23.561945$$

STORE AS A STORE AS B

use a candidates TEST...

\therefore The maximum # of mosquitoes is ≈ 1039 after ≈ 7.854 days

Rounded

x	0	A	B	31
T(x)	1000	1039	842	964

8. [No Calculator] Suppose $\int_1^3 f(x) dx = 3$, $\int_1^5 f(x) dx = -13$, and $\int_1^5 g(x) dx = 7$. Find each of the following:

a) $\int_3^5 g(x) dx = \boxed{0}$

b) $\int_5^3 f(x) dx = -\int_1^5 f(x) dx$
 $= -(-13)$
 $= \boxed{13}$

c) $\int_1^5 [g(x) - f(x)] dx$
 $= \int_1^5 g(x) dx - \int_1^5 f(x) dx$
 $= 7 - (-13) = \boxed{20}$

d) $\int_2^5 f(x) dx = \int_1^5 f(x) dx - \int_1^2 f(x) dx$
 $= -13 - (3)$
 $= \boxed{-16}$

e) $\int_1^5 [3f(x) - g(x)] dx$
 $= 3 \int_1^5 f(x) dx - \int_1^5 g(x) dx$
 $= 3(-13) - (7)$
 $= -39 - 7$
 $= \boxed{-46}$

f) $\int_1^5 \frac{g(x)}{4} dx = \frac{1}{4} \int_1^5 g(x) dx$
 $= \frac{1}{4} (7)$
 $= \boxed{\frac{7}{4}}$

9. [No Calculator] Suppose $H(x) = \int_2^x \ln(t+5) dt$ for the interval $[2, 10]$.

a) Use MRAM to approximate $H(10)$ using 4 equal subdivisions.

$$H(10) = \int_2^{10} \ln(t+5) dt \approx \boxed{2 \cdot \ln(3+5) + 2 \cdot \ln(5+5) + 2 \cdot \ln(7+5) + 2 \cdot \ln(9+5)}$$

$$= 2 \cdot \ln(8) + 2 \cdot \ln(10) + 2 \cdot \ln(12) + 2 \cdot \ln(14) = 2 \cdot \ln(13440)$$

b) When is $H(x)$ decreasing? Justify your response.

if $H'(x) < 0$, then $H(x)$ is decreasing.

$H'(x) = \ln(x+5)$... $\therefore H(x)$ is decreasing on $[-5, -4)$

c) If the average rate of change of $H(x)$ on $[2, 10]$ is k , what is the value of $\int_2^{10} \ln(t+5) dt$ in terms of k .

$$k = \frac{H(10) - H(2)}{10 - 2} = \frac{\int_2^{10} \ln(t+5) - \int_2^2 \ln(t+5)}{8} = \frac{\int_2^{10} \ln(t+5) dt}{8} \quad \therefore \int_2^{10} \ln(t+5) = 8k$$

10. [No Calculator] Let $H(x) = \int_0^x f(t) dt$, where f is the continuous function with domain $[0, 12]$ shown below.

a) Find $H(0)$

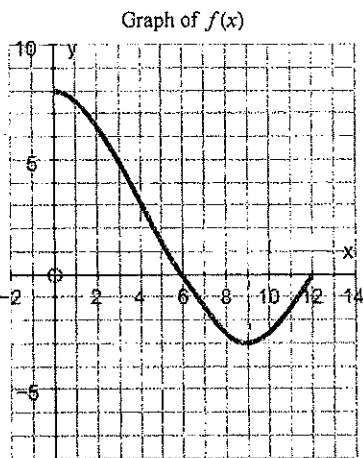
$$H(0) = \int_0^0 f(t) dt = \boxed{0}$$

b) Is $H(12)$ positive or negative? Explain.

$H(12) = \int_0^{12} f(t) dt > 0$ because there is more area above the x-axis from 0 to 6 than below the x-axis from 6 to 12.

c) Find $H'(x)$ and use it to evaluate $H'(0)$.

$$\boxed{H'(x) = f(x)} \quad \therefore \boxed{H'(0) = f(0) = 8}$$



d) When is $H(x)$ increasing? Justify your answer.

if $H'(x) > 0$, then $H(x)$ is increasing. Since $H'(x) = f(x)$

$$\boxed{H(x) \text{ is increasing on } [0, 6]}$$

e) Find $H''(x)$.

$$\boxed{H''(x) = f'(x)}$$

f) When is $H(x)$ concave up? Justify your answer.

$H(x)$ is concave up when $H''(x) > 0$. Since $H''(x) = f'(x)$,

$$\boxed{H \text{ is concave up on } (9, 12)}$$

g) At what x -value does $H(x)$ achieve its maximum value? Justify your answer.

$$H(x) = 0 \text{ when } f(x) = 0 \therefore \text{this occurs at } x = 6 \neq x = 12$$

$H'(x)$ is never undefined

use a candidates test

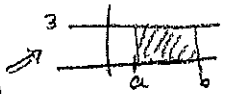
$$H(6) > H(12) \text{ because } \int_6^{12} f(t) dt < 0$$

\therefore the maximum value of $H(x)$ is achieved when $x = 6$

x	0	6	12
$H(x)$	0	8	0

11. [No Calculator] If $\int_a^b f(x) dx = a + 2b$, then $\int_a^b [f(x) + 3] dx =$

$$\int_a^b f(x) dx + \int_a^b 3 dx$$



A $a + 2b + 3$

B $3b - 3a$

C $4a - b$

D $5b - 2a$

E $5b - 3a$

$$a + 2b + 3(b - a)$$

given

$$= a + 2b + 3b - 3a$$

$$= -2a + 5b$$

12. [No Calculator] Let $f(x) = \int_{-2}^{x^2-3x} e^t dt$. At what value of x is $f(x)$ a minimum?

A none

B 0.5

C 1.5

D 2

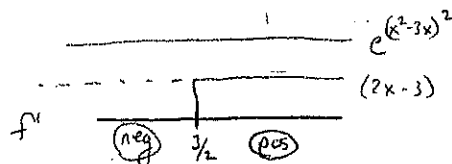
E 3

$$f'(x) = e^{(x^2-3x)^2} \cdot (2x-3)$$

$$f'(x) = 0 \text{ when } 2x-3=0 \text{ since } e^{(x^2-3x)^2} \neq 0$$

$$\text{so, } f'(x) = 0 \text{ when } x = 3/2.$$

since $e^{(x^2-3x)^2} > 0$ for all values of x , the sign of f' depends solely on the $(2x-3)$ fac



since $f' < 0$ on $(-\infty, 3/2)$
and $f' > 0$ on $(3/2, \infty)$
 f has a minimum at $x = 3/2$.

13. [Calculator] If $f(x) = \int_a^x \ln(2 + \sin t) dt$, and $f(3) = 4$, what does $f(5) = ?$

$$f(3) = \int_a^3 \ln(2 + \sin t) dt = 4$$

$$f(5) = \int_a^5 \ln(2 + \sin t) dt$$

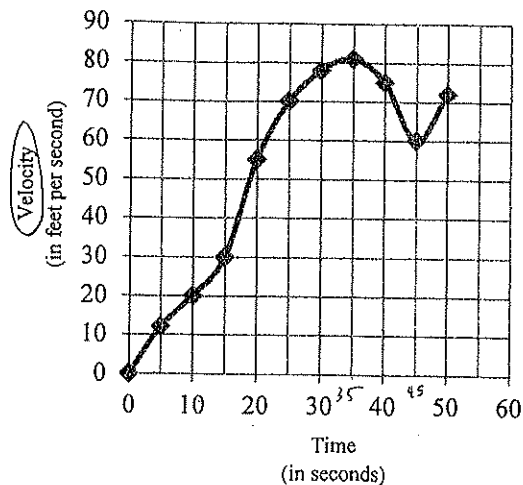
$$\underbrace{\int_a^3 \ln(2 + \sin t) dt}_{\text{given}} + \underbrace{\int_3^5 \ln(2 + \sin t) dt}_{\text{calculate}} = \underbrace{\int_a^5 \ln(2 + \sin t) dt}$$

} BTW... this holds true regardless of where a is!

$$4 + .5550851972 = f(5)$$

4.5550851972 = f(5)

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

$$a(t) = v'(t) \quad \text{so when } v(t) \text{ is increasing, } a(t) > 0$$

$$a(t) > 0 \text{ on } (0, 35) \cup (45, 50)$$

b) Find the average acceleration of the car, in ft/sec^2 , over the interval $0 \leq t \leq 50$.

average (Rate of change in velocity)

$$\frac{v(50) - v(0)}{50 - 0} = \frac{72 - 0}{50} = 14.4 \text{ ft/sec}^2$$

c) Approximate $\int_0^{50} v(t) dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

$$\int_0^{50} v(t) dt \approx \boxed{10(12) + 10(30) + 10(70) + 10(81) + 10(60)}$$

$$= 120 + 300 + 700 + 810 + 600$$

$$= 2530$$

This car has traveled 2530 feet from time = 0 to time = 50 second