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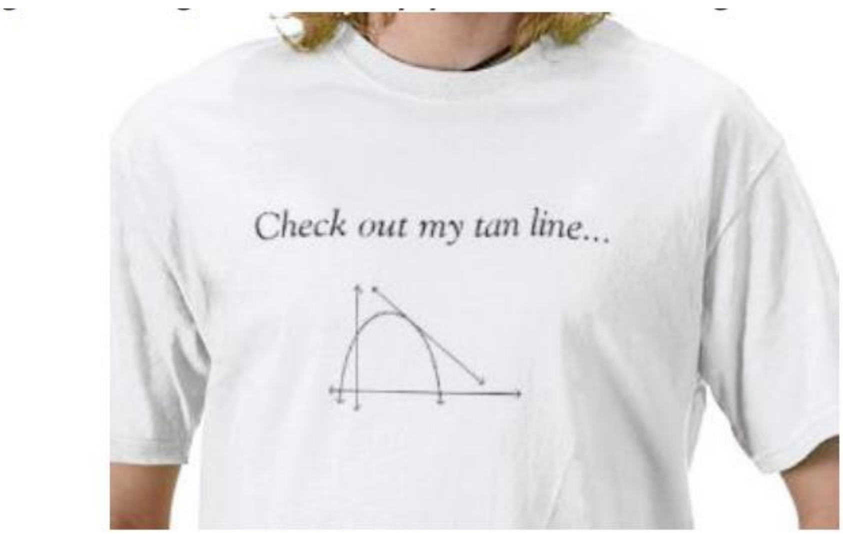
Lesson 1 tangent line approximation

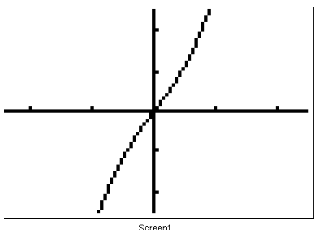
http://www.chaoticgolf.com/vodcasts/calc/lesson4_5b/lesson4_5b.html

watch Mr Leckie –linearization   53 second to 6:40



CW- next page-HW –WS with 3AP problems

1. Write the equation of the tangent line of the function
   $f(x) = x^3+2x$ at x=1 -use it to estimate f(.9)



2. Write the equation of the tangent line of the function
   $f(x) = \sqrt{x}$ at x=16

   Use that tangent line to approximate the value of f(16.5)

tangent line approximation –Class work

1

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

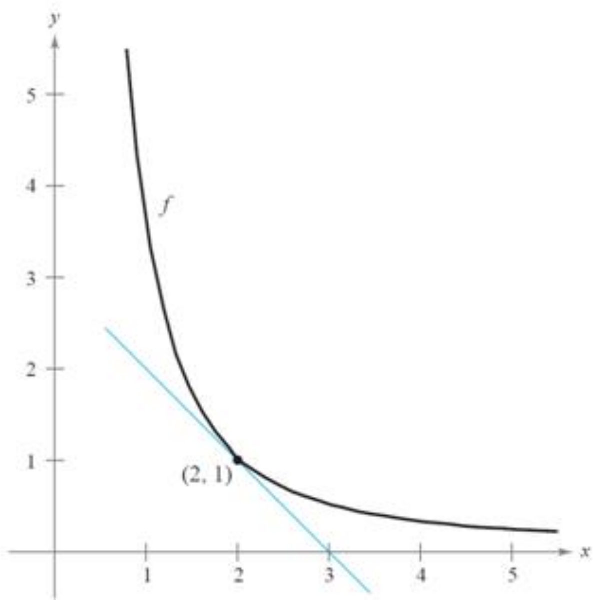
using the tangent line of f(1) estimate the value of f(1.2)

2. Find the equation of the tangent line to y=sinx at the origin, and use it to find an estimation of sin 0.12

3. Let f be a differentiable function. Estimate f(2.1) given that f(2)=1 and f'(2) =3

4.

Use differentials and the graph of f to approximate (a) $f(1.9)$ and (b) $f(2.04)$.



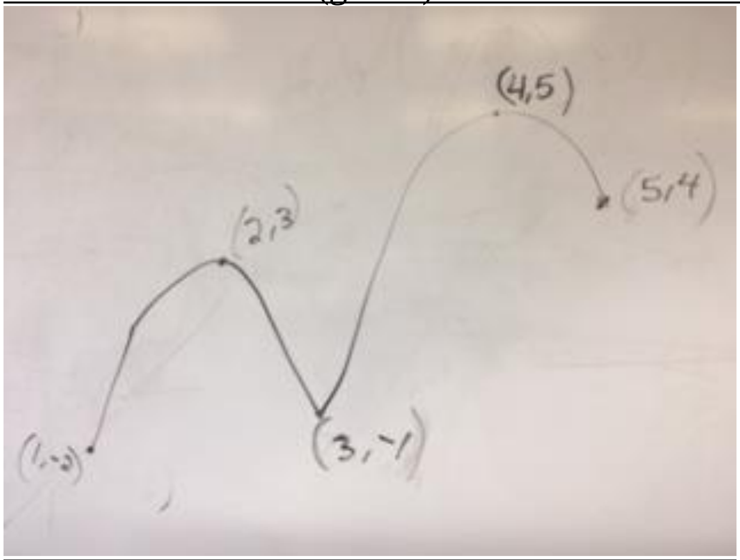
Lesson 2 EVT- HW pp. 198 # 1-18 odd – p.219 1-5 odd
Sec. 5.1 – Extreme Values of Functions

No calculator

The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

What is the absolute (global) maximum? What is the absolute minimum



Extreme value theorem -If f is continuous on a closed interval [a,b] then f has both a minimum and a maximum on the interval

Definition of Absolute Extrema … The BIGGEST y and smallest y in the interval
Let f be defined on a closed interval I containing c.

1. *f(c)* is the minimum of *f* on the interval I if $f(c) \leq f(x)$ for all x in I.
2. *f(c)* is the maximum of *f* on the interval I if $f(c) \geq f(x)$ for all x in I.

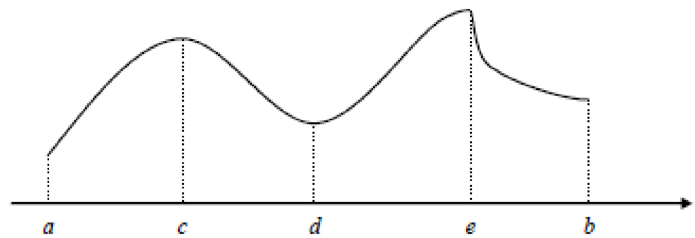
*Absolute Extrema occur at either endpoints of a closed interval or critical values of *f*.

Definition of Relative Extrema

1. If there is an open interval containing c on which *f(c)* is a maximum, then *f(c)* is called a **relative or local maximum**
2. If there is an open interval containing c on which *f(c)* is a minimum, then *f(c)* is called a **relative or local minimum**

Basically relative extrema exist when the value of the function is larger or smaller than all other function values relatively close to that value.

1. Suppose you were given the function below. Label $f(a)$ through $f(e)$ as absolute or relative extrema.



<u>Definition of a Critical Point (Critical number or critical value)</u>
Let *f* be defined at c. If *f'(c)* = 0 <u>or</u> *f'(c)* is undefined (DNE), then c is a critical number or point of f.

<u>Relative Extrema Occur Only at Critical Point</u>
If *f* has a relative minimum or relative maximum at x= c, then c is a critical number of *f*.

However, the converse is not necessarily true. Just because the derivative is equal to zero (or undefined) does not mean there is a relative max or min there. It is possible for the derivative to equal zero (or undefined) and there be NO extrema there.
(Ex. $f(x) = x^3$ at x= 0)

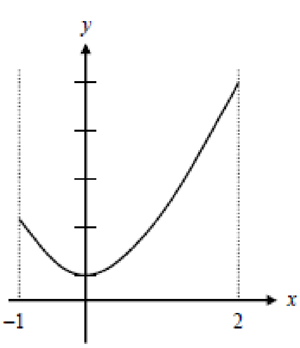
<u>*Guidelines for finding Extrema (absolute max and min) on a closed interval</u>
To find the extrema of a continuous function f on a closed interval [a,b] use the following steps

1. Find the critical numbers of f in (a,b). Do this by____________________________
2. Evaluate f at each critical number in (a,b)
3. Evaluate f at each endpoint of [a,b]
4. The least of these <u>y-values</u> is the absolute minimum, the greatest is the absolute max

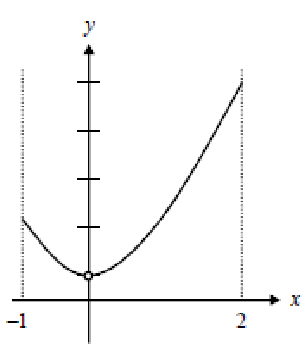
The critical numbers (or critical points) are _____ values, while the maximums/minimums of the function are __________ values.
Say you find that (2, 70) is the maximum of your graph, then you would write, the minimum of the function is 70, and it occurs at x = 2.

2. Using the graphs provided, find the minimum and maximum values on the given interval. *If there is no maximum or minimum value*, explain which part of the hypothesis of the Extreme Value Theorem is not satisfied.

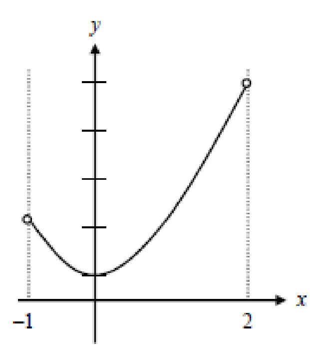
(a) [–1, 2]



(b) [–1, 2]



(c) (–1, 2)



1. Find the absolute max and absolute minimum of $f(x)=\sin x$ on the interval $[0,2\pi]$

2. Find the extrema of $f(x) = -x^2+3x$ on the interval [0,3]

3. Find the extrema of $f(x) = \frac{x^2}{x^2-3}$ on the interval [-1,1]

Lesson 3

Find where a function is increasing and decreasing-Find the critical values and identify if it is a relative (local) max or min- justify
HW- (old book, next page)-WS p. 181 -#11-31 odd 43-47 odd

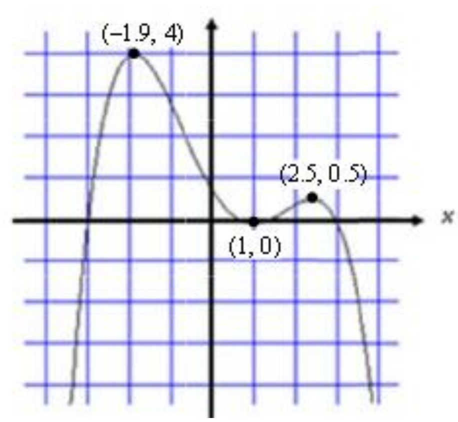
1. Find the critical value(s)

   $f(x) = \ln x^2$ $g(x) = \sqrt{4-x^2}$

2. Determine the absolute maximum and the absolute minimum for $y=8x-x^2$ over the interval [0,6]

How to justify (in the least number of words):
Absolute max or min, you need the chart to show that you checked the endpoints and critical values- then state the abs max is ___(y value)   absolute min is ____(y-value).
relative min- x=3 is a relative min because f'(x) changed from negative to positive
relative max- x=2 is a relative max because f'(x) changed from positive to negative
f(x) is increasing (0,2) because f'(x) is positive
f(x) is decreasing (1,3) because f'(x) is negative

EX 1   *Graphically   answer in interval notation and justify*   ***h(x)***



Where is the function increasing?

Where is the function decreasing?

Find the x value of the relative minimum

Find the x values of the relative maximum

EX 2 From an equation - express in interval notation and **justify**

$\mathbf{f(x)} = \frac{2}{3}x^3 - 2x^2 - 30x + 5$

Find the intervals where f(x) is increasing? Justify

Find the intervals where f(x) is decreasing? Justify

Find relative min and max and justify

EX 3 *$F(x) = x^3 - 3x^2 + 4$*

Find the intervals where f(x) is increasing and justify

Find the intervals where f(x) is decreasing and justify

Find relative min and max and justify

In Exercises 11–32, find the critical numbers of f (if any). Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. Use a graphing utility to confirm your results.

11. $f(x) = x^2 - 6x$

12. $f(x) = x^2 + 8x + 10$

13. $f(x) = -2x^2 + 4x + 3$

14. $f(x) = -(x^2 + 8x + 12)$

15. $f(x) = 2x^3 + 3x^2 - 12x$

16. $f(x) = x^3 - 6x^2 + 15$

17. $f(x) = x^2(3 - x)$

18. $f(x) = (x + 2)^2(x - 1)$

19. $f(x) = \frac{x^5 - 5x}{5}$

20. $f(x) = x^4 - 32x + 4$

21. $f(x) = x^{1/3} + 1$

22. $f(x) = x^{2/3} - 4$

23. $f(x) = (x - 1)^{2/3}$

24. $f(x) = (x - 1)^{1/3}$

25. $f(x) = 5 - |x - 5|$

26. $f(x) = |x + 3| - 1$

27. $f(x) = x + \frac{1}{x}$

28. $f(x) = \frac{x}{x + 1}$

29. $f(x) = \frac{x^2}{x^2 - 9}$

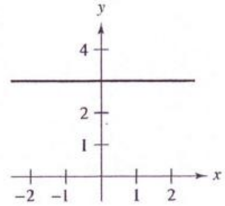
30. $f(x) = \frac{x + 3}{x^2}$

31. $f(x) = \frac{x^2 - 2x + 1}{x + 1}$

32. $f(x) = \frac{x^2 - 3x - 4}{x - 2}$

Think About It **In Exercises 43–48, the graph of f is shown in the figure. Sketch a graph of the derivative of f. To print an enlarged copy of the graph, go to the website** ***www.mathgraphs.com.***

43.



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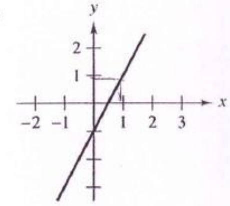

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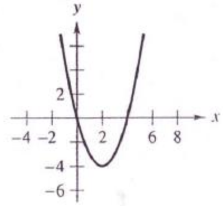

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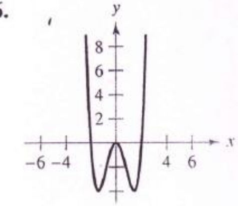

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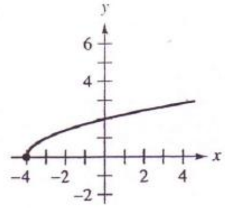

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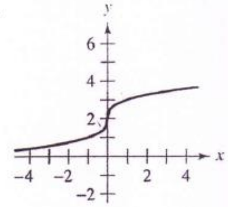


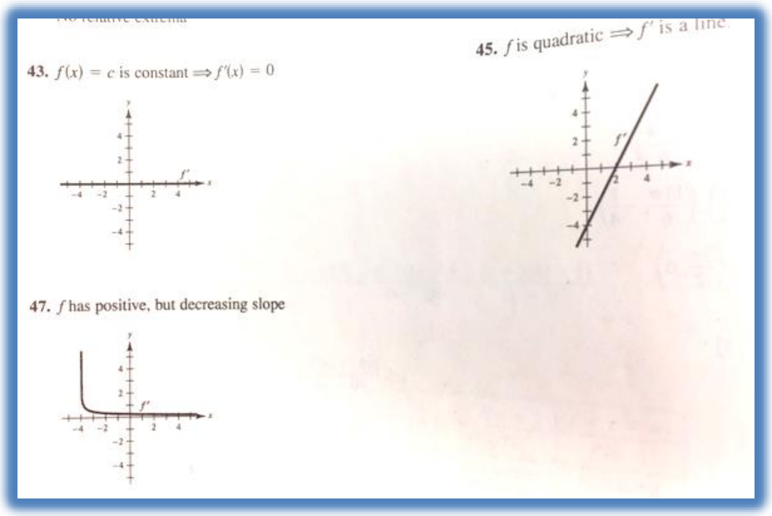
Lesson 4

go over all homework and ch 4 test–, if time #28 from last nights homework
HW-Additional problems worksheet-

$$f(x) = -3x^5 + 5x^3$$

Find the x-values of the relative min and max and justify

Find where increasing and decreasing and justify



WARNING!!!!
$f\,'(c) = 0$ or undefined will give us the *possible* extrema of f. Do not assume that every critical value is an extreme value.

ANOTHER WARNING
You must use calculus to justify an extreme value. You may NOT just draw a graph and label extrema.

Example: Graph $y = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

NOTE: On the AP Exam, it is **not** sufficient to simply draw the chart and write the answer. You must give a *written explanation!*

$y' = 3x^2 - 6x$

First derivative test:

y'	+	0	−	0	+
		0		2	

There is a local maximum at $(0,4)$ *because* $y' > 0$ *for all* x *in* $(-\infty, 0)$ *and* $y' < 0$ *for all* x *in* $(0,2)$.

There is a local minimum at $(2,0)$ *because* $y' < 0$ *for all* x *in* $(0,2)$ *and* $y' > 0$ *for all* x *in* $(2,\infty)$.

Additional Problems WS- show work (Do calculus) and justify when asked

1. Find all critical values of the function $f(x) = 2xe^{2x}$.

2. Find all critical values of the function $f(x) = \ln(x^2 + 4)$.

3. Given $f'(x) = \dfrac{(x+1)x^2}{(x-1)^{1/3}}$, and the function *f(x)* is continuous

   a) On which interval(s) is the function *f(x)* increasing? Justify
   b) On which interval(s) is *f(x)* decreasing? Justify
   c) At what x-value(s) is there a relative minimum? Justify
   d) At what x-value(s) is there a relative maximum? Justify

4. Find the absolute maximum and absolute minimum of $f(x) = e^{-x}\sin x$ on the interval $[0,2\pi]$. Justify

5. $f(x) = \sin^2 x$ on the interval $[0,2\pi]$.
   Determine the interval(s) on which f(x) is increasing and the interval(s) on which it is decreasing. Justify

6. $f(x) = \sin x + \cos x$
   Find all critical numbers on the interval $[0,2\pi]$. Then determine if each critical number is a relative minimum, relative maximum, or neither. Justify

Lesson 5

5.3 Concavity and inflection points

HW pp. 219 # 7-13, 21-29, odd,

Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

(a) On what intervals is f increasing?

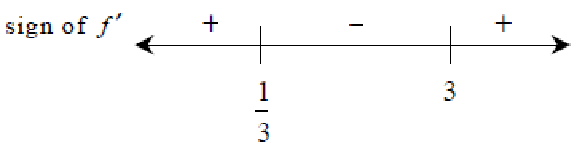
(b) On what intervals is the graph of f concave downward?

(c) Find the value of k for which f has 11 as its relative minimum.

(a) $f'(x) = 3x^2 - 10x + 3$

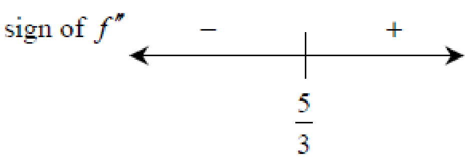
$= (3x-1)(x-3)$

$f'(x) = 0$ at $x = \frac{1}{3}$ and $x = 3$



f is increasing on $\left(-\infty, \frac{1}{3}\right]$ and on $[3, \infty)$

(b) $f''(x) = 6x - 10$



The graph is concave down on $\left(-\infty, \frac{5}{3}\right)$

(c) From (*a*), f has its relative minimum at $x = 3$, so

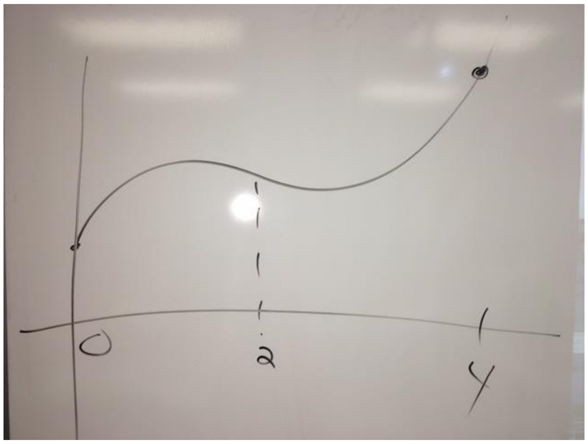
$f(3) = (3)^3 - 5(3)^2 + 3(3) + k$

$= -9 + k = 11$

$k = 20$

f'' positive $\Rightarrow$ Concave UP

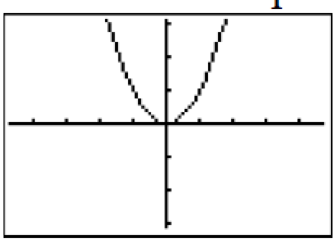
f'' negative $\Rightarrow$ Concave DOWN



The second derivative is the change in slope. If the function is concave up- the slope is increasing- it is going from neg to 0 to positive. Concave down- the slope is decreasing- it is going from positive to negative

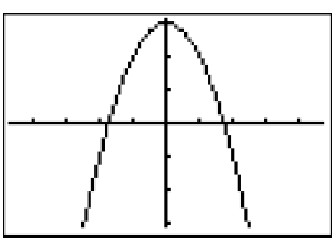
Using the Second Derivative

Let f be differentiable on an open interval I. The graph of f is concave upward on I if $f''(x) > 0$ for all x in I.



Concave up – like a cup

If $f''(x) < 0$, then the graph of f is concave down.



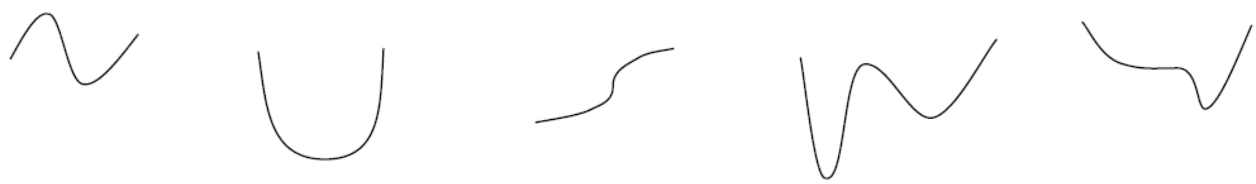
Concave down – like a frown

If the graph of f changes from concave up to concave down or vice-versa, at $x = c$, then the point $(c,\ f(c))$ is called a point of inflection.

Points of Inflection

> *Definition*
>
> A point where the graph of a function has a tangent line (even if it's a vertical tangent line) AND where the concavity changes is a **point of inflection**.

Example: Using each picture, estimate each point of inflection, if any, and sketch the tangent line at that point.



> Since points of inflection occur when the graph changes _______________, and a graph changes concavity when the __________________________________ changes from positive to negative (or vice-versa), then if we wanted to find the points of inflection of a graph, we only need to focus on when the second derivative equals 0 (or does not exist)
>
> ***IMPORTANT*** ♬: Just because the second derivative equals zero (or does not exist) you are NOT guaranteed that the function has a point of inflection. The second derivative MUST change signs (meaning concavity changed) in order for a point of inflection to exist!

Points of inflection can occur where $f''(x) = 0$ or where $f''(x)$ is undefined. [This does not mean that they will occur, only that these are the only possible points of inflection.]
If given the graph of the first derivative, then you can find points of inflection if the graph of the first derivative

changes from either increasing to decreasing or decreasing to increasing at some value $x = c$.

A tangent line is below the graph if the graph is concave up at that value of x. Likewise, a *tangent line is above the graph* if the graph is concave down at that value of x.

Another great use of the second derivative is the "Second Derivative Test". It can be used to find extrema. ♪ It only works sometimes – not always but can be handy.

Let f be a function such that $f'(c) = 0$ and the second derivative exists on an open interval containing $x = c$. Remember, c is the critical value!!!

So you are evaluating the second derivative at the critical values.
(1) If $f''(c) > 0$, then f has a relative minimum value at the point $(c,\ f(c))$
(2) If $f''(c) < 0$, then f has a relative maximum value at the point $(c,\ f(c))$
(3) If $f''(c) = 0$, then the Second Derivative Test fails and you must resort to the First Derivative Test to find your extrema.

Increasing and positive are not interchangeable

Draw- where it changes from down to up- where it switched concavity- inflection pts

Ex 2- Find the inflection point(s) and determine the concavity of the function. Find the x-values of any relative min and max of the function

$F(x) = 2x^3 - 3x^2 - 12x + 5$

$F'(x) = 6x^2 - 6x - 12$
$F''(x) = 12x - 6$ set =0 inflection x=1/2

½, is an inflection pt because f" changes sign
The graph is concave up on the interval $(\frac{1}{2}, \infty)$ because f" is positive
The graph is concave down on the interval $(-\infty, \frac{1}{2})$ because f" is negative

Since $f''(x)$ changes from negative to positive values at $x = 0$, then the point $(0, 0)$ is a point of inflection for the graph of f.

("AP" speak)

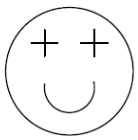
Concavity deals with how a graph is curved. A graph that is concave up looks like  , while a graph that is concave down looks like  . We can use the SECOND derivative to determine the concavity of a function.

> *Definition of Concavity*
>
> Let $y = f(x)$ be a differentiable function on an interval *I*. The graph of $f(x)$ is **concave up** on *I* if f' is increasing on *I*, and **concave down** on *I* if f' is decreasing on *I*.

If the first derivative is increasing, then the second derivative must be __________________. If the first derivative is decreasing, then the second derivative must be __________________. Thus instead of using the definition of concavity to determine whether the function is concave up or down, we can use the following test.

> *Concavity TEST*
>
> The graph of a twice-differentiable function $y = f(x)$ is **concave up** on any interval where $y'' > 0$, and **concave down** on any interval where $y'' < 0$.

The Concavity Test can be summed up by the following pictures … While this is a humorous (and hopefully helpful) way to remember concavity, please understand that this is NEVER to be used as a justification on ANY test!



f'' positive $\Rightarrow$ Concave UP

f'' negative $\Rightarrow$ Concave DOWN

Lesson 6

Concavity –relative min and max from 2nd derivative test
CW/HW- AP problems 1-6 (don't use 5.3 worksheet- algebra too hard)

Fill in the blanks

1 F'(x) >0 then f(x) is ________________

2 F'(x) <0 then f(x) is _________________

3 F'(c) =0 or undefined then c is a ___________________

4 If F'(x) changes from + to – at C then c is a _______________

5 If F'(x) changes from -to +at C then c is a ____________________

6. There is a critical value at x=2, and the function is concave up on the interval (-∞,4) therefore your critical value is a ________________

Example

F(x)= $2x^3-3x^2-12x+5$

Find the critical values-

Use the second derivative to decide if your critical value is a relative max or min

Second Derivative Test for Local Extrema

If $f'(c)=0$ (which makes $x=c$ a critical point) AND $f''(c)<0$, then f has a local MAXIMUM at $x=c$.
If $f'(c)=0$ (which makes $x=c$ a critical point) AND $f''(c)>0$, then f has a local MINIMUM at $x=c$.

Important ♬: If the second derivative is equal to zero (or undefined) then the Second Derivative Test is INCONCLUSIVE.

Remember the happy (and sad) faces? If a critical point happens to occur in an interval where the graph of the function is CONCAVE UP, then that critical point is a relative MINIMUM. If a critical point happens to occur in an interval where the graph of the function is CONCAVE DOWN, then that critical point is a relative MAXIMUM.

Fill in the blanks

1 F'(x) >0 then f(x) is increasing

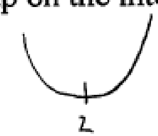
2 F'(x) <0 then f(x) is decreasing

3 F'(c) =0 or undefined then c is a critical value

4 If F'(x) changes from + to – at C then c is a relative max

5 If F'(x) changes from -to +at C then c is a relative min

6. There is a critical value at x=2, and the function is concave up on the interval (-∞,4) therefore your critical value is a relative min



Example

F(x)= $2x^3-3x^2-12x+5$

Find the critical values-

$F'(x)= 6x^2-6x-12$

$6(x^2-x-2)$

$(x-2)(x+1)$

$x=2$

$x=-1$

Use the second derivative to decide if your critical value is a relative max or min

$F''(x)= 12x-6$

$0=12x-6$

$6=12x$

$1/2=x$

– | + at $\frac{1}{2}$

$(-\infty, \frac{1}{2})$ concave down −1 relative max

$(\frac{1}{2}, \infty)$ concave up 2 relative min

Lesson 7

MVT and Rolles

p. 206 1-7,(skip 5)11,15,17

Secants Versus Tangents 2011ch2daytwo.doc

Mean Value Theorem [another existence theorem]

If f is continuous on the closed interval $[a, b]$ AND differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

What's this mean?

If we draw a secant between the endpoints of the closed interval, then somewhere in the interior of the interval the slope of the secant will equal the slope of a tangent line. Or, at some interior point the average rate of change equals the instantaneous rate of change.

Stress continuous and differentiable

Slope of the secant line (endpoints) = to the slope of the tangent line at f(c)

Notice that the function must be continuous AND differentiable.

My MVT JustificationTemplate:

By the Mean Value Theorem, there is a $c, a < c < b$, such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Since $\frac{f(b) - f(a)}{b - a} =$ ________, then there is a $c, a < c < b$, such that $f'(c) =$ _____.

get on the parkway at 12 noon, at exit 163, I arrive down at Seaside Heights exit 83 at 1:30, what was my average speed?

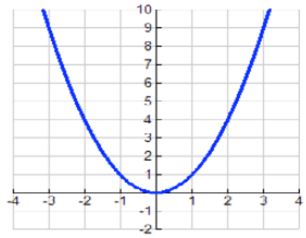
Is this a secant line or a tangent line?

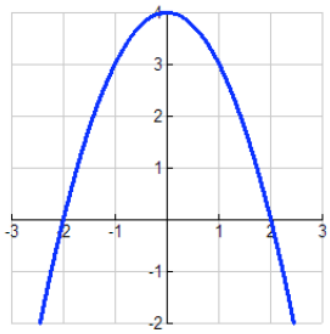
At what time was I driving at that speed? (what line is this?)

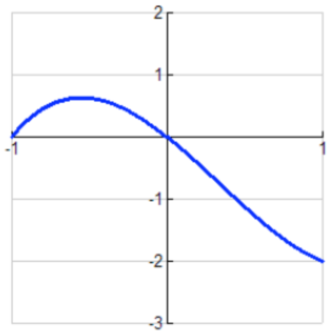
Demonstrate graphically using

http://math.dartmouth.edu/~klbooksite/2.10/210.html start with applet- show visually- -give formal definition - do an example

Handout for Mean Value Theorem







At what value(s) of x does $f(x) = x^3 - x^2 - 2x$ *satisfy the Mean Value Theorem on the interval* $[-1, 1]$

Always start by writing out the MVT statement!!!

Notice that the function must be continuous AND differentiable.

My MVT JustificationTemplate:

By the Mean Value Theorem, there is a $c, a < c < b,$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Since $\frac{f(b) - f(a)}{b - a} =$ ________, then there is a $c, a < c < b,$ such that $f'(c) =$____.

Have students draw a graph that begins at (1,3) and ends at (5,3)
Put five examles on board.,, make sure one of them is not differentiable

Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ AND differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$. [Graphically speaking, there is a horizontal tangent somewhere in the interior of the interval.]

Example 1

Let $f(x) = x^4 - 2x^2$

Find all values of c in the interval [-2,2] such that f'(c) =0

First ask if it satisfies the condition of rolles theorem-Differentiable and continuous- and f(a)=f(b)

Since f(-2) = 8 and f(2) =8 then f(-2)=f(2) you can conclude that there exists at least one c in (-2,2) such that f'(c) = 0

<u>Example 2</u>

Find the 2 x-intercepts of $f(x) = -x^2+7x-10$ and show that $f'(x) = 0$ at some point between the intercepts

<u>Example 3</u>

Find x values where the tangent line is parallel to the secant line on the closed interval [-1,1]

Lesson 8

Go over HW-
Worksheet with 7 MVT and IVT and Rolles-problems-
(answers on website) HW- complete

$f(x) = x^3-3x+3$ find and justify :

1. Find the absolute max on the interval [0,2]

2. the x-value of the relative extrema- (max and min)

3. the (s) of inflection-

4. interval(s) where the function is increasing-

5. interval(s) where the function is decreasing-

6. interval(s) where the function is concave up

Absolute max is 5, justify by showing that you checked EP and C.V f(0)=3 f(2)=8 f(1)=1
relative min at x=1 because the f'(x) changed from neg to pos
relative max at x=-1because the f'(x) changed from pos to neg
inflection point (0,3)because f''(x) changed from negative to positive
increasing (-∞,-1) and (1,∞) because f'(x) is positive
decreasing (-1,1) because f'(x) is negative
concave up (0, ∞) because f''(x) is positive

<u>Review the conditions of rolles thm.-</u>

Differentiable and continuous and f(a) = f(b) that means the y's are the same – then apply the theorem which states that there must be a place (c) where the derivative=0

Review the conditions of Mean Value Theorem

Continuous and differentiable

There exists a place (c) where the secant line (average slope-end to end) = tangent line (instantaneous slope)

Notice that the function must be continuous AND differentiable.

My MVT JustificationTemplate:

By the Mean Value Theorem, there is a $c, a < c < b$, such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Since $\frac{f(b) - f(a)}{b - a} =$ ________, then there is a $c, a < c < b$, such that $f'(c) =$____.

By the IVT, there must exist some c, with $1 < c < 2$, such that $f(1) < f(c) < f(2)$. Which means that there must exist a c, $1 < c < 2$, such that $-1 < f(c) < 5$. Hence, there exists at least one root on [1, 2].

Lesson 9-review-

Do now a worksheet-known as review 1

HW- Review 2- true and false below as pair

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?

2. Describe the concavity of the function-find the inflection point
$F(x) = 3x^4$

Worksheet 2: Classifying Critical Points, Part II

Determine if each of the following statements is true or false. If you decide a statement is false, provide a counterexample to show why it is false and then rewrite the statement in order to make it true. Unless otherwise specified, assume each function is defined and continuous for all real numbers.

1. A critical point (or critical number) of a function f of a variable x is the x-coordinate of a relative maximum or minimum value of the function.
2. A continuous function on a closed interval can have only one maximum value.
3. If $f''(x)$ is always positive, then the function f must have a relative minimum value.
4. If a function f has a local minimum value at $x = c$, then $f'(c) = 0$.
5. If $f'(2) = 0$ and $f''(2) < 0$, then $x = 2$ locates a relative maximum value of f.
6. If $f''(2) < 0$, then $x = c$ is a point of inflection for the function f and cannot be the x-coordinate of a maximum or minimum point on the graph of f.
7. If a function f is defined on a closed interval and $f'(x) > 0$ for all x in the interval, then the absolute maximum value of the function will occur at the right endpoint of the interval.
8. The absolute minimum value of a continuous function on a closed interval can occur at only one point.
9. If $x = 2$ is the only critical point of a function f and $f''(2) > 0$, then $f(2)$ is the minimum value of the function.
10. To locate the absolute extrema of a continuous function on a closed interval, you need only compare the y-values of all critical points.
11. If $f'(c) = 0$ and $f'(x)$ decreases through $x = c$, then $x = c$ locates a local minimum value for the function.
12. Absolute extrema of a continuous function on a closed interval can occur only at endpoints or critical points.

50 point test- Applications of Derivatives-

Tangent line approximation
Mean Value Theorem
Rolle's Theorem
Finding absolute Min and Max
Finding relative Min and Max
Finding where a function is increasing or decreasing
Critical values, Concavity, Inflection points

1 Find all critical values of the given function: $F(x)= \ln x^2$

2 Find the relative max, min and absolute max, min of
$F(x) = x^3-2x^2+3$ on the interval [-2,2]

3 Find the absolute max and min of the following
$F(x) = e^{-x} \sin x$ on the interval $[0,2\pi]$

4 $F(x) = e^x$ on [0,1] Use MVT-find all c between a and b for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

6. Use the tangent line approximation at x=1 to estimate f(1.1) for the following function $F(x) =2x^4+3x$

7. Let f be the function given by $f(x)=3x^4+x^3-21x^2$.

(a) Write an equation of the line tangent to the graph of f at the point $(2,-28)$.

(b) Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

(c) Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

Lesson 10

Review day-5.3 homework