AP Calculus AB Review Sheet for Chapter 3 test

1. A spring is bobbing up and down so that its position at any time $t \geq 0$ is given by $s(t) = -4\sin t$.

   a) What is the initial position of the spring?

   b) Which way is the particle moving to start? Justify your response.

   c) At $t = \frac{5\pi}{4}$, is the spring moving up or down? Justify your response.

   d) Is the spring speeding up or slowing down at $t = \frac{5\pi}{4}$? Justify your response.

2. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

3. If $x(t) = t^2 - 8t + 12$ is a position of a particle moving along the x axis at time t, then

   a) Find the average velocity for the first 3 seconds.
   b) Find the velocity at $t = 4$ seconds.
   c) When is the object stopped?
   d) When is the acceleration of the object 0?
   e) When does the object change direction?
   f) When does the object slow down?
   g) When is the object moving left?

4. $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$

   A) 0 B) 1 C) $\sin x$ D) $\cos x$ E) nonexistent

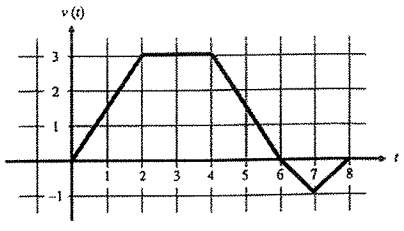
5. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

   A) $y = 2x + 1$ B) $y = x + 1$ C) $y = x$ D) $y = x - 1$ E) $y = 0$

6. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

   A) $\sec x \csc x$ B) $\sec x - \csc x$ C) $\sec x + \csc x$ D) $\sec^2 x - \csc^2 x$ E) $\sec^2 x + \csc^2 x$

7. A bug begins to crawl up a vertical wire and it's velocity at time t is given in the graph below.
   Find:
   a) When does the particle change direction?
   b) When is the particle moving down?
   c) Is the particle speeding up or slowing down at t = 6.5?



8. [Calculator] A particle moves along a line so that at time t, $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

A) –5.19 B) 0.74 C) 1.32 D) 2.55 E) 8.13

9. If $f(x) = \frac{x}{\tan x}$, then find $f'(2)$.

10. [**Calculator**]. Find the equation of the tangent line to the graph of $f(x) = (4-\sin x)^2$ at $x = 0$.

11. Find $\lim_{x \to \frac{\pi}{4}} \frac{\sec x - \sec(\frac{\pi}{4})}{x - \frac{\pi}{4}}$. This is a non-calculator question!! THINK!

12. Find y' if $y = 4e^x + 10\ln x - \csc x + \sqrt[3]{x^2} + \frac{8}{x^2} + 4x - 100$

13. Given the data in the table find the following: (remember: v(t)=s'(t))

t	1	2	3	4	5	6	7
S(t)	45	56	78	90	35	30	25

a) v(3) b) v(4.5) c) Find the average velocity on the interval [0,5].

14.

x	f(x)	g(x)	f'(x)	g'(x)
-1	0	-1	2	1

Given the functions f and g and their derivatives at x = -1 find h'(-1).

a) $h(x) = \frac{f(x)}{g(x)}$ b) $h(x) = f(x)g(x)$ c) $h(x) = 3f(x) - 4g(x)$

Also do in the book: page 149; 61

KEY

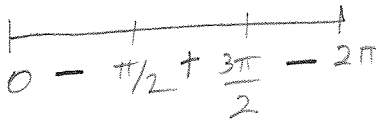
AP Calculus AB Review Sheet for Chapter 3 test

1. A spring is bobbing up and down so that its position at any time $t \geq 0$ is given by $s(t) = -4\sin t$.

a) What is the initial position of the spring? $s(0) = 0$

b) Which way is the particle moving to start? Justify your response. $v(t) = -4\cos t$

Down since $v(t) < 0$



c) At $t = \frac{5\pi}{4}$, is the spring moving up or down? Justify your response.

up since $v(5\pi/4) > 0$

d) Is the spring speeding up or slowing down at $t = \frac{5\pi}{4}$? Justify your response. $a(t) = 4\sin t$

$v(5\pi/4) = +$ $a(5\pi/4) = -$ slowing down since the are different signs

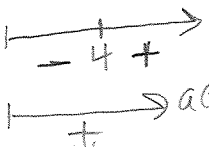
2. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx} = \sec x \tan x$ $\frac{d^2y}{dx^2} = \sec x \tan^2 x + \sec^3 x$

3. If $x(t) = t^2 - 8t + 12$ is a position of a particle moving along the x axis at time t, then

$v(t) = 2t - 8$ $a(t) = 2$

a) Find the average velocity for the first 3 seconds. -5 m/s
b) Find the velocity at $t = 4$ seconds. 0 m/s
c) When is the object stopped? $t = 4$ sec
d) When is the acceleration of the object 0? Never
e) When does the object change direction? $t = 4$
f) When does the object slow down? $0 < t < 4$ $v(t)$ & $a(t)$ are diff sign
g) When is the object moving left? $0 < t < 4$ since $v(t) < 0$



4. $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$ asking for the deriv of $\sin x$ which is $\cos x$

A) 0 B) 1 C) $\sin x$ D) $\cos x$ E) nonexistent

5. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is $y' = 1 - \sin x$ $m_T = 1 - \sin(0)$

A) $y = 2x + 1$ B) $y = x + 1$ C) $y = x$ D) $y = x - 1$ E) $y = 0$

$y - 1 = 1(x - 0)$
$y = x + 1$

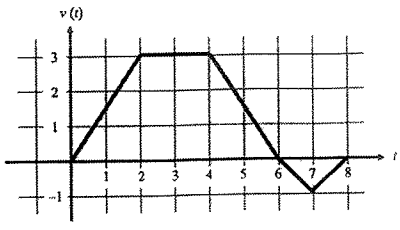
6. If $y = \tan x - \cot x$, then $\frac{dy}{dx} = \sec^2 x + \csc^2 x$

A) $\sec x \csc x$ B) $\sec x - \csc x$ C) $\sec x + \csc x$ D) $\sec^2 x - \csc^2 x$ E) $\sec^2 x + \csc^2 x$

7. A bug begins to crawl up a vertical wire and it's velocity at time t is given in the graph below.
Find:

a) When does the particle change direction? $t = 6$
b) When is the particle moving down? $6 < t < 8$
c) Is the particle speeding up or slowing down at t = 6.5?

speeding up $v(t)$ & $a(t)$ have the same sign!



$s(t) = -4\cos t - \frac{1}{2}t^2 + 10$

$v(t) = 4\sin t - t$

$a(t) = 4\cos t - 1$

8. [Calculator] A particle moves along a line so that at time t, $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

A) –5.19 B) 0.74 C) 1.32 D) 2.55 E) 8.13

$4\cos t - 1 =$ USE CALC

$t \approx 1.31$

$v(1.318) = 2.554$

9. If $f(x) = \frac{x}{\tan x}$, then find $f'(2)$.

CALC!

$\frac{\tan x(1) - x\sec^2 x}{\tan^2 x} = \frac{\tan x - x\sec^2 x}{\tan^2 x} \approx -2.876$

10. [**Calculator**]. Find the equation of the tangent line to the graph of f(x) = (4-sinx)² at x = 0.

USE CALC TO FIND $f'(0)$!! MATH #8

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11. Find $\lim_{x \to \frac{\pi}{4}} \frac{\sec x - \sec(\frac{\pi}{4})}{x - \frac{\pi}{4}}$. This is a non-calculator question!! THINK!

(★) They want the derivative of $y = \sec x$ at $x = \pi/4$!

$y' = \sec x \tan x$

$y'(\pi/4) = \sec \pi/4 \tan \pi/4 = \sqrt{2}$

12. Find y' if $y = 4e^x + 10\ln x - \csc x + \sqrt[3]{x^2} + \frac{8}{x^2} + 4x - 100$

$y' = 4e^x + \frac{10}{x} + \csc x \cot x + \frac{2}{3}x^{-1/3} - 16x^{-3} + 4$

13. Given the data in the table find the following: (remember: v(t)=s'(t))

t	1	2	3	4	5	6	7
S(t)	45	56	78	90	35	30	25

a) v(3) b) v(4.5) c) Find the average velocity on the interval [0,5].

$v(3) = s'(3) \approx \frac{90-56}{4-2} = 17$

$s'(4.5) \approx \frac{35-90}{5-4} = -55$

$\frac{s(5)-s(0)}{5-0} = \frac{35-45}{5} = -2$

14.

x	f(x)	g(x)	f'(x)	g'(x)
-1	0	-1	2	1

Given the functions f and g and their derivatives at x = -1 find h'(-1).

$h'(-1) = 3f'(-1) - 4g'(-1) = 2$

a) $h(x) = \frac{f(x)}{g(x)}$ b) $h(x) = f(x)g(x)$ c) $h(x) = 3f(x) - 4g(x)$

b) $h'(-1) = f'(-1)g(-1) + f(-1)g'(-1) = (2)(-1) + 0(1) = -2$

Also do in the book: page 149; 61

a) $h'(-1) = \frac{g(-1)\cdot f'(-1) - f(-1)\cdot g'(-1)}{g^2(-1)} = \frac{(-1)(2) - (0)}{(-1)^2} = -2$