Some people hate the word "average". As luck would have it, many people dislike the use of the word "average" in AP Calculus. Here is a look at the different concepts that use the word "average" for AP Calculus AB.

AVERAGE RATE OF CHANGE

If given a function or a table of values of a function and asked to find the average rate of change for that function on the closed interval [a, b], then

The average rate of change for f(x) on [a, b] is $\frac{f(b)-f(a)}{b-a}$

Example:

Find the average rate of change of the function f(x) on [0, 2] if

$f(x)=2x^2-2$

Average rate of change =

$\frac{f(2)-f(0)}{2-0}$

$= \frac{(8-2)-(0-2)}{2-0}$

$= 4$

Average rate of change = slope of secant between two given points

[But, instantaneous rate of change = slope of tangent at some particular point]

To find the average velocity of a moving object/particle you need the position function to be given to you.

Average velocity for the interval [a, b] will equal

$\frac{s(b)-s(a)}{b-a}$ where $s(t)$ is the position function

To find the average acceleration of a moving object/particle you need the velocity function to be given to you

Average acceleration for the interval [a, b] will equal

$$\frac{v(b)-v(a)}{b-a}$$

The **AVERAGE VALUE OF A FUNCTION** is ***NOT*** the same as average rate of change.

When you find an "average" of something, you sum up the given values,then divide by the number of values. For example, when finding the average of three quizzes, you would add up the scores and then divide by three.

To find the Average Value of a Function on [a, b] use the following;

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Example:

Find the average value of the function $f(x)=2x^2-2$ on the interval [0, 2]

$$\frac{1}{2-0}\int_0^2 \left(2x^2-2\right)dx=$$

$$\frac{1}{2}\left[\frac{2}{3}\right]\left(2^2\right)-2(2)=$$

$$\frac{1}{2}\left(\frac{4}{3}\right)=$$

$$\frac{2}{3}$$

To find the average velocity of a particle you need to be given the velocity function and the closed interval.

Average velocity for the interval [a, b] would be equal to

$$\frac{1}{b-a}\int_a^b v(t)dt$$

To find the average acceleration of a particle you need to be given the acceleration function and the closed interval.

Average Acceleration for the interval [a, b] would be equal to

$$\frac{1}{b-a}\int_a^b a(t)dt$$

One more thing to ponder:

We already know that $\int_a^b a(t)\,dt = \int_a^b v'(t)\,dt$ AND

$\int_a^b v'(t)\,dt = v(b) - v(a)$

So finding the average acceleration can be found using either

$\frac{v(b) - v(a)}{b - a}$ OR $\frac{1}{b-a}\int_a^b a(t)\,dt$ because

$\frac{1}{b-a}\int_a^b a(t)\,dt = \frac{1}{b-a}\int_a^b v'(t)\,dt$ which is the same as

$\frac{1}{b-a}[v(b) - v(a)]$ which is the same as $\frac{v(b) - v(a)}{b - a}$

Hence, you can find average acceleration using two different methods. Which one you chose is based on what function you are given. If not given a function but a table of values, then you will probably only have one choice. What method would you chose for this problem [2006AB4]

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (ft/second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity v(t) after being launched from an initial height of 0 feet at time t = 0 seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above. Find the average acceleration of Rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

You should look back at all of the various problems that we did over the year to be sure that you understand the difference between average rate of change and average value.